Program of the 9th edition of the conference

Functional Equations in LIMoges

FELIM 2016

March 29-31, 2016

Organised by

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Salle de Conférences XLIM 3

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FELIM 2016, Functional Equations in LIMoges

Tuesday, March 29

8:45-9:00 Welcome

Chair : F. Chyzak

9:00-9:50 **George Labahn**: *Rational Invariants of Finite Abelian Groups and their applications*

10:00-10:30 Coffee break, Discussions

10:30-10:55 **Alban Quadrat**: *An algebraic analysis approach to linear systems of differential time-varying delay equations*

11:00-11:25 **Islam Boussaada**: *Multiplicity and stable manifolds of Time-delay systems: A missing Link*

11:30-11:55 **Yacine Bouzidi**: *Computer algebra methods for the stability analysis of differential systems with commensurate time-delays*

12:00-14:30 Lunch break

Chair : G. Chèze

14:30-14:55 **Philippe Dumas**: *Fast computation of the Nth term of an algebraic series in positive characteristic*

15:00-15:25 **Tristan Vaccon**: *On p-adic differential equations with separation of variables*

15:30-16:00 Coffee break, Discussions

Chair : C. Mitschi

16:00-16:25 **Thierry Combot**: *Integrability of the one dimensional Schroedinger equation*

16:30-16:55 **Thomas Dreyfus**: *Towards an algorithm to compute the differential Galois group*

17:00-17:25 **Thomas Cluzeau**: *Computing the Lie algebra of the differential Galois group of a linear differential system*

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Wednesday, March 30

Chair : M. Barakat

9:00-9:50 **Vladimir Bavula**: *The algebras of polynomial integro-differential operators, their ideals and automorphisms*

10:00-10:30 Coffee break, Discussions

10:30-10:55 **Clemens Raab**: *A tensor approach to operator algebras*

11:00-11:25 **Jamal Hossein Poor**: *Tensor representation of the algebra of integro-differential operators with linear substitutions*

11:30-11:55 **Anton Panferov**: *Linear differential-algebraic systems with selected unknowns*

12:00-14:30 Lunch break

14:30-16:00 **Excursion**: visit of the Museum of Fine Arts

16:00-16:30 Coffee break, Discussions

Chair : D. Robertz

16:30-16:55 **Frédéric Chyzak**: *Computing solutions of linear Mahler equations*

17:00-17:25 **Louis Dumont**: *Efficient Algorithms for Mixed Creative Telescoping*

17:30-17:55 **Georg Grasegger**: *Rational General Solutions of First-Order Algebraic ODEs - Existence and Computation*

20:00- **Dinner**

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Thursday, March 31

Chair : A. Quadrat

9:00-9:50 **Mohamed Barakat**: *Programming abstract mathematics*

10:00-10:30 Coffee break, Discussions

10:30-10:55 **François Ollivier**: *Matrices extatiques et calcul de sorties linéarisantes rationnelles*

11:00-11:25 **Jordy Palafox**: *Isochronous center and correction of vector fields*

11:30-11:55 **Sergei Abramov**: *On the unimodularity testing for operator matrices*

12:00-14:30 Lunch break

Chair : S. Abramov

14:30-14:55 **Michel Petitot**: *Stochastic Petri nets and computer algebra*

15:00-15:25 **Sergey Paramonov**: *On some algorithmically undecidable problems connected with partial differential equations*

15:30-16:00 **Changgui Zhang**: *An analytic viewpoint on mock theta functions of Ramanujan*

16:00- Coffee break, Discussions

Abstracts of the talks

Contents

1 Rational invariants of finite abelian groups and their applications

George Labahn
University of Waterloo, Canada.

We give a constructive procedure for determining a generating set of rational invariants of the linear action of a finite abelian group in the non-modular case and investigate its use in the symmetry reductions of polynomial and dynamical systems. Finite abelian subgroups of $GL(n, K)$ can be diagonalized which allows the group action to be accurately described by an integer matrix of exponents. We make use of integer linear algebra to construct both a minimal generating set of invariants and the substitution to rewrite any invariant in terms of this generating set. The set of invariants provide a symmetry reduction scheme for polynomial systems whose solution set is invariant by a finite abelian group action.

This is joint work with E. Hubert (INRIA Méditerranée).

2 An algebraic analysis approach to linear systems of differential time-varying delay equations

Alban Quadrat
Inria, Lille-Nord Europe, France.

No algebraic approach seems to exist in the literature of control theory for the study of linear differential time-delay systems in the case of a (sufficiently regular) time-varying delay. Contrary to constant delays, the ring of differential time-varying delay operators is not an Ore algebra. Based on the concept of skew polynomial rings developed by Ore in the 30s, the purpose of this talk is to construct the ring of differential time-varying delay operators as an Ore extension and to analyze its properties. A characterization of classical algebraic properties of this ring, such as its noetherianity, homological and Krull dimensions and the existence of Gröbner bases, are given in terms of the time-varying delay function. In particular, we show that classical hypotheses on the time-varying delay function made in the literature of control theory can be mathematically explained. Finally, the algebraic analysis approach to linear systems theory allows us to study linear differential time-varying delay systems (existence of autonomous elements, controllability, parametrizability, flatness, behavioral approach, equivalences, decomposition, Serre's reduction, ...) through methods coming from module theory, homological algebra and constructive algebra.

This is joint work with R. Ushirobira (INRIA, Lille-Nord Europe).

3 Multiplicity and stable manifolds of time-delay systems: A missing link

Islam Boussaada
IPSA & L2S, CNRS-CentraleSupélec-Université Paris Sud, France.

Multiple spectral values in dynamical systems are often at the origin of complex behaviors as well as unstable solutions. However, in some recent studies, an unexpected property is emphasized. More precisely, an example of delay system is constructed, where the maximal multiplicity of an appropriate delay-dependant negative spectral value leads to a negative spectral abscissa and, as a consequence, the asymptotic stability of the corresponding steady state solution holds. In algebraic terms, the manifold corresponding to such a multiple root defines a stable manifold for the steady state. Furthermore, for the illustrative examples we consider, we show that, under mild assumptions, such a multiple spectral value is nothing but the spectral abscissa. Motivated by the potential implication of such a property in control systems applications, this study is devoted to better explore the connexion between those manifolds.

This is joint work with S. I. Niculescu (L2S, CNRS-CentraleSupélec-Université Paris Sud) and H. Unal (L2S & INRIA Saclay).

4 Computer algebra methods for the stability analysis of diffeential systems with commensurate time-delays

Yacine Bouzidi
INRIA, Lille-Nord Europe, France.

In this presentation, we study the stability of linear differential systems with commensurate delays. Within the frequency-domain approach, it is well-known that the asymptotic stability of such systems is ensured by the condition that all the roots of the corresponding quasipolynomial have negative real parts. A classical approach for checking this condition (see [1]) consists in computing the set of critical zeros of the quasipolynomial, i.e., the roots (and the corresponding delays) of the quasipolynomial that lie on the imaginary axis, and then analyzing the variation of these roots with respect to the variation of the delay. Following this approach, based on solving algebraic systems techniques, we propose a certified and efficient symbolic-numeric algorithm for computing the set of critical roots of a quasipolynomial. Moreover, using recent algorithmic results developed by the computer algebra community, we present an efficient algorithm for the computation of Puiseux series at a critical zero which allows us to finely analyze the stability of the system with respect to the variation of the delay [2]. Explicit examples are given in order to illustrate our algorithms.

This is joint work with A. Poteaux (CRISTAL, Université de Lille 1-CNRS) and A. Quadrat (INRIA, Lille-Nord Europe).

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5 Fast computation of the Nth term of an algebraic series in positive characteristic

Philippe Dumas
INRIA Saclay, France.

For an integer N and a prime number p, we want to compute efficiently the Nth term f_N of an algebraic series $f(x) \in x\mathbb{F}_p[[x]]$. The input power series $f = \sum_i f_i x^i$ is represented by an annihilating polynomial $E \in \mathbb{F}_p[x, y]$ assumed to satisfy the regularity condition $E_y(0, 0) \neq 0$.

A first idea is to use the method of undetermined coefficients. This requires a number of arithmetic operations in $\mathbb{F}_p$ of order N^d, where d is the degree in y of E. The formal Newton iteration allows to speed up this computation and to achieve a quasi-linear arithmetic complexity $\tilde{O}(N)$, where the soft-O notation $\tilde{O}(\cdot)$ hides polylogarithmic factors, as well as the dependence in d.

But there is a far more efficient way to compute f_N. It uses the automatic character of an algebraic series with coefficients in a finite field [1, Cor. 4.5]. Every digit of the expansion of N in base p provides a transition in an automaton; starting from the initial state, the value returned by the automaton is exactly f_N. This computation needs essentially $\log_p N$ operations $\mathbb{F}_p$.

However, the precomputation of the automaton appears to be very costly. It relies on the construction of an algebraic equation of a special form, called a Mahler equation, based on the substitution operator $g(x) \mapsto g(x^p) = g(x)^p$. The mere size of this Mahler equation is of order p^d, which precludes its use.

A better approach is based on Furstenberg's theorem [4] which asserts that the algebraic series f is the diagonal of a bivariate rational function. This yields an alternative algorithm for computing f_N. The main computations are performed in a space of bivariate polynomials whose partial degrees are bounded linearly in those of E [2, Th. 32].

We thus prove, and this is our main contribution, that the diagonal data structure for f permits to compute f_N in $\tilde{O}(p) + O(\log_p N)$, where the hidden constants depend only on the partial degrees of E.

This is joint work with A. Bostan (INRIA Saclay) and G. Christol (IMJ).

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6 On p-adic differential equations with separation of variables

Tristan Vaccon
JSPS - Rikkyo University, Japan.

Real or complex numbers are not the only fields on which the study of differential equations is of interest. For instance, there are polynomial or power series over finite fields that can be characterized by differential equations, such as some isogenies between elliptic curves. Yet, solving effectively differential equations over finite fields raise some special issues coming from positive characteristic: for example, over the finite field of order p (p a prime number), it is not possible to integrate the monomial X^{p-1}.

To overcome such issues, a classical strategy is to lift to the ring of p-adic integers, Z_p, which is of zero characteristic, solve the equation and reduce modulo p. The main hurdle to overcome is then the handling of precision in the lifting.

With X. Caruso and D. Roe, we have designed a method, called the method of differential precision, to handle the precision in a p-adic context in a way that is essentially optimal. In a joint work with P. Lairez (T.U., Berlin), we apply this method to study of p-adic differential equations with separation of variables. This allows an analysis of the p-adic methods to recover a polynomial over a finite field from its Newton sums and to compute normalized isogenies between elliptic curves defined over a finite field.

7 Integrability of the one dimensional Schrœdinger equation

Thierry Combot
Université de Bourgogne, France.

We present a definition of integrability for the one dimensional Schrœdinger equation containing all known one dimensional quantum integrable systems, i.e. systems for which the spectrum can be explicitely computed. We introduce a new class of functions, built in a similar way as Liouvillian functions, but containing functions for which the monodromy group can be algebraically computed, as the hypergeometric function. For functions in this class, the monodromy group can be explicitely computed, and the Schrœdinger equation boundary conditions can be restated, most of the time, in terms of monodromy and Stokes matrices. We then make a classification of quantum potentials with eigenfunctions in this class, finding many new examples of quantum integrable systems, some of them having possibly physical applications.

8 Toward an algorithm to compute the differential Galois group

Thomas Dreyfus
Université Lyon, France.

To a linear differential equation we may associate a group, that measures the algebraic relations between the solutions of the system. Computing such a group in general is a complicated task. In this talk, we will explain how to compute it effectively in a particular case. As an application, we will show how our methods may be applied in order to apply effectively the Morales Ramis Simo theorem.

This is joint work with J.-A. Weil (Université de Limoges, XLIM)

9 Computing the Lie algebra of the differential Galois group of a linear differential system

Thomas Cluzeau
Université de Limoges, XLIM, France.

We consider a linear differential system $[A] : \mathbf{y}' = A\,\mathbf{y}$, where A has coefficients in the differential field $\mathbb{C}(x)$. The differential Galois group G of $[A]$ is a linear algebraic group which measures the algebraic relations among solutions. Although there exist general algorithms to compute G, none of them is either practical or implemented. In this talk we propose an algorithm to compute the Lie algebra $\mathfrak{g}$ of G. To do that, we show how to transform the system into a *reduced form* using a Lie algebra conjugation algorithm. The algorithm is implemented in MAPLE.

This is joint work with M. Barkatou (Université de Limoges, XLIM), L. Di Vizio (Université de Versailles-St Quentin), and J.-A. Weil (Université de Limoges, XLIM).

10 The algebras of polynomial integro-differential operators, their ideals and automorphisms

Vladimir Bavula
University of Sheffield, United Kingdom.

We consider various properties of the algebras of polynomial integro-differential operators (for arbitrary many variables), classifications of their two-sided ideals and automorphisms.

11 A tensor approach to operator algebras

Clemens Raab
RICAM, Austrian Academy of Sciences, Austria.

Algebras of linear operators of various type are used to solve and compute with linear functional equations, e.g. differential, difference, or integral equations. We propose a general framework for treating such algebras of linear operators as quotients of tensor algebras. Relations among operators are formulated in terms of analogs of Gröbner bases.

This is joint work with G. Regensburger (RICAM, Austrian Academy of Sciences) and J. Hossein Poor (RICAM) (see also the talk by J. Hossein Poor).

12 Tensor representation of the algebra of integro-differential operators with linear substitutions

Jamal Hossein Poor
RICAM, Austria.

The algebra of integro differential operators has already been introduced in the study of boundary problems of differential equations. In addition, this algebra can be extended by adjoining linear substitutions to prepare a suitable framework for studying boundary problems of delay differential equations. In this talk, we show how to apply the tensor approach to these two algebras of linear operators in order to obtain normal forms.

This is joint work with G. Regensburger (RICAM, Austrian Academy of Sciences) and C. Raab (RICAM, Austrian Academy of Sciences) (see also the talk by C. Raab).

13 Linear differential-algebraic systems with selected unknowns

Anton Panferov
Joint-Stock Company "Research Institute of Precision Instruments", Russia.

We consider linear differential-algebraic systems for which some components of vector of unknowns are selected. We present the algorithm ExtrAB which for a full rank linear differential system S of the form $A_1y' + A_0y = 0$ (where A_1, A_0 are matrices above a differential field K of characteristic 0) produces, first, a normal differential system S_d (i.e. a system of the form $\tilde{y}' = A\tilde{y}$), whose unknowns are a part of the selected unknowns of the original system and some of their derivatives, and, second, an algebraic system S_a, by means of which other selected unknowns can be linearly expressed only via the selected unknowns from S_d. Suppose that we are interested in solutions whose selected components belong to some differential extension of K. Then the projection of the space of such solutions of the original system onto the selected unknowns coincides with the similar projection of the space of such solutions of S_d and S_a. Furthermore if the system S_d has a solution, whose selected components belong to some differential extension of K then all other components of this solution also belong to the same extension. The sizes of the systems S_d and S_a obtained by ExtrAB are as minimal as possible.

The algorithm is implemented in Maple.

14 Computing solutions of linear Mahler equations

Frédéric Chyzak
INRIA Saclay, France.

Mahler equations relate evaluations of the same function f at iterated bth powers of the variable. They arise in particular in the study of automatic sequences and in the complexity analysis of divide-and-conquer algorithms. Recently, the problem of solving Mahler equations in closed form has occurred in connection with number-theoretic questions. A difficulty in the manipulation of Mahler equations is the exponential blow-up of degrees when applying a Mahler operator to a polynomial. In this work, we present and analyze polynomial-time algorithms for solving linear Mahler equations for series, polynomials, and rational functions.

This is joint work with T. Dreyfus (Université Lyon 1), P. Dumas (INRIA Saclay) and M. Mezzarobba (CNRS).

15 Efficient algorithms for mixed creative telescoping

Louis Dumont
INRIA Saclay, France.

Creative Telescoping is a powerful computer algebra paradigm-initiated by Doron Zeilberger in the '90s- for dealing with definite integrals and sums with parameters. We address the mixed continuous-discrete case, and focus on the integration of bivariate hypergeometric-hyperexponential terms. We design a new creative telescoping algorithm operating on this class of inputs. The new algorithm, as other most recent creative telescoping algorithms, relies on a reduction strategy. This approach is efficient and avoids the costly computation of the normalized certificate. Its analysis reveals tight bounds on the size of the telescoper.

This is joint work with B. Salvy (INRIA Grenoble-Rhône-Alpes, LIP) and A. Bostan (INRIA Saclay).

16 Rational general solutions of first-order algebraic ODEs - existence and computation

Georg Grasegger
RICAM, Austrian Academy of Sciences, Austria.

In this talk we present a decision algorithm for computing rational general solutions of a class of first-order algebraic ordinary differential equations (AODEs). The main approach intrinsically uses algebraic geometry in particular rational parametrizations of plane curves. The interpretation of the AODE as an algebraic equation gives on the one hand the chance to use theory on algebraic curves and on the other hand allows the transformation to a simpler ODE. The class of AODEs we are considering is related to the kind of parametrization that the corresponding curve admits. In the famous collection of Kamke almost all first-order AODEs are in this class. Furthermore, given any first-order AODE we can decide by the same algorithm whether a rational general solution exists in which the arbitrary constant appears rationally. In the affirmative case we compute such a solution.

This is joint work with N. Thieu Vo (RISC, Johannes Kepler University Linz) and F. Winkler (RISC, Johannes Kepler University Linz).

17 Programming abstract mathematics

Mohamed Barakat
University of Siegen, Germany.

What does it mean to program a category? Can one program the derived category formalism and what is this good for? I will try to answer these questions by showing how far we got and where we are heading.

18 Matrices extatiques et calcul de sorties linéarisantes rationnelles

François Ollivier
LIX École polytechnique, CNRS, France

Les systèmes différentiels plats sont communs dans de nombreux domaines de la pratique industrielle. La connaissance de fonctions particulières de l'état du système (et éventuellement des commandes et de leurs dérivées), les "sorties linéarisantes", permettent de paramétrer simplement les trajectoires, simplifiant grandement le contrôle. Cette notion a été étudiée par Cartan et Hilbert et remonte aux travaux de Monge. On ne sait pas décider si un système général est plat, mais on dispose de conditions nécessaires qui expriment les sorties linéarisantes comme des intégrales première communes à plusieurs champs de vecteur. Le calcul du rang d'une matrice extatique permet alors de tester l'existence de sorties linéarisantes rationnelles d'un degré donné, et le calcul du noyau de les obtenir explicitement.

C'est un travail en collaboration avec G. Chèze (MIT, Université de Toulouse).

19 Isochronous center and correction of vector fields

Jordy Palafox
Université de Pau et des Pays de l'Adour, France.

In this talk, I will present new results obtain with J. Cresson (UPPA) about centers of planar polynomial Hamiltonian systems in the real case. In particular we will focus on isochronous centers. Our main concern is the following conjecture stated by Jarque and Villadelprat in [4]: Let X be a real polynomial Hamiltonian vector field of the form

$$X = -\partial_y H \partial_x + \partial_x H \partial_y,$$

where H is a real polynomial in the variables x and y. The maximum degree of the polynomials $\partial_x H$ and $\partial_y H$ is the degree of the Hamiltonian vector field.

Conjecture: *Every center of a planar polynomial Hamiltonian system of even degree is nonisochronous.*

The conjecture is known to be true for quadratic systems thanks to a result of Loud in [6] and in the quartic case by a result of Jarque-Viladelprat in [4]. The conjecture is open for the other cases despite partial results in this direction obtain by B. Schuman in [8]. The proof of Jarque and Villadelprat is based on a careful study of the bifurcations set and seems difficult to extend to an arbitrary degree.

Using the formalism of *moulds* introduced by Jean Ecalle (see [2]) and a particular object attached to a vector field called the *correction* defined by Ecalle and Vallet in [3], we obtain a partial answer to the conjecture for arbitrary degree. It is well known that isochronicity of a real center is equivalent to its linearizability (see [1], theorem 3.3, p.12). A main property of the correction is that it gives a very useful criterion for linearizability. Indeed, a vector field is linearizable if and only if its correction is zero. As the correction possesses an algorithmic and explicit form which is easily calculable using mould calculus we are able to give more informations on the isochronous set. In particular, we prove using complex representation (see [5]) of the real vector field:

Theorem. *Let X be a real Hamiltonian vector field of even degree $2n$ given by :*

$$X = i(x\partial_x - \overline{x}\partial_{\overline{x}}) + \sum_{i=1}^{2n} (P_i(x,\overline{x})\partial_x + \overline{P_i(x,\overline{x})}\partial_{\overline{x}})$$

for $x \in \mathbb{C}$, with $P_i(x,\overline{x}) = \sum_{j=0}^{i} p_{i-j-1,j} x^{i-j}\overline{x}^j$. If X satisfies one of the following conditions:

a) there exists $1 \le r < n-1$ *such that* $p_{i,i} = 0$ *for* $i = 1, \ldots, r-1$ *and* $p_{r,r} > 0$*,*

b) $p_{i,i} = 0$ *for* $i = 1, \ldots, n-1$*,*

then the vector field is nonisochronous.

The fact that homogeneous Hamiltonian vector fields are nonisochronous (see [7]) is also easily deduced from our proof.

In this talk, we will give the main steps of the proof.

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20 On the unimodularity testing for operator matrices

Sergei Abramov
Federal Research Center "Computer Science and Control" of the Russian Academy of Science, Russia.

We consider $n \times n$-matrices whose entries are scalar ordinary differential operators of order $\leq d$ over a constructive differential field K. We give a complexity analysis of the unimodularity testing for an operator matrix and of constructing the inverse matrix if it exists. Some new algorithms for solving these problems are proposed. Besides the complexity as the number of arithmetic operations we consider the number of differentiations in the worst case.

This talk is prepared jointly with A. Storjohann (University of Waterloo).

21 Stochastic Petri nets and computer algebra

Michel Petitot
Université Lille 1, France.

The formalism of chemical reaction systems (equivalently to stochastic Petri nets) is often used in biology, dependability etc. The generating function of the probabilities associated with the master equation is solution of an evolution equation of the type of the Schrödinger equation.

In this first part, we will focus on the calculation of the stationary distribution for a system involving only one chemical species. By using a theorem (in real algebraic geometry) due to Enestrom and Kakeya, we show that generally the stationary generating function is a holomorphic function over the whole complex plane. This result allows us to set up a very precise mixed calculus (symbolic-numeric) of the stationary distribution and associated moments.

Keywords: Stochastic Petri nets, Stochastic models in biology, Weyl algebra, Generating series, Computer algebra.

This is a joint work with C. Versari (Université Lille 1) and K. Batmanov (Université Lille 1).

22 On some algorithmically undecidable problems connected with partial differential equations

Sergey Paramonov
Lomonosov Moscow State University, Russia.

We consider the problems of testing the existence of solutions of certin forms for partial linear differential equations with polynomial coefficients. We prove undecidability of such problems for rational function solutions (this result holds also for difference equations) and for formal Laurent series solutions. Also we prove undecidability of problems of testing the uniqueness of analytic solution and of testing the existence of indefinitely differentiable solution for partial differential equation with boundary conditions.

23 An analytic viewpoint on mock theta functions of Ramanujan

Changgui Zhang
Université Lille 1, France.

Ramanujan discovered a list of functions analytic inside the unit disc and possessing exponential singularities near roots of unity. These functions like as Theta-functions but are not theta-functions, even not false theta-functions previously considered by Rogers. Ramanujan called them mock-theta functions, and the exact meaning is not clear. In this talk, we will make use of the analytic theory of q-difference equation to study these functions, in order to arrive at a possible meaning of mock theta. Algorithms for computing the asymptotic expansions at a root of unity will also be presented for some of these functions.