# Calculus of Variations and Applications

# Report of Contributions

Contribution ID: **1** Type: **not specified**

# Sylvia Serfaty: Vortex lines in 3D Ginzburg-Landau with magnetic field

*Monday, June 19, 2023 9:00 AM (50 minutes)*

In joint work with Carlos Román and Etienne Sandier, we study the onset of vortex lines in the three-dimensional Ginzburg-Landau model of superconductivity. We discuss the critical field at which the first lines appear, which is naturally connected to an "isoflux" problem. We study the optimal number of lines, their interaction, and derive a (Gamma)-limit problem for their arrangement.

Contribution ID: **2** Type: **not specified**

# Lucia De Luca: Stability results for fractional parabolic flows

*Monday, June 19, 2023 10:10 AM (50 minutes)*

We present an abstract method for studying the stability of parabolic flows, exploiting the Gamma-convergence of the corresponding energy functionals. We apply such a result to analyse the behavior of the s-fractional heat flows, as s tends to 0 and to 1, and of the s-Riesz flows, as s tends to 0 and to d (where d is the dimension of the ambient space). Time permitting, we present also stability results for the corresponding geometric flows.

Contribution ID: **3** Type: **not specified**

# Vincent Millot: Torus and split solutions of the Landau-de Gennes model for nematic liquid crystals

*Monday, June 19, 2023 11:00 AM (50 minutes)*

In this talk, I will present the Q-tensor model of Landau-de Gennes for nematic liquid crystals in the so called Lyutsyukov regime dealing with maps with values in the 4-dimensional sphere. This model describes stable configurations of a liquid crystal as minimizers of a Ginzburg-Landau type energy in which the potential well is the real projective plane, seen as a submanifold of S4. In the case where the 3D domain is the unit ball and the Dirichlet boundary data is radially symmetric (equivariantly), one may expect that a minimizer inherits such symmetry. Simulations show that this is not the case and a certain toroidal structure is expected to appear. If (equivariant) axial symmetry is imposed to reduce the complexity of the problem, another type of « singular » solutions appears, the split solutions. By means of regularity results on this model, I will discuss the existence / geometry of torus and split solutions and explain the strong dependence of the type of solutions with respect to the boundary condition and the shape of the domain. This talk is based on recent works in collaboration with Federico Dipasquale and Adriano Pisante.

Contribution ID: **4** Type: **not specified**

# Sergio Conti: Laminates with variable volume fraction in shape-memory alloys

*Monday, June 19, 2023 2:00 PM (50 minutes)*

I will discuss a singularly perturbed variational model for single laminates in shape-memory alloys, with boundary conditions that induce a position-dependent volume fraction. The scaling of the minimum value of the (geometrically linear) energy with respect to the surface energy density is determined by an explicit upper bound and an ansatz-free lower bound, both for a Dirichlet and for a Neumann problem. The lower bound builds upon a rigidity estimate for functions of bounded deformation. This talk is based on joint work with R. V. Kohn and O. Misiats.

Contribution ID: **5** Type: **not specified**

# Leonie Schmeller: Gel models for phase separation at finite strains

*Monday, June 19, 2023 2:50 PM (30 minutes)*

Hydrogels are crosslinked polymer networks saturated in a liquid solvent and can be modeled as a two-phase system employing the phase field approach. During swelling and squeezing, they undergo enormous volume changes, which requires finite strain models for realistic considerations. We
analytically investigate the two-phase model for phase separation in a geometrically nonlinear elas-
tic material. The coupled system of PDEs consists of a Cahn-Hilliard equation and a quasi-static mechanical force balance of the deforming gel. The phase field and the mechanics are coupled by a
multiplicative decomposition of the deformation gradient, and time-dependent Dirichlet boundary conditions are imposed on the deformation field. Based on a time-incremental scheme, we derive existence theory of solutions in a suitable weak formulation. Using techniques from the calculus of
variations and nonlinear PDE theory, we obtain further an existence result for the time-continuous problem under suitable assumptions.
This is a joint work with Marita Thomas within the DFG priority program SPP 2171 Dynamic wetting of flexible, adaptive and switchable substrates, project \# 422786086 and within the MATH+ project AA2-9.

Contribution ID: **6** Type: **not specified**

# Andreas Vikelis: Measure-valued solutions for non-associative finite plasticity

*Monday, June 19, 2023 3:50 PM (30 minutes)*

The variational treatment of evolutionary non-associative elasto-plasticity at finite strains remains unexplored. In this direction, following the concept of energetic solutions, we present an existence result for measure-valued solutions of the quasistatic evolution problem which are stable and balance the energy. In particular, we apply a modification of the standard time-discretization scheme, considering Young measures generated by piecewise constant interpolants of time-discrete solutions of a properly defined minimization problem. A key point in our analysis is the limit passage in the dissipation energy. The later calls for time-continuity properties of the stresses which are not expected in the quasistatic framework. To overcome this obstacle we introduce a regularization of the generalized stress in the definition of our energetic solutions.
Joint work with Ulisse Stefanelli.

Contribution ID: **7** Type: **not specified**

# Barbara Zwicknagl: Variational models for pattern formation: from helimagnets to shape-memory alloys

*Monday, June 19, 2023 4:20 PM (50 minutes)*

We consider continuum variational models for pattern formation in helimagnetic compounds. The energy functional consists of a multi-well bulk energy regularized by a higher order interfacial energy, and arises from a frustrated spin model in the sense of Gamma-convergence. We derive the scaling law for the minimal energy in the case of incompatible boundary conditions. The scaling law indicates the formation of various branching-type patterns in certain parameter regimes. We in particular outline relations to well-studied variational models for martensitic microstructures. This talk is based on joint works with Janusz Ginster and Melanie Koser (both Humboldt-Universität zu Berlin).

Contribution ID: **8** Type: **not specified**

# Simone Di Marino: The shape of Kantorovich potentials: on the existence of minimizers for the Lieb-Oxford inequality

*Tuesday, June 20, 2023 9:00 AM (50 minutes)*

We explain the connection between the classical Lieb-Oxford inequality and multimarginal optimal transport with repulsive cost. We can see that the first order condition is linked with the Kantorovich potential, and we show, through a detailed analysis of the shape of the potentials, that if a minimizer exists, then it should be compactly supported, extending the case N=1 which was already settled by Lieb and Oxford in their original contribution.
This is a work in preparation with R. Lelotte (U. Paris-Dauphine)

Contribution ID: **9** Type: **not specified**

# Manuel Friedrich: Equilibrium configurations for epitaxially strained crystalline films

*Tuesday, June 20, 2023 10:10 AM (50 minutes)*

In this talk, we revisit results obtained on the existence of minimizers and relaxation for energies related to epitaxially strained crystalline films. We first extend the analysis to the framework of three-dimensional linear elasticity. Afterwards, we discuss a rigorous relation between models in nonlinear and linearized elasticity for both continuum and atomistic energies. Based on joint works with Vito Crismale, Leonard Kreutz, and Konstantinos Zemas.

Contribution ID: **10** Type: **not specified**

# Paul Pegon: Asymptotics for optimal quantization in branched optimal transport

*Tuesday, June 20, 2023 11:00 AM (50 minutes)*

The problem of optimal quantization of measures consists in finding the best approximation of a given measure by an atomic measure with a fixed number of atoms, usually expressed through Wasserstein distances. One can formulate the same problem considering instead the irrigation distances of branched optimal transport, where the transport cost behaves as a concave power of the mass and depends on all the trajectories of the particles. We study the asymptotic behaviour of optimal quantizers for absolutely continuous measures as the number of atoms grows to infinity. We compute the limit distribution of the corresponding point clouds and show in particular a branched transport version of Zador's theorem. Moreover, we establish the asymptotic quasi-uniformity of optimal quantizers in terms of separation distance and covering radius of the atoms, when the measure is uniform. This is a joint work with Mircea Petrache.

Contribution ID: **11** Type: **not specified**

# Liangjun Weng: A constrained mean curvature type flow and isoperimetric type inequalities

*Tuesday, June 20, 2023 2:20 PM (30 minutes)*

In this talk, we will discuss the isoperimetric inequality and its high-order version – Alexandrov-Fenchel inequality, which dates back to Queen Dido in the ancient Carthage era. We introduce the quermassintegrals for compact hypersurfaces with capillary boundary from the variational viewpoint. Then by using a constrained mean curvature type flow, we can obtain some new isoperimetric type inequalities for compact hypersurfaces with capillary boundary.

Contribution ID: **12** Type: **not specified**

# Eloi Martinet: Numerical maximization of Neumann eigenvalues on domains on the sphere

*Tuesday, June 20, 2023 2:50 PM (30 minutes)*

We consider the numerical optimization of the first three eigenvalues of the Laplace-Beltrami operator of domains on the sphere with Neumann boundary conditions. We adress two approaches : one is a shape optimization procedure via the level-set method and the other one is a relaxation of the initial problem leading to a density method. These computation gives some strong insight on the optimal shapes of those eigenvalue problems and shows a rich variety of shapes regarding the proportion of the surface area of the sphere occupied by the domain.

Contribution ID: **13** Type: **not specified**

# Alice Marveggio: Uniqueness and stability of planar multiphase mean curvature flow beyond a circular topology change

*Tuesday, June 20, 2023 3:50 PM (30 minutes)*

The evolution of a network of interfaces by mean curvature flow features the occurrence of topology changes and geometric singularities. As a consequence, classical solution concepts for mean curvature flow are in general limited to short-time existence theorems, which include singular times only for some stable shrinkers such as the circle. At the same time, the transition from strong to weak solution concepts (e.g. Brakke solutions) may lead to non-uniqueness of solutions.

Following the relative energy approach à la Fischer-Hensel-Laux-Simon and introducing a suitable notion of gradient-flow calibration for a shrinking circle, we prove a quantitative stability estimate holding up to the singular time. This implies a weak-strong uniqueness principle for weak BV solutions to planar multiphase mean curvature flow beyond circular topology changes.

Furthermore, we expect our method to have further applications to other types of shrinkers, as well as to prove quantitative convergence of diffuse-interface (Allen-Cahn) approximations for mean curvature flow.

This is work in progress with Julian Fischer, Sebastian Hensel and Maximilian Moser.

Contribution ID: **14** Type: **not specified**

# Tim Laux: The large-data limit of the MBO scheme for data clustering

*Tuesday, June 20, 2023 4:20 PM (50 minutes)*

The MBO scheme is an efficient algorithm for data clustering, the task of partitioning a given dataset into several meaningful clusters. In this talk, I will present the first rigorous analysis of this scheme in the large-data limit.
The starting point for the first part of the talk is that each iteration of the MBO scheme corresponds to one step of implicit gradient descent for the thresholding energy on the similarity graph of the dataset. It is then natural to think that outcomes of the MBO scheme are (local) minimizers of this energy. We prove that the algorithm is consistent, in the sense that these (local) minimizers converge to (local) minimizers of a suitably weighted optimal partition problem.
To study the dynamics of the scheme, we use the theory of viscosity solutions. The main ingredients are (i) a new abstract convergence result based on quantitative estimates for heat operators and (ii) the derivation of these estimates in the setting of random geometric graphs.
To implement the scheme in practice, two important parameters are the number of eigenvalues for computing the heat operator and the step size of the scheme. Our results give a theoretical justification for the choice of these parameters in relation to sample size and interaction width.
This is joint work with Jona Lelmi (U Bonn).

Contribution ID: **15** Type: **not specified**

# Maria Giovanna Mora: Explicit minimizers for anisotropic Coulomb energies

*Wednesday, June 21, 2023 9:00 AM (50 minutes)*

Nonlocal interaction energies play a pivotal role in describing the behavior of large systems of particles, in a variety of applications. Traditionally, the focus of the mathematical literature on nonlocal energies has been on radially symmetric potentials, which model interactions depending on the mutual distance between particles. The mathematical study of anisotropic potentials, despite their natural occurrence in modeling interactions where a preferred direction of interaction is present, has on the other hand been very limited until recently. In this talk we will consider a general class of anisotropic energies of Coulomb type in three dimensions and give a complete characterization of their minimizers, under the sole assumption of non-negativity for the Fourier transform of the interaction kernel.

Contribution ID: **16** Type: **not specified**

# Joao Machado: 1D approximation of measures in Wasserstein spaces

*Wednesday, June 21, 2023 10:10 AM (30 minutes)*

Given a Borel probability measure, we seek to approximate it with a measure uniformly distributed over a 1-dimensional set. With this end, we minimize the Wasserstein distance of this fixed measure to all probability measures uniformly distributed to connected 1 dimensional sets and a regularization term given by their length. To show existence of solution to this problem, one cannot easily resort to the direct method in the calculus of variations due to concentration of mass effects. Therefore, we propose a relaxed problem in the space of probability measures which always admits a solution. In the sequel, we show that whenever the initial measure has $L^1$ density w.r.t. the 1-Hausdorff measure (in particular for absolutely continuous measures w.r.t. Lebesgue) then the solution will be a rectiable measure. This allows us to perform a blow-up argument that, in dimension 2, shows that the solution has a uniform density, being therefore a solution to the original problem. Finally, in any dimension, we manage to prove that solutions to the relaxed problem are Ahlfors regular.

Contribution ID: **17** Type: **not specified**

# Annette Dumas: Existence and Lipschitz regularity of the trajectories minimizing the total variation in a congested setting

*Wednesday, June 21, 2023 10:40 AM (30 minutes)*

The problem I will present is motivated by the study of a Mean Field Game model whose theory was simultaneously introduced by Lasry and Lions and by Caines, Huang and Malhamé in 2006. The model consists in studying a population in a city where each agent jumps to move from one place to another. Each inhabitant minimizes a cost composed of the number of jumps and an increasing function of the density of the population. The solution to this problem is a probability measure on the trajectories which is a Nash equilibrium.

The probability measure Q on the trajectories can be seen as a trajectory of the density of the population which leads us to the minimization of a variational problem which depends on the $L^1$-norm of the speed of the density which is linked to the number of jumps and the additional cost which is associated with the increasing function of the density. Density constraints are also added to the problem. We will see that the solution exists, is unique and is Lipschitz in time, despite the discontinuous trajectories taken by each agent. With additional hypothesis on the data, boundedness or continuity in space can be obtained with Dirichlet conditions in time.

The aspect of the solutions are given by the Euler-Lagrange equations which show that in space, either the solution is constant, or it follows the critical points of the cost. Numerical simulations are carried out on a simple example by using the fast dual proximal gradient method from Beck which validates the theoretical framework.

Contribution ID: **18** Type: **not specified**

# Antoine Lemenant: Epsilon-regularity for Griffith minimizers

*Wednesday, June 21, 2023 11:10 AM (50 minutes)*

In this talk I will present some recent advances concerning the $C^1$ regularity of minimizers for the vectorial free-discontinuity problem of Griffith. In particular I will try to explain the strategy of proof inspired by the Reifenberg-flat theory, relying on a geometric stopping time argument on the flatness, coupled with a general extension lemma, which was employed in our latest result valid for any dimension $N > 2$. This is a recent joint work with C. Labourie, and generalizes, with a different proof, a previous 2 dimensional result obtained in collobaration with J.F. Babadjian and F. Iurlano.