

Annette Dumas: Existence and Lipschitz regularity of the trajectories minimizing the total variation in a congested setting

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The problem I will present is motivated by the study of a Mean Field Game model whose theory was simultaneously introduced by Lasry and Lions and by Caines, Huang and Malhamé in 2006. The model consists in studying a population in a city where each agent jumps to move from one place to another. Each inhabitant minimizes a cost composed of the number of jumps and an increasing function of the density of the population. The solution to this problem is a probability measure on the trajectories which is a Nash equilibrium.

The probability measure Q on the trajectories can be seen as a trajectory of the density of the population which leads us to the minimization of a variational problem which depends on the L^1 -norm of the speed of the density which is linked to the number of jumps and the additional cost which is associated with the increasing function of the density. Density constraints are also added to the problem. We will see that the solution exists, is unique and is Lipschitz in time, despite the discontinuous trajectories taken by each agent. With additional hypothesis on the data, boundedness or continuity in space can be obtained with Dirichlet conditions in time.

The aspect of the solutions are given by the Euler-Lagrange equations which show that in space, either the solution is constant, or it follows the critical points of the cost. Numerical simulations are carried out on a simple example by using the fast dual proximal gradient method from Beck which validates the theoretical framework.