

# Joao Machado: 1D approximation of measures in Wasserstein spaces

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Given a Borel probability measure, we seek to approximate it with a measure uniformly distributed over a 1-dimensional set. With this end, we minimize the Wasserstein distance of this fixed measure to all probability measures uniformly distributed to connected 1 dimensional sets and a regularization term given by their length. To show existence of solution to this problem, one cannot easily resort to the direct method in the calculus of variations due to concentration of mass effects. Therefore, we propose a relaxed problem in the space of probability measures which always admits a solution. In the sequel, we show that whenever the initial measure has  $L^1$  density w.r.t. the 1-Hausdorff measure (in particular for absolutely continuous measures w.r.t. Lebesgue) then the solution will be a rectifiable measure. This allows us to perform a blow-up argument that, in dimension 2, shows that the solution has a uniform density, being therefore a solution to the original problem. Finally, in any dimension, we manage to prove that solutions to the relaxed problem are Ahlfors regular.