Random Matrices, Free Probability and Determinantal Processes

Detailed Scientific Program

Lille – 2,3,4 of May 2016

Monday, May 2

10:00–10:45, **Welcoming**.

10:45–12:00, **Kurt Johansson**, *Determinantal point processes* (lecture 1).

Abstract: I will give an introduction to some aspects of the theory of determinantal point processes. The presentation is intended to be introductory but will be a little directed to so called grand canonical determinantal processes, e.g. free fermions in a quadratic well at positive temperature. In connection with this example I will also discuss some more recent results. I also plan to say something about dimer models on bipartite graphs as an example of an interesting class of discrete determinantal processes.

12:00–14:00, **Lunch break**.

14:00–14:45, **Mireille Capitaine**, *Outliers for Hermitian polynomials in unitarily invariant random matrices and a spiked deterministic matrix.*

Abstract: Given a self-adjoint polynomial $P(X, Y)$ in two noncommuting indeterminates, we investigate the asymptotic spectral properties of the random matrix $P(A_N, U_N B_N U_N^*)$, where A_N and B_N are deterministic $N \times N$ Hermitian matrices having compactly supported limiting spectral measures, A_N has a fixed number of spiked eigenvalues outside the support of its limiting spectral measure and U_N is a random $N \times N$ unitary Haar matrix. This is a joint work in progress with S. Belinschi and H. Bercovici.

14:45–15:30, **Folkmar Bornemann**, *Singular values and evenness symmetry: on making Pfaffian processes determinantal.*

Abstract: As a unifying framework for examining several properties that nominally involve eigenvalues, we present a particular structure of the singular values of the Gaussian orthogonal ensemble (GOE): the even-location singular values are distributed as the positive eigenvalues of a Gaussian ensemble with chiral unitary symmetry, while the odd-location singular values, conditioned on the even-location ones, can be algebraically transformed into a set of independent chi-distributed random variables. We discuss applications to the distribution of the GOE determinant and to higher order gap probabilities avoiding much of the technical machinery previously used to analyze these applications such as Pfaffians, skew-orthogonal polynomials, martingales, Meijer G-function. A generalization to other classical orthogonal ensembles is discussed.

This is joint work with M. La Croix (MIT) and P. Forrester (Melbourne).

15:30–16:00, **Coffee break**.

16:00–17:15, **Neil O'Connell**, *From longest increasing subsequences to random polymers, and related topics* (lecture 1).

Abstract: The longest increasing subsequence problem has a long and interesting history, due in no small part to its deep connections to representation theory and the theory of Young tableaux. The latter has vast generalisations, with many applications across random matrix theory, integrable systems and statistical physics. In these lectures I will give an introduction to this topic and outline some recent developments.

Tuesday, May 3

09:00–10:15, **Jesper Møller**, *Statistics for determiantal point processes* (lecture 1).

Abstract: Statistical models and methods for DPPs defined on 1) the d-dimensional Euclidean space and 2) the d-dimensional unit sphere will be discussed, where in the examples $d = 2$. The presentation will be based on the following papers.

- F. Lavancier, J. Møller and E. Rubak (2014). Determinantal point process models and statistical inference: Extended version. Available at arXiv: 1205.4818.
- F. Lavancier, J. Møller and E. Rubak (2015). Determinantal point process models and statistical inference. Journal of Royal Statistical Society: Series B (Statistical Methodology), 77, 853-877. Doi 10.1111/rssb.12096.
- J. Møller, M. Nielsen, E. Porcu and E. Rubak (2015). Determinantal point process models on the sphere. Research Report 13, Centre for Stochastic Geometry and Advanced Bioimaging. Submitted for journal publication.
- J. Møller and E. Rubak (2016). Determinantal point processes and functional summary statistics on the sphere. Research Report 2, Centre for Stochastic Geometry and Advanced Bioimaging. Available at arXiv:1601.03448. Conditionally accepted by Spatial Statistics where the final title is expected to be Functional summary statistics for point processes on the sphere and determinantal point processes on the sphere.

10:15–10:45, **Coffee break**.

10:45–12:00, **Kurt Johansson**, *Determinantal point processes* (lecture 2).

12:00–14:00, **Lunch break**.

14:00–14:45, **Sandrine Péché**, *Universal and non universal features in RMT.*

Abstract: We will discuss through deformed matrix ensembles what can lead to non universal features in the asymptotic spectral properties of large random matrices. The approach makes use of special ensembles of random matrices which induce determinantal random point fields.

14:45–15:30, **Alice Guionnet**, *Discrete Beta-models.*

Abstract: We will discuss discrete analogues of beta models that arise in tiling problems and show how to analyze the fluctuations of their linear statistics by Nekrasov's equations.

15:30–16:00, **Coffee break**.

16:00–16:45, **Alex Kulesza**, *Machine learning with determinantal point processes.*

Abstract: In machine learning, it is a constant challenge is to find models that accurately characterize real-world data while still permitting computationally efficient algorithms for learning and prediction. In this talk I will show how determinantal point processes fill one of the long-standing gaps in our coverage by efficiently modeling "repulsive" or "diversifying" data that cannot be handled by traditional methods. I will describe several of the modeling extensions and computational tricks that we use to make learning practical, and show results for a variety of applications, including document summarization, human pose detection in images, and automatic extraction of news timelines.

16:45–17:00, **Break**.

17:00–17:45, **Arno Kuijlaars**, *Propagation of singular behavior in UE and GUE sums.*

Abstract: I will discuss sums of Hermitian random matrices $M + \sqrt{\tau}H$ where M is a matrix from a unitary ensemble of the form

$$\frac{1}{Z_n} e^{-n Tr V(M)}$$

and H is a scaled GUE matrix. For very special choices of the potential V the eigenvalues of M has a density that vanishes at an interior point, or vanishes to higher order at an edge point. We show that the singular behavior persists for the eigenvalues of $M + \sqrt{\tau}H$ for τ up to a critical value τ_{cr}. In addition, the local scaling limits at the singular point continue to hold as well. This is joint work with Tom Claeys, Karl Liechty and Dong Wang.

Wednesday, May 4

09:00–10:15, **Jesper Møller**, *Statistics for determiantal point processes* (lecture 2).

10:15–10:45, **Coffee break**.

10:45–12:00, **Neil O'Connell**, *From longest increasing subsequences to random polymers, and related topics* (lecture 2).

12:00–14:00, **Lunch break**.

14:00–14:45, **Florent Benaych-Georges**, *Kernel spectral clustering of large dimensional data.*

Abstract: In this joint work with Romain Couillet (Central-Supelec), we give an analysis of kernel spectral clustering methods in the regime where the dimension p of the data vectors to be clustered and their number n grow large at the same rate. We demonstrate, under a k-class Gaussian mixture model, that the normalized Laplacian matrix associated with the kernel matrix asymptotically behaves similar to a so-called spiked random matrix. Some of the isolated eigenvalue-eigenvector pairs in this model are shown to carry the clustering information upon a separability condition classical in spiked matrix models. We evaluate precisely the position of these eigenvalues and the content of the eigenvectors, which unveil important properties concerning spectral clustering, in particular in simple toy models. Our results are then compared to the practical clustering of images from the MNIST database, thereby revealing an important match between theory and practice.

14:45–15:30, **Djalil Chafaï**, *About the spectral edges.*

Abstract: We will present some results about the convergence of extremal eigenvalues of random matrices, including a joint work with Konstantin Tikhomirov http://arxiv.org/abs/1509.02231.

15:30–16:00, **Coffee break**.

16:00–16:45, **Laurent Decreusefond**, *Distances between determinantal processes.*

Abstract: Distances between distributions of random configurations can be defined in various manners. We introduce the Wasserstein and Kolmogorov-Rubinstein variants of these definitions and show how they can be computed for determinantal point processes.

15:30–16:00, **Break**.

17:00–17:45, **Alexander Bufetov**, *Quasi-Symmetries of Determinantal Point Processes.*

Abstract: The classical De Finetti Theorem (1937) states that an exchangeable collection of random variables is a mixture of Bernoulli sequences.

The first result of the talk is that determinantal point processes on $\mathbb{Z}$ induced by integrable kernels are quasi-invariant under the action of the infinite symmetric group. The Radon-Nikodym derivative is a regularized multiplicative functional on the space of configurations. A key example is the discrete sine-process of Borodin, Okounkov and Olshanski.

The second result is a continuous counterpart of the first: namely, it is proved that determinantal point processes with integrable kernels on $\mathbb{R}$, a class that includes processes arising in random matrix theory such as Dyson's sine-process, or the processes with the Bessel kernel or the Airy kernel studied by Tracy and Widom, are quasi-invariant under the action of the group of diffeomorphisms of the line with compact support.

While no analogues of these results in higher dimensions are known, in joint work with Yanqi Qiu it is shown that for determinantal point processes corresponding to Hilbert spaces of holomorphic functions on the complex plane $\mathbb{C}$ or on the unit disk $\mathbb{D}$, the quasi-invariance under the action of the group of diffeomorphisms with compact support also holds.

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