ID de Contribution: **8** Type: **Non spécifié**

Non-homogeneous fluids with low regularity density and dynamical interpolation methods

jeudi 16 mars 2023 14:30 (1h 45m)

We are interested in the global existence and uniqueness issue for systems of PDEs describing the evolution of non-homogeneous viscous fluids.
In particular, we focus on the non-homogeneous incompressible Navier-Stokes system (INS) and on the system for pressureless gases (PNS). Despite their strong formal similarity – coupling between a mass conservation equation and a parabolic-type equation, same scaling invariance and same energy identity – those systems enjoy very different properties.

For (INS), we know from a short note of Kazhikov in 1974 that any initial datum consisting of a finite energy velocity field and a density bounded away from vacuum generates a global weak solution. However, apart from the case of constant density in dimension 2, the question of the uniqueness of such solution remains open. Moreover, we know many results on regular solutions in Sobolev and Besov spaces.
On the contrary, the number of results available for the (PNS) system is much lower, and establishing global results, even for small data, looks more delicate.

In this talk, we present global results with uniqueness for those two systems in some two-dimensional domains (under quite mild assumptions on the domain), which hold for densities having large variations but being bounded away from vacuum.
The initial velocity field belongs to a critical space, constructed by real interpolation, which is very close to the energy space. The velocity has to satisfy a smallness condition with respect to the viscosity coefficient for (PNS), whereas it can be arbitrarily large for (INS).

For the two systems, the constructed solution possesses a continuous differentiable flow, a fact which ensures the persistence of C^1 regularity of the discontinuity interfaces of the density and allows, via a lagrangian change of coordinates, to prove uniquess and stability of the solution with respect to perturbations of the initial datum.

The proof mainly relies on energy estimates with time-dependent weights, combined with a robust dynamical interpolation method. The choice of the functional spaces for the initial velocity is not the same for (PNS), which verifies a (almost) maximum principle, and for (INS), for which such property is false.

Orateur: DANCHIN, Raphaël (LAMA, Université Paris-Est Créteil)