

Actions of Large Groups, Geometric Structures, and the Zimmer Program

Rapport sur les contributions

ID de Contribution: 1

Type: **Non spécifié**

On global rigidity of Anosov G-actions

lundi 10 juin 2024 09:30 (1 heure)

I will present some recent progress on the global rigidity for Anosov actions of semisimple Lie groups of higher rank. This is joint work with Spatzier, Vinhage and Xu.

Orateur: DAMJANOVIC, Danijela (KTH)

ID de Contribution: 2

Type: **Non spécifié**

Constructing super-expanders from actions of higher rank lattices

lundi 10 juin 2024 11:00 (1 heure)

Super-expanders are sequences of finite, d -regular graphs that satisfy some nonlinear form of spectral gap with respect to all uniformly convex Banach spaces. This notion vastly strengthens the classical notion of expander. In this talk I will explain some recent constructions of super-expanders, coming from actions of higher rank lattices on Banach spaces and on manifolds. I will also review some recent constructions of (usual) expanders, for which we do not know whether they are super-expanders.

Orateur: DE LAAT, Tim (University of Münster)

ID de Contribution: 3

Type: **Non spécifié**

Arithmeticity for Smooth Maximal Rank Positive Entropy Actions of \mathbb{R}^k

lundi 10 juin 2024 14:00 (1 heure)

We prove an arithmeticity theorem in the context of nonuniform measure rigidity. Adapting machinery developed by A. Katok and F. Rodriguez Hertz [J. Mod. Dyn. 10 (2016), 135–172; MR3503686] for \mathbb{Z}^k systems to \mathbb{R}^k systems, we show that any maximal rank positive entropy system on a manifold generated by $k \geq 2$ commuting vector fields of regularity C^r for $r > 1$ is measure theoretically isomorphic to a constant time change of the suspension of some action of \mathbb{Z}^k on the $(k+1)$ -torus or the $(k+1)$ -torus modulo $\{\text{id}, -\text{id}\}$ by affine automorphisms with linear parts hyperbolic. Further, the constructed conjugacy has certain smoothness properties. This in particular answers a problem and a conjecture from a prequel paper of Katok and Rodriguez Hertz, joint with B. Kalinin [Ann. of Math. (2) 174 (2011), no. 1, 361–400; MR2811602].

Orateur: UZMAN, Alp (University of Utah)

ID de Contribution: 4

Type: **Non spécifié**

Zimmer's embedding theorem for tractor solutions and conformal actions of simple groups on closed pseudo-Riemannian manifolds

lundi 10 juin 2024 15:30 (1 heure)

Zimmer's embedding theorem concerns actions of connected Lie groups by automorphisms of differential-geometric structures and has yielded important restrictions on which groups can act on a manifold with a given structure. It has a useful version for Cartan geometries which generalizes rather easily to tractor solutions on parabolic-type geometries. Tractor solutions are parallel sections of associated vector bundles for connections which can encode a very wide array of geometric PDEs. An application for the conformal-to-Einstein tractor connection is a rigidity theorem for conformal actions of $SU(p',q')$ on closed (p,q) -pseudo-Riemannian manifolds in the real-analytic setting: $2p' \leq p+1$ and if $2p'=p+1$, then the metric is conformally flat. This is work in progress with K. Neusser.

Orateur: MELNICK, Karin (University of Luxembourg)

ID de Contribution: 5

Type: **Non spécifié**

Continuity of a measurable conjugacy between linear cocycles

mardi 11 juin 2024 09:30 (1 heure)

We consider Hölder continuous $GL(d, \mathbb{R})$ -valued cocycles over hyperbolic and partially hyperbolic diffeomorphisms. We discuss results on continuity of a measurable conjugacy between two cocycles. We focus on perturbations of constant cocycles and on cocycles with one Lyapunov exponent. We also mention related results on continuity of measurable invariant geometric structures. As an application, we bootstrap regularity of a conjugacy between an Anosov toral automorphism and its perturbation.

Orateur: SADOVSKAYA, Victoria (PennState University)

ID de Contribution: 6

Type: **Non spécifié**

Higher-rank lattices actions on conformal structures

mardi 11 juin 2024 11:00 (1 heure)

In this talk I will discuss both Lie groups and lattices actions by conformal transformation of a pseudo-Riemannian manifold, related to the Lorentzian Lichnerowicz' conjecture.

I will first discuss dynamics of $SL(2, \mathbb{R})$ -actions on closed Lorentzian manifolds, and then I will detail recent advances for higher-rank lattices conformal actions, and ongoing works with Thierry Barbot in the specific Lorentzian case.

Orateur: PECASTAING, Vincent (Université Côte d'Azur)

ID de Contribution: 7

Type: **Non spécifié**

Rigidity of lattice actions on boundaries

mercredi 12 juin 2024 14:00 (1 heure)

Lattices in linear semi-simple Lie groups have a natural action on their (generalized) Furstenberg boundaries, e.g. a lattice in $SL_n(\mathbb{R})$ acting on the different flag varieties. For such natural actions, I will discuss the local rigidity question—*are these actions ‘stable’ under small deformations in the homeomorphism group of the boundary?* We will answer this question for higher rank uniform lattices by showing that any small deformation is semi-conjugate to the natural action. This is joint work with Chris Connell, Thang Nguyen, and Ralf Spatzier. Time permitting, I will discuss how local rigidity fails when we replace the Furstenberg boundary with the visual boundary of the symmetric space.

Orateur: ISLAM, Mitul (MPI Leipzig)

ID de Contribution: 8

Type: **Non spécifié**

Poisson–Voronoi tessellations and fixed price in higher rank

mercredi 12 juin 2024 15:30 (1 heure)

We define and motivate the Poisson point process, which is, informally, a “maximally random” scattering of points in space. We introduce the ideal Poisson–Voronoi tessellation (IPVT), a new random object with intriguing geometric properties when considered on a semisimple symmetric space (the hyperbolic plane, for example). In joint work with Mikolaj Fraczyk and Sam Mellick, we use the IPVT to prove a result on the relationship between the volume of a manifold and the number of generators of its fundamental group. We give some intuition for the proof. No prior knowledge on Poisson point processes, fixed price, or higher rank will be assumed.

Orateur: WILKENS, Amanda (University of Texas)

ID de Contribution: 9

Type: **Non spécifié**

Higher rank lattice actions with positive entropy

lundi 10 juin 2024 16:40 (1 heure)

In this talk, I will discuss about smooth higher rank lattice actions on manifolds with positive entropy. From dynamical information, we can detect information on groups and manifolds. For instance, when lattices in $SL(n, \mathbb{R})$ act on an n -dimensional manifold with positive entropy, we can see that the lattice is abstractly commensurable with $SL(n, \mathbb{Z})$.

This is joint work with Aaron Brown.

Orateur: LEE, Homin (Northwestern University)

ID de Contribution: **10**

Type: **Non spécifié**

Systems with rank one factors

mercredi 12 juin 2024 09:30 (1 heure)

Lie group actions with rank one factors have natural families of perturbations arising from modifying each factor independently. I will explain ways to obtain semi-rigid settings, in particular, various characterizations of product systems. Partially based on work with R. Spatzier, and work in-progress with A. Uzman.

Orateur: VINHAGE, Kurt (University of Utah)

ID de Contribution: 11

Type: **Non spécifié**

Non tame cocycle rigidity above affine unipotent abelian actions on the torus

mercredi 12 juin 2024 11:00 (1 heure)

Cocycle rigidity with tame solutions is a crucial ingredient in KAM theory. We are interested in cocycle rigidity above affine unipotent abelian actions on the torus with Diophantine translation data. We consider *unlocked* actions whose rank one factors are non vanishing translations (the locked actions do not have any kind of stability).

It follows from Katok and Robinson's observations that when one generator of the action is of step less or equal to 2 then cocycle rigidity with tame solutions holds. Moreover, Damjanovic, Fayad and Saprykina proved that in this case *almost* cocycles also have *almost* solutions (with a low regularity control on the error), and from there concluded KAM-rigidity of these actions.

In a joint work with S. Durham, we find examples of affine \mathbb{Z}^2 -actions on the torus above which smooth cocycle rigidity holds but is not tame. The linear part of the action is generated by unipotent matrices of step 3. Our examples show that KAM-rigidity for higher rank actions by affine unipotent toral actions does not hold in general when no element of the actions is of step less or equal to 2.

Orateur: FAYAD, Bassam (University of Maryland)

ID de Contribution: 12

Type: **Non spécifié**

Rigidity of commensurators and its applications

jeudi 13 juin 2024 09:30 (1 heure)

I will discuss a question raised independently by Greenberg and Shalom: Can an infinite discrete subgroup of a simple Lie group have dense commensurator and not be a lattice? I will explain the surprising connections between this question and other long-standing open problems, and discuss recent progress on special cases of the question. This is joint work with (subsets of) Brody, Fisher, and Mj.

Orateur: VAN LIMBEEK, Wouter (University of Illinois Chicago)

ID de Contribution: 13

Type: **Non spécifié**

Global smooth rigidity for toral automorphisms

jeudi 13 juin 2024 11:00 (1 heure)

Suppose f is a diffeomorphism on torus whose linearization A is weakly irreducible. Let H be a conjugacy between f and A . We prove the following: 1. if A is hyperbolic and H is weakly differentiable 2. if A is partially hyperbolic and H is C^1 -holder. Then H is C^∞ . Our result shows that the conjugacy in all local and global rigidity results for irreducible A is C^∞ . This is a joint work with B. Kalinin and V Sadovskaya.

Orateur: WANG, Zhenqi (Michigan State University)

ID de Contribution: 14

Type: **Non spécifié**

Rigidity of some higher rank partially hyperbolic actions

jeudi 13 juin 2024 14:00 (1 heure)

Smooth rigidity of higher rank abelian and lattice actions with some hyperbolicity has been studied extensively. When the manifold is a nilmanifold, results by Rodriguez Hertz, Wang, and Brown, Rodriguez Hertz, Wang show that: If the action contains an Anosov diffeomorphism then the action is globally rigid. I will discuss rigidity of higher rank partially hyperbolic actions on nilmanifolds. In particular, I will discuss global rigidity of abelian and higher rank lattice actions that contain one fibered partially hyperbolic element.

Orateur: SANDFELDT, Sven (KTH)

ID de Contribution: 15

Type: **Non spécifié**

Strong primeness for equivalence relations arising from Zariski dense subgroups

jeudi 13 juin 2024 15:30 (1 heure)

In this talk, I will describe an ongoing joint work with Daniel Drimbe in which we show that equivalence relations arising from essentially free ergodic probability measure preserving actions of Zariski dense discrete subgroups of simple algebraic groups are strongly prime. As a consequence, we obtain a unique prime factorization result for direct products of such equivalence relations. This extends and strengthens Zimmer's primeness result for equivalence relations arising from actions of lattices in simple Lie groups (1981). The key novelty in our approach relies on a combination of ergodic theory of algebraic group actions and Popa's intertwining theory for equivalence relations.

Orateur: HOUDAYER, Cyril (ENS Paris)

ID de Contribution: 16

Type: **Non spécifié**

Lichnerowicz conjecture(s)

jeudi 13 juin 2024 16:40 (1 heure)

A priori, the conformal group of a compact Riemannian manifold has no reason to be compact, since it only preserves angles and not distances. A posteriori, however, it turns out that this group is compact, with a single exception: the round sphere! The Lichnerowicz conjecture refers to similar rigidity statements in the cases of pseudo-Riemannian conformal and projective structures.

Orateur: ZEGHIB, Abdelghani (ENS Lyon)

ID de Contribution: 17

Type: **Non spécifié**

Rigidity for boundary actions and classification in low dimensions

vendredi 14 juin 2024 09:30 (1 heure)

The plan of the talk is to describe joint work with A. Brown and Z. Wang on the smooth classification of actions of lattices in $SL(n, \mathbb{R})$ on $n-1$ dimensional manifolds. The method is also amenable to show rigidity for some boundary actions.

Orateur: RODRIGUEZ-HERTZ, Federico (PennState University)

ID de Contribution: **18**

Type: **Non spécifié**

Coarse embeddings and group actions preserving rigid geometric structures

vendredi 14 juin 2024 11:00 (1 heure)

We will see how the notion of coarse embeddings allows to better understand discrete group actions preserving a rigid geometric structure. The focus will mainly be on isometric and conformal actions. In particular, we will discuss a Tits alternative for isometry groups of compact Lorentzian manifolds.

Orateur: FRANCES, Charles (University of Strasbourg)

ID de Contribution: **19**

Type: **Non spécifié**

Miklós Abért

Orateur: ABÉRT, Miklós (Rényi Institute)