

# H-matrix Solver for BEM: a task-based approach

Thursday, February 4, 2016 9:50 AM (35 minutes)

For the numerical simulation of wave propagation in acoustics, Airbus Group Innovations relies on integral equations solved with the Boundary Elements Method. The advantages of this approach are well known: mainly

accuracy, and simpler (surfacic) mesh. The main algorithm drawback is the need to cope with a dense matrix whose size can be quite large for wave propagation problems, where the mesh step is governed by the wave-length

of the physical problem treated (in frequency domain).

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Since the late 90's, fast methods have been introduced to deal with these limitations. First, the Fast Multipole Method (FMM) allowed to compute fast matrix-vector products (in  $O(n \log^2(n))$  instead of  $O(n^2)$  for the standard algorithm), and hence to design fast solvers using iterative methods. Lately, H-Matrix methods have gained wide acceptance by introducing fast direct solvers, allowing to solve systems in  $O(n \log^2(n))$  - or less - without the hassle of using iterative solvers (unknown convergence rate and difficulty to find a good preconditionner).

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H-Matrix [1] is a lossy, hierarchical storage scheme for matrices that, along with an associated arithmetic, provides a rich enough set of approximate operations to perform the matrix addition, multiplication, factorization (e.g. LU or LDLT) and inversion. It relies on two core ideas : (a) nested clustering of the degrees of freedom, and of their products; and (b) adaptative compression of these clusters. Several choices exist in the literature for these two ingredients, the most common being Binary Space Partitioning for the clustering and Adaptative Cross Approximation for the compression.

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Together, they allow for the construction of a fast direct solver [2], which is especially important for BEM applications as it gracefully handles a large number of Right-Hand Sides (RHS). They also provide a kernel-independent fast solver, allowing one to use the method for different physics.

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Airbus Group Innovations has recently implemented the H-Matrix arithmetic and successfully applied it to a wide range of industrial applications in electromagnetism and acoustics. Furthermore, these algorithms are hard to efficiently parallelize, as the very scarce literature on the subject shows [3]. We developed a parallel solver that goes beyond the aforementioned reference, using innovative techniques on top of a state-of-the-art runtime system StarPU [4][5]. This enables the solving of very large problems, with a very good efficiency. In this presentation, we show some results on the accuracy of this method on several challenging applications, and

its fast solving time and efficient use of resources.

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References

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