

Second-Kind Single Trace Boundary Integral Equations

vendredi 5 février 2016 14:00 (35 minutes)

For second-order linear transmission problems involving a single closed interface separating two homogeneous materials, a well-posed second-kind boundary integral formulation has been known for a long time. It arises from a straightforward combination of interior and exterior Calderon identities. Apparently, this simple approach cannot be extended to “composite” settings involving more than two materials.

The key observation is that the same second-kind boundary integral equations (BIE) can also be obtained through a multi-potential representation formula. We can attach a potential to each boundary of a material sub-domain, add them all up to a multi potential, and then we notice that, thanks to a null-eld property, the sum provides a representation of the field solution, when its traces a plugged into the potentials. Taking traces yields a BIE on the skeleton of the sub-domain partition. The skeleton traces of the unknown field will solve it.

Using the fact that multi-potentials for a single homogeneous material must vanish, the BIE can be converted into second-order form: for the scalar case (acoustics) its operator becomes a compact perturbation of the identity in L^2 . Galerkin matrices arising from piecewise polynomial Galerkin boundary element (BEM) discretization will be intrinsically well-conditioned.

The new second-kind boundary element method has been implemented both for acoustic and electromagnetic scattering at composite objects. Numerical tests confirm the excellent mesh-size independent conditioning of the Galerkin BEM matrices and the resulting fast convergence of iterative solvers like GMRES. Furthermore, by simple post-processing, we obtain discrete solutions of competitive accuracy compared to using BEM with the standard first-kind BIE.

Well-posedness of the new second-kind formulations is an open problem, as is the compactness of the modulation of the identity in the case of Maxwell’s equations. Reassuringly, computations have never hinted at a lack of stability.

References

[1] X. Claeys, R. Hiptmair, and E. Spindler, Second-kind boundary integral equations for scattering at composite partly impenetrable objects, Tech. Rep. 2015-19, Seminar for Applied Mathematics, ETH Zurich, Switzerland, 2015. Submitted to BIT.

[2] X. Claeys, R. Hiptmair, and E. Spindler, A second-kind galerkin boundary element method for scattering at composite objects, BIT Numerical Mathematics, 55 (2015), pp. 33-57.

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