New Trends in Integral Equations

Thursday, February 4, 2016 - Friday, February 5, 2016

Ecole Polytechnique

Book of Abstracts

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New results in BEM for Electromagnetic Compatibility

Electromagnetic Compatibility (EMC) modelling is a major issue in aeronautic industry, with applications like Lightning Indirect Effects, High-Intensity Radiated Fields, antenna coupling, wifi… These simulations represent many challenges for the numerical solvers: very large frequency band (starting from DC), need for local models (wires, slots, equipment…), low levels of currents and fields inside cavities, presence of geometric and data singularities and the need of robust adaptive meshing procedures… and finally the model size requires efficient fast and parallel solvers. We present new results on these issues.

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Boundary Element Solutions of Electromagnetic Problems Based on Helmholtz Projectors

Integral equation solvers are widely used for simulating electromagnetic scattering and radiation from metallic and penetrable objects. Long popular in academic circles, these solvers have been in recent years incorporated into several commercial electromagnetic analysis and design tools, after the advent of fast multipole and related algorithms. The effectiveness of these solvers notwithstanding, boundary element methods are often plagued by several issues related to ill-conditioning and numerical instabilities both for low and high frequencies.

This talk will focus on a new family of schemes which can effectively solve several of these issues. These solvers are based on implicit discrete Helmholtz decompositions obtained via suitably defined projectors. The use of these projectors, we will briefly delineate, allows to obtain electric/magnetic/combined Calderon equations with peculiarly favorable properties (both in frequency and time domain), efficient wavelet preconditioners, as well as Calderon-like approaches that do not require the use of dual boundary elements.

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Fast Boundary Elements Methods and applications

Fast convolution on unstructured grids have been developed for many applications (e.g. electrostatics, magnetostatics, acoustics, electromagnetics, etc.). The goal is to reduce the complexity of matrix-vector products, from O(N^2) to O(N log N). In this presentation, we describe a new efficient numerical method called Sparse Cardinal Sine Decomposition (SCSD), based on a suitable Fourier decomposition of the Green kernel, sparse quadrature formulae and Type-III Non Uniform Fast Fourier Transforms (type-III NUFFT). This talk summarizes this new way, provide comparisons between SCSD, FMM and H-Matrix, and gives numerical results from our new Boundary Element solver, MyBEM.

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A new a posteriori error estimate for the BEM

A posteriori error estimates are tools which enable a measure of the numerical error. They are required to be equivalent to the norm of the error and to be locally computable. They are norms of values which can be computed from the numerical solution and the problem parameters. In the context of BEM, there is a loss of locality of the norms and therefore of the estimates. Standard localization techniques partially solve the issue but lead to a loss of control between the norm of the error and the estimate.

We introduce a new localization technique based on the computation of the residual (the norm of the residual is an equivalent norm of the error). By applying a well-chosen isomorphism to the residual, we can "carry" it in some functional space where the norm is local (typically L2). We rst introduce the concept for the BEM in 3D-acoustics, then we give an introduction on what this -estimation would look like for the EFIE.

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A low dispersive Trefftz DG method based on a local BEM for the solution of the Helmholtz equation

We first briefly present the dispersion phenomenon and show how it damages the numerical solution of wave problems over large distances of propagation. We then introduce a Trefftz Discontinuous Galerkin (TDG) symmetric formulation for the Helmholtz equation with piecewise constant coefficients. Recall that Trefftz methods are discretization processes based on the use of exact interior either local or global solutions. The Trefftz method considered in this work is based on using locally a Boundary Element Method (BEM), thus avoiding the restriction on the type of the local waves used in the usual TDG methods. We show then that accurate local approximations of the Dirichlet-to-Neumann map have a direct impact on the reduction of the dispersion error. We then present some numerical tests bringing out that the obtained procedure can completely rub out the dispersion error contrary to the best Interior Penalty DG (IPDG) methods.

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Volume integral equation formulation for anisotropic elastodynamic scattering: solvability, application to small-inclusion asymptotics

In contrast with the vast existing literature on the mathematical aspects of boundary integral equations and their application to scattering by impenetrable obstacles characterized by Dirichlet, Neumann or impedant boundary conditions, comparatively few studies are available regarding the mathematical properties of volume integral equation (VIE) formulations. This communication addresses the solvability of VIEs arising in elastodynamic scattering by penetrable obstacles. The elasticity tensor and mass density are allowed to be smoothly heterogeneous inside the obstacle and may be discontinuous across the background-obstacle interface, the background elastic material being homogeneous. Both materials may be anisotropic, within certain limitations for the background medium. The VIE associated with this problem is first derived, relying on known properties of the background fundamental tensor. To avoid difficulties associated with existing radiation conditions for anisotropic elastic media, we also propose a definition of the radiating character of transmission solutions. The unique solvability of the volume integral equation (and of the scattering problem) is established. For the important special case of isotropic background properties, our definition of a radiating solution is found to be equivalent to the Sommerfeld-Kupradze radiation conditions. Moreover, usefulness of this result for the derivation and justification of small-inclusion asymptotic approximations is discussed.

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An H-matrix based direct solver for the Boundary Element Method in 3D elastodynamics

The main advantage of the Boundary Element Method (BEM) is that only the domain boundaries are discretized leading to a drastic reduction of the total number of degrees of freedom.

In traditional BE implementation the dimensional advantage with respect to domain discretization methods is offset by the fully-populated nature of the BEM
coefficient matrix. In the present work, we propose a fast method to solve the BEM system in 3-D frequency-domain elastodynamics.

Using the H-matrix arithmetic and low-rank approximations (performed with Adaptive Cross Approximation), we derive a fast direct solver.
We assess the numerical efficiency and accuracy on the basis of numerical results obtained for problems having known solutions.
In particular, we study the efficiency of low-rank approximations when the frequency is increased.
The efficiency of the method is also illustrated to study seismic wave propagation in 3-D domains.

This is a joint work with Patrick Ciarlet and Luca Desiderio

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Connecting Integral Equation and Unified Transform Methods for Wave Scattering

In this talk we discuss the application of the unified transform method, due to A.S. Fokas and co-authors, to interior and exterior problems for time harmonic waves. Like integral equation methods, the method reduces to the solution of a problem on the boundary of the domain in question. We discuss numerical implementations, restricting the solution space to a finite-dimensional subspace, and explain how some implementations can be interpreted as Galerkin methods, the convergence of which can be established by standard arguments. We focus particularly on implementations that are based on approximation by plane waves (and generalised, evanescent plane waves), and explain that these can be implemented in such a way that the numerical solution is precisely the best approximation from the plane wave subspace. In the case of scattering by diffraction gratings we note that this

particular unified transform method is in fact precisely the SS* method proposed previously (though with a less precise analysis) in Arens, Chandler-Wilde and De Santo (2006), where it is derived as a first kind integral equation formulation. Further details can be found in [1,2].

[1] Acoustic scattering: high frequency boundary element methods and unified transform methods. S N Chandler-Wilde & S Langdon, in "Unified Transform for Boundary Value Problems: Applications and Advances", A S Fokas & B Pelloni (editors), SIAM, 2015.

[2] When all else fails, integrate by parts" - an overview of new and old variational formulations for linear elliptic PDEs. E A Spence in "Unified Transform Method for Boundary Value Problems: Applications and Advances", A S Fokas & B Pelloni (editors), SIAM, 2015.

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Quasi-local multi-trace formulations

In the context of time harmonic wave scattering by piecewise homogeneous penetrable objects, we present a new variant of the multi-trace boundary integral formulations (MTF). This new approach differs from the so-called "local" MTF by the presence of regularisation terms involving boundary integral operators and localised at junctions i.e. points where at least three subdomains abut. It lends itself to much more standard analysis: all operators are continuous in standard Sobolev trace spaces, a Garding inequality can be proved, which implies quasi-optimal approximation property for conformal Galerkin discretisations. As regards numerical performances, this new formulation also appears slightly more accurate compared to pre-existing MTF, while the speed of convergence of iterative solvers remains comparable.

References:

[1] X.Claeys, Quasi-local multi-trace boundary integral formulations, Numer. Methods Partial Differential Equations, 31(6):2043–2062, 2015.

[2] X.Claeys and R. Hiptmair and C. Jerez-Hanckes, Multi-trace boundary integral equations, chapter in Direct and Inverse Problems in Wave Propagation and Applications, 51–100, Radon Ser. Comput. Appl. Math., 14, De Gruyter, Berlin, 2013.

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Analytical preconditioners for the solution of three-dimensional surface scattering problems

The numerical solution of time-harmonic scattering problems remains challenging in the high fre-

quency regime due to its specific computational bottlenecks. The techniques based on integral equations lead to the resolution of linear systems where the involved matrices are dense and usually badly conditioned. The improvement of these methods is a timely research area. One possibility to reduce the computational cost is to precondition iterative solvers (to speed up the convergence) and on the other hand to use fast methods to compute the matrix-vector products needed at each iteration.

We propose an analytical preconditioner taking inspiration of On-Surface Radiation Condition techniques. This preconditioner is an accurate approximation to the Dirichlet-to-Neumann map. The associated integral equations are of the second kind. Moreover, the proposed preconditioner shows highly desirable advantages: sparse structure, ease of implementation and low additional computational cost.

In this talk, we present first the principle of the method in the acoustic case. We show numerical simulations for various configurations. Next, we explain how to extend the approach to other types of waves, namely elastic waves (joint work with Stéphanie Chaillat et Frédérique Le Louër).

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Solution schemes and approximation tools devoted to the simulation of electromagnetic testing by the boundary element method

Electromagnetic testing is widely used for the characterization of a medium as to the detection of defects. In particular, the eddy current non-destructive testing of tubes in steam generators is determinant to diagnose the integrity of heat exchangers in nuclear industry. A valuable support to the mastering of these processes is brought by modeling and the finite boundary element method (BEM) is an appropriate simulation tool to many inspection configurations. The department of imaging and simulation for the control at CEA LIST is developing a BEM code devoted to these applications, mostly for eddy current testing. Some of these tools are, or will be, integrated into the CIVA software platform, whose target users are experts in non-destructive testing (non numericians).

In this talk, we will present our research work and review the technical options related to this activity. We will start with an overview of a recent study carried out in collaboration with IRSN (the French public expert in nuclear and radiological risks) to illustrate our practical use of BEM. We will then introduce low frequency formulations that are studied in collaboration with the research group POEMS. We will in particular discuss a multi-step algorithm for solving the transmission problem (known as PMCHWT), which is stable over a wide range of parameters that are relevant to electromagnetic testing, and in particular go beyond eddy currents. The third part of the talk will focus on the discretization tools developed for the BEM code at LIST. They are based on the use of basic interpolation techniques to simplify the construction of complex approximation spaces that meet our needs for light but accurate computations (such as the Helmholtz decomposition of high-order edge functions).

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Quasi-optimal domain decomposition methods for time-harmonic acoustic and electromagnetic wave problems

In terms of computational methods, solving three-dimensional time-harmonic acoustic or electromagnetic wave problems is known to be challenging, especially in the high frequency regime and in the presence of inhomogenous media. The brute-force application of the finite element method in this case leads to the solution of very large, complex and possibly indefinite linear systems. Direct sparse solvers do not scale well for such problems, and Krylov subspace iterative solvers can exhibit slow convergence, or even diverge. Domain decomposition methods provide an alternative, iterating between subproblems of smaller sizes, amenable to sparse direct solvers. In this talk I will present a class of non-overlapping Schwarz domain decomposition methods that exhibit quasi-optimal convergence properties, i.e., with a convergence that is optimal for the evanescent modes and significantly improved compared to competing approaches for the remaining modes [1, 2]. These improved properties result from a combination of an appropriate choice of transmission conditions and a suitable localization of the optimal, integral operators associated with the complementary of each subdomain [3]. The resulting algorithms are well suited for high-performance, large scale parallel computations in complex geometrical configurations when combined with appropriate preconditionners [4].

[1] Y. Boubendir, X. Antoine and C. Geuzaine, A quasi-optimal non-overlapping domain decomposition algorithm for the Helmholtz equation. Journal of Computational Physics 231 (2), pp. 262-280, 2012.

[2] M. El Bouajaji, B. Thierry, X. Antoine and C. Geuzaine. A quasi-optimal domain decomposition algorithm for the time-harmonic Maxwell's equations. Journal of Computational Physics 294, pp. 38-57, 2015.

[3] M. El Bouajaji, X. Antoine, C. Geuzaine. Approximate local magnetic-to-electric surface operators for time-harmonic Maxwell's equations. Journal of Computational Physics 279 (15), 241-260, 2014.

[4] A. Vion and C. Geuzaine. Double sweep preconditioner for optimized Schwarz methods applied to the Helmholtz problem. Journal of Computational Physics 266, 171-190, 2014.

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Second-Kind Single Trace Boundary Integral Equations

For second-order linear transmission problems involving a single closed interface separating two homogeneous materials, a well-posed second-kind boundary integral formulation has been known for a long time. It arises from a straightforward combination of interior and exterior Calderon identities. Apparently, this simple approach cannot be extended to "composite" settings involving more than two materials.

The key observation is that the same second-kind boundary integral equations (BIE) can also be obtained through a multi-potential representation formula. We can attach a potential to each boundary of a material sub-domain, add them all up to a multi potential, and then we notice that, thanks to a null-eld property, the sum provides a representation of the field solution, when its traces a plugged into the potentials. Taking traces yields a BIE on the skeleton of the sub-domain partition. The skeleton traces of the unknown field will solve it.

Using the fact that multi-potentials for a single homogeneous material must vanish, the BIE can be converted into second-order form: for the scalar case (acoustics) its operator becomes a compact perturbation of the identity in L^2. Galerkin matrices arising from piecewise polynomial Galerkin boundary element (BEM) discretization will be intrinsically well-conditioned.

The new second-kind boundary element method has been implemented both for acoustic and electromagnetic scattering at composite objects. Numerical tests confirm the excellent mesh-size independent conditioning of the Galerkin BEM matrices and the resulting fast convergence of iterative solvers like GMRES. Furthermore, by simple post-processing, we obtain discrete solutions of competitive accuracy compared to using BEM with the standard first-kind BIE.
Well-posedness of the new second-kind formulations is an open problem, as is the compactness of the modulation of the identity in the case of Maxwell's equations. Reassuringly, computations have never hinted at a lack of stability.

References

[1] X. Claeys, R. Hiptmair, and E. Spindler, Second-kind boundary integral equations for scattering at composite partly impenetrable objects, Tech. Rep. 2015-19, Seminar for Applied Mathematics, ETH Zurich, Switzerland, 2015. Submitted to BIT.

[2] X. Claeys, R. Hiptmair, and E. Spindler, A second-kind galerkin boundary element method for scattering at composite objects, BIT Numerical Mathematics, 55 (2015), pp. 33-57.

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Hybrid numerical-asymptotic methods for wave scattering problems

Linear wave scattering problems (e.g. for acoustic, electromagnetic and elastic waves) are ubiquitous in science and engineering applications. However, conventional numerical methods for such problems (e.g. FEM or BEM with piecewise polynomial basis functions) are prohibitively expensive when the wavelength of the scattered wave is small compared to typical lengthscales of the scatterer (the so-called "high frequency" regime). This is because the solution possesses rapid oscillations which are expensive to capture using conventional approximation spaces. In this talk we outline recent progress in the development of "hybrid numerical-asymptotic" methods, which incur significantly reduced computational cost. These methods use approximation spaces containing oscillatory basis functions, carefully chosen to capture the high frequency asymptotic behaviour. In particular we discuss some of the interesting challenges arising from nonconvex, penetrable and three-dimensional scatterers.

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Preconditionning integral equations on the unit disc in 3d

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Modelling tools for scattering and antennas problems

The Surface Integral Equation is one of the most used methods in the simulation of electromagnetic problems. Its implementation combined with a fast multipole algorithm (MLFMM) and iterative solvers leads efficient and accurate methods for the analysis of radar cross section (RCS) for a target.

Different methods (MLFMM, ACA, Domain Decomposition Method …) and formulations EFIE (Electric Field Integral Equation), MFIE (Magnetic Field Integral Equation), CFIE (Combined Field Integral Equation) and GCSIE (integral formulations inherently well-conditioned using a regularizing operator) were implemented at ONERA.

In this talk, we will present various applications obtained with our tools like radar cross section (RCS), antenna radiation, ….

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High Order Impedance Boundary Condition for 3D Integral Equations

Impedance Boundary Conditions (IBC) are widely used in computational
electromagnetics to model thin coatings on perfectly conducting (PEC) objects.
The IBC is used to reduce drastically the number of unknowns of the integral
equations models and to obtain a better conditioned linear system that can be
more efficiently iteratively solved.

Integral equations models can be built that mix homogeneous regions, whose
material characteristics are defined by their electrical permittivities and magnetic
permeabilities, PEC regions and IBC. These models are very efficient for the
modeling of complex structures.

Usually, the impedance of a coating is assumed to be the same for all incidence angles and polarizations (standard IBC). This assumption is valid for coatings with high refractive index or significant losses. For coatings with smaller refractive index or lower losses, the accuracy of the IBC can be greatly improved by approximating it as a partial derivative equation ([1] D.J. Hoppe, Y. Rahmat-Samii, Impedance Boundary Conditions in Electromagnetics, Taylor and Francis, 1995). The second order IBC can be written as:

$E_t + b_1\nabla_\Gamma div E_t - b_2 rot_\Gamma rot_\Gamma E_t = a_0 J + a_1\nabla_\Gamma div J - a_2 rot_\Gamma rot_\Gamma J$
E_t and J are respectively the tangent electric field and the electric current. Integral equations formulations for structures with homogeneous regions, PEC regions and SIBC can be generalized to take into account second order IBC.
In [1], the second order IBC is discretized with spline basis functions for 2D and body of revolution problems. Here, the second order IBC is applied to 3D problems. The currents J and $M = Exn$ are discretized with the lower order HDiv functions that can be used to model objects both smooth or with sharp edges.

A first set of validations will be presented that show the increased accuracy of high order IBC over standard IBC while the computational effort for solving the overall integral equations model remains similar.

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Fast boundary element methods in industrial applications

In this talk we consider direct and indirect boundary integral formulations for the solution of electro– and magnetostatic field computations in industrial applications. Particular interest is on the use of floating potentials in a multi–dielectric setting with high permittivity.

This talk summarizes joint work with A. Blaszczyk and Z. Andjelic (ABB), and with D. Amann, G. Of, and P. Urthaler (TU Graz).

References

[1] Z. Andjelic, G. Of, O. Steinbach, P. Urthaler: Fast boundary element methods for industrial applications in magnetostatics. In: Fast Boundary Element Methods in Engineering and Industrial Applications (U. Langer, M. Schanz, O. Steinbach, W. L. Wendland eds.), Lecture Notes in Applied and Computational Mechanics, vol. 63, Springer, Heidelberg, pp. 111–143, 2012.

[2] Z. Andjelic, G. Of, O. Steinbach, P. Urthaler: Boundary element methods for magnetostatic field problems: A critical view. Comput. Visual. Sci. 14

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in industrial applications by boundary element methods. J. Math.
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H-matrix Solver for BEM: a task-based approach

For the numerical simulation of wave propagation in acoustics, Airbus Group Innovations relies on integral
equations solved with the Boundary Elements Method. The advantages of this approach are well known: mainly
accuracy, and simpler (surfacic) mesh. The main algorithm drawback is the need to cope with a dense matrix
whose size can be quite large for wave propagation problems, where the mesh step is governed by the wavelength
of the physical problem treated (in frequency domain).

Since the late 90's, fast methods have been introduced to deal with these limitations. First, the Fast Multipole
Method (FMM) allowed to compute fast matrix-vector products (in O(n log2(n)) instead of O(n2) for the
standard algorithm), and hence to design fast solvers using iterative methods. Lately, H-Matrix methods
have gained wide acceptance by introducing fast direct solvers, allowing to solve systems in O(n log2(n)) - or
less - without the hassle of using iterative solvers (unknown convergence rate and difficulty to find a goodpreconditionner).

H-Matrix [1] is a lossy, hierarchical storage scheme for matrices that, along with an associated arithmetic,
provides a rich enough set of approximate operations to perform the matrix addition, multiplication,
factorization (e.g. LU or LDLT) and inversion. It relies on two core ideas : (a) nested clustering of the degrees of
freedom, and of their products; and (b) adaptative compression of these clusters. Several choices exist in the
literature for these two ingredients, the most common being Binary Space Partitioning for the clustering and
Adaptative Cross Approximation for the compression.

Together, they allow for the construction of a fast direct solver [2], which is especially important for BEM
applications as it gracefully handles a large number of Right-Hand Sides (RHS). They also provide a
kernel-independent fast solver, allowing one to use the method for dierent physics.

Airbus Group Innovations has recently implemented the H-Matrix arithmetic and successfully applied it to

a wide range of industrial applications in electromagnetism and acoustics. Furthermore, these algorithms are
hard to eciently parallelize, as the very scarce literature on the subject shows [3]. We developed a parallel
solver that goes beyond the aforementioned reference, using innovative techniques on top of a state-of-the-art
runtime system StarPU [4][5]. This enables the solving of very large problems, with a very good eciency. In
this presentation, we show some results on the accuracy of this method on several challenging applications, and
its fast solving time and ecient use of resources.

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Special Issue: Euro-Par 2009, vol. 23, pp. 187198, Feb. 2011.

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Windowed Green function method for layered-media scattering

We present a new Windowed Green Function (WGF) method for the numerical integral-equation solution of problems of electromagnetic scattering by obstacles in presence of dielectric or conducting half-planes. The WGF method, which is based on use of integral kernels that can be expressed directly in terms of the free-space Green function, does not require evaluation of expensive Sommerfeld integrals. The proposed approach is fast, accurate, flexible and easy to implement. In particular straightforward modifications of existing solvers suffice to incorporate the WGF capability. The proposed method can be up to thousands of times faster, for a given accuracy, than a corresponding method based on the layer-Green-function.