Nonlinear Analysis & PDE in Lille

Schedule

Wednesday 1st

12:30 - *Lunch*

14:00 - **Didier Smets**: *Binormal curvature flow for unbounded curves*

In this talk, we will first recall the notion of binormal curvature flow for curves in the three dimensional euclidean space, as well as the theory for weak solutions for lipschitz closed curves that was developed jointly with R.L. Jerrard. We will then consider possible extensions to the case of unbounded curves, and compare these to other already existing approaches.

15:00 - **Laurent Thomann**: *Almost sure scattering for the 1D nonlinear Schrödinger equation*

I will present some results on the 1D nonlinear Schrödinger equation with a nonlinearity of degree $p > 1$. I will define measures on the space of the initial data for which we can describe the nontrivial evolution by the linear Schrödinger flow and show that their nonlinear evolution is absolutely continuous with respect to this linear evolution. We deduce from this precise description decay estimates implying the globally well-posedness of the equation for $p > 1$ with scattering for $p > 3$. This is joint work with Nicolas Burq (University Paris-Saclay).

16:00 - *Coffee break*

16:30 - **Radu Ignat**: *Separation and interaction energy between domain walls in a nonlocal variational model*

We analyse a nonconvex variational model from micromagnetics with a nonlocal energy functional, depending on a small parameter $\epsilon > 0$. The model gives rise to transition layers, called Néel walls, and we study their behaviour in the limit $\epsilon \to 0$. The analysis has some similarity to the theory of Ginzburg-Landau vortices. In particular, it gives rise to a renormalised energy that determines the interaction (attraction or repulsion) between Néel walls to leading order. But while Ginzburg-Landau vortices show attraction for winding numbers of the same sign and repulsion for those of opposite signs, the pattern is reversed in this model. First, we show that the Néel walls stay separated from each other and second, we determine the interaction energy between them. The theory gives rise to an effective variational problem for the positions of the walls, encapsulated in a Gamma-convergence result. This is a joint work with R. Moser (University of Bath).

19:30 - *Dinner*

Thursday 2nd

9:00 - **Jean-Michel Coron**: *On the time for the local controllability for KdV and water tank control systems*

We are interested in the local null-controllability of the nonlinear KdV equation equipped the Dirichlet boundary conditions using the Neumann boundary control on the right. Lionel Rosier proved in 1997 that this KdV system is small-time locally controllable for all non-critical lengths and that the uncontrollable space of the linearized system is of finite dimension when the length is critical. Concerning critical lengths, Emmanuelle Crépeau and I proved in 2004 that the same result holds when the uncontrollable space of the linearized system is of dimension 1, and later Eduardo Cerpa in 2007, and then Eduardo Cerpa and Emmanuelle Crépeau in 2009 established that the local controllability holds for large time for all other critical lengths. We show that, for an explicit large class of critical lengths, the nonlinear KdV system is not small-time locally controllable. We also show that for a water control system one needs more time for the steady state controllability for the nonlinear systems than for the linearized system. These two results are joint works with Armand Koenig and Hoai-Minh Nguyen.

10:00 - **Mihai Mariş**: *Some non-homogeneous Gagliardo-Nirenberg inequalities and standing waves to a biharmonic non-linear Schrödinger equation*

We develop a new method, based on the Tomas-Stein inequality, to establish non-homogeneous Gagliardo-Nirenberg-type inequalities in the Euclidean space. Then we use these inequalities to study standing waves to a fourth-order Schrödinger equation with mixed dispersion. We are interested in the solutions that minimize the energy when the L^2-norm (the mass) is kept fixed. We prove optimal results on the existence of minimizers in the mass-subcritical case and in the mass-critical case. In the mass-supercritical case, the infimum of the energy when the mass is kept fixed is always $-\infty$. However, we are able to show that there are local minimizers. The existence of those local minimizers is significantly more difficult than the study of global minimizers in the mass-subcritical and mass-critical cases. They give global in time solutions for the corresponding fourth order Schrödinger equation which have small H^2-norm but do not scatter. If the mass does not exceed some threshold, our results on "best" local minimizers are optimal. This is joint work with Antonio J. Fernández (Madrid), Louis Jeanjean (Besançon), and Rainer Mandel (Karlsruhe).

11:00 - *Coffee break*

11:30 - **Nicholas Alikakos**: *On the triple junction problem for the vector Allen-Cahn on the plane without symmetry hypotheses*

We investigate the Allen-Cahn system $\Delta u - W_u(u) = 0$, $u : \mathbb{R}^2 \to \mathbb{R}^2$, where $W \in \mathcal{C}^2(\mathbb{R}^2, [0, +\infty))$ is a potential with three global minima. We establish the existence of an entire solution u, which possesses a triple junction structure. The main strategy is to study the global minimizer u_ε of the blow-down (rescaled) variational problem over the unit disc with appropriate Dirichlet boundary conditions. An energy lower bound is derived that plays a crucial role in estimating the location and size of the diffuse interface. We do not impose any symmetry hypotheses on the solution. This is a joint work with Zhiyuan Geng (BCAM, Bilbao).

12:30 - *Lunch*

14:00 - **Jean-Claude Saut**: *On fractionary KdV equations*

We will survey recent results on the fractionary KdV equations that are good models to study the influence of a weak dispersive perturbation on the dynamics of the Burgers equation. They fall in two categories. In the first one, one expects global well-posedness (with soliton resolution) or finite type blow-up (not of shock type however). In the second one, shock formation is possible but also long time existence of small solutions. We will also comment on the Whitham equation that have some relevance in the modeling of water waves.

15:00 - **Giacomo Canevari**: *Variational convergence for the Ginzburg-Landau functional on complex line bundles*

The Ginzburg-Landau functional was originally proposed as a model for superconductivity in Euclidean domains. However, invariance with respect to gauge transformations — which is one of the most prominent features of the model — suggests that the functional can be naturally defined in the setting of complex line bundles, where it can be regarded as an Abelian Yang-Mills-Higgs theory. In this talk, we shall consider the Ginzburg-Landau functional on an Hermitian line bundle over a closed Riemannian manifold, in the so-called "non-self dual scaling", which is naturally inherited from superconductivity theory. We shall focus on the variational aspects of the problem. More precisely, we will discuss a Γ-convergence result for sequences whose energy grows at most logarithmically in the Ginzburg-Landau coupling parameter. As we shall see, the London equation for superconductivity plays a significant role in our analysis. The talk is based on a joint work with Federico Dipasquale (Università Federico II, Napoli) and Giandomenico Orlandi (Verona).

16:00 - *Coffee break*

16:30 - **Étienne Sandier**: *Vortex filaments in the Ginzburg-Landau model of superconductivity*

In this ongoing work we study the onset of superconductivity in a 3D domain. A first result improves on previous work by Alama-Bronsard-Montero, Baldo-Jerrard-Orlandi-Soner and C. Román by computing a more precise estimate of the so-called first critical field. We also bound the number of filaments present if the applied magnetic field is close enough to the first critical field. Finally we describe the geometry of the filaments using an interaction energy derived in the spirit of recent results of Contreras-Jerrard. This is a joint work with C. Román et S. Serfaty.

19:30 - *Dinner*

Friday 3rd

9:00 - **Sylvia Serfaty**: *Dipole transition in the two-component plasma*

We study the two-dimensional two-component plasma or Coulomb gas with N positive charges and N negative charges interacting logarithmically in the plane. We obtain an expansion of the free energy and large deviations principles that show a transition from a system of free charges to a system of bound dipoles. This is based on joint work with Thomas Leblé and Ofer Zeitouni, and with Jeanne Boursier.

10:00 - **David Chiron**: *Travelling and stationary states for Nonlinear Schrödinger models*

We shall present various existence results of travelling and stationary states for nonlinear Schrödinger models. The first ones are about the nonlinear Schrödinger equation (also called Gross-Pitaevskii equation). We shall present existence results of smooth branches of travelling waves, for small speeds (joint works with Eliot Pacherie) when they exhibit vortices and for speed close to the speed of sound, when they rescale as KP-I solitary waves. The second ones concern the Gross-Clark model describing an impurity in a Bose condensate (joint works with Joe Alhelou and Mihai Mariş). This last model couples a Gross-Pitaevskii equation for the condensate with a linear Schrödinger one for the impurity.

11:00 - *Coffee break*

11:30 - **Giandomenico Orlandi**: *A variational approach to hyperbolic obstacle problems*

We prove existence of weak solutions to the obstacle problem for wave equations (including the semilinear and fractional case) by using a suitable approximating variational scheme in the spirit of minimizing movements. We discuss some applications and open questions. This is joint works with Le Van Phu Cuong (Verona), Mauro Bonafini (Verona) and Matteo Novaga (Pisa).

12:30 - *Lunch*

14:00 - **Frédéric Hélein**: *Dynamical mechanisms for Kaluza-Klein theories*

I will present a theory based on a variational principle leading to solutions of Einstein-Maxwell or, more generally, Einstein-Yang-Mills systems in the spirit of Kaluza-Klein theories. In these theories, no fibration is assumed: the fields are defined on a spacetime Y of dimension $4 + r$ without a priori structure, where r is the dimension of the structure group. If the latter is compact and simply connected, the classical solutions allow the construction of a manifold X of dimension 4 which can be interpreted as the physical space-time, so that Y acquires the structure of a principal bundle over X, and provide solutions of the Einstein-Yang-Mills systems. If the structure group is $U(1)$, a case which corresponds to the Einstein-Maxwell system, the situation is slightly degenerate and hence requires some extra hypothesis.