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A pyramid-shaped blow-up set for the 2d semilinear wave equation

We consider the semilinear wave equation with subconformal power nonlinearity in two space dimensions. We construct a finite-time blow-up solution with a pyramid-shaped blow-up surface and an isolated characteristic blow-up point at the origin. Our solution is symmetric with respect to both axes, and anti-symmetric with respect to both bisectrices. The blow-up surface is differentiable outside the bisecrtices. On the bisectrices, it only has directional derivatives.

As for the asymptotic behavior in similariy variables, the solution converges to the classical one-dimensional soliton outside the bisectrices, and to a genuinely two dimensional stationary solution, on the bisectrices, outside the origin. At the origin, it behaves like the sum of 4 solitons localized on the two axes, with opposite signs for neighbors.

This is the first example of a blow-up solution with a characteristic point in higher dimensions, showing a really two dimensional behavior. Moreover, the points of the bisectrices outside the origin give us the first example of non characteristic points where the blow-up surface is non differentiable.