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Undecidably semilocalizable metric measure spaces and Radon-Nikodymification of arbitrary measure spaces

The questions raised here grew from the desire to give an integral representation for members of the dual of BV, the Banach space of functions of bounded variation. This potentially has application to the calculus of variations since BV dual contains subgradients of energy functionals to be minimized. The question quickly links to that of identifying the dual of $L^1(\mathbb{R}^m, \mathcal{A}, \mathcal{H}^{m-1})$ where $\mathcal{H}^{m-1}$ is the Hausdorff measure. Whether the corresponding canonical map $\Upsilon : L^\infty \to (L^1)^*$ is injective or not depends upon the σ-algebra $\mathcal{A}$. For $\mathcal{A}$ being the σ-algebra of measurable sets in the sense of Caratheodory, the surjectivity of Υ is undecidable in ZFC. This calls for trying to associate with every measure space (X, Σ, μ), in a universal way, a new measure space $(\hat{X}, \hat{\Sigma}, \hat{mu})$ with respect to which the Radon-Nikodym theorem holds – alternatively such that the corresponding $\hat{\Upsilon}$ is an isometric isomorphism – and $L^1(X, \Sigma, \mu) \cong L^1(\hat{X}, \hat{\Sigma}, \hat{mu})$. I will explain how this is better stated in a specific category whose objects are "measurable spaces with negligibles". In that context, the existence of the universal "Radon-Nikodymification" is obtained via several applications of Zorn's Lemma and, therefore, is not much of practical use in general. In a particular case that pertains to BV dual, specifically when μ is an integral geometric measure (instead of Hausdorff measure), I will show that $\hat{X}$ can be described explicitly as a fibered space of $\mathcal{R}^m$ whose fiber above x consists of germs of rectifiable sets through x. Part of these results have been obtained in collaboration with Philippe Bouafia.

Auteur principal: Prof. DE PAUW, Thierry (Université Paris Cité)

Orateur: Prof. DE PAUW, Thierry (Université Paris Cité)