Analyse des EDP, géométrie et physique

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Toulouse

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Singularities of degenerate complex Monge-Ampère equation, according to Alireza Bahraini

Auteur: Daniel Bennequin[1]

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Conjugating a variety of methods, analytic series, Schauder a priori estimates, global continuity, Alireza Barhaini obtained recently a precise description of the singular behavior of the solution of a complex Monge-Ampère equation, degenerating along a smooth divisor in a compact Kähler manifold. Applications are given to the Bogomolov-Miyaoka-Yau inequality and to the Kodaira-Hodge theory on a class of singular complex structures, in view of the proof of the existence of Lagrangian surfaces in projective complex surfaces of the general type.

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Asymptotics for two-dimensional elliptic Allen-Cahn systems

Auteur: Fabrice Bethuel[1]

[1] *Laboratoire Jacques-Louis Lions, Sorbonne Université*

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The formation of codimension-one interfaces for multi-well gradient-driven problems is well-known and established in the scalar case, where the equation is often referred to as the Allen-Cahn equation. The vectorial case in contrast is quite open. This lack of results and insight is to a large extent related to the absence of known monotonicity formula. I will focus on the elliptic case in two dimensions, and presents some results which extend to the vectorial case in two dimensions most of the results obtained for the scalar case. I will also emphasize some specific features of the vectorial case.

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The physical origin of the Batalin-Vilkovisky approach

Auteur: Christian Brouder[1]

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This talk describes a (hypothetical) historical path that lead Batalin and Vilkovisky to the discovery of their (wonderful) approach, starting from the quantization of scalar fields, to the quantization of electrodynamics and non-Abelian gauge fields.

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Eigenfunctions and representation theory

Auteur: Fran Burstall[1]

[1] *University of Bath*

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Originally discussed by Strichartz in the '80's, there has been some recent interest from the harmonic map community in eigenfunctions of the Laplacian on symmetric spaces whose squares are also eigenfunctions. The state of the art until recently has been the construction of local examples on classical symmetric spaces. In this talk, I shall show how some simple ideas from representation theory explain the known examples and provide new results.

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Domain wall dynamics in notched ferromagnetic nanowires

Auteur: Gilles Carbou[1]

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Ferromagnetic nanowires have promising applications in data storage. In such devices, the information is encoded by Domain Walls (DW) which are thin zones of magnetization reversal. The magnetization behavior is described by the non linearLandau-Lifschitz model. In this talk, we investigate the stability of DW configurations. In particular we highlight DW pinning properties by notches patterned along the wire, and DW depinning effect of applied magnetic field.

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Stability and Boundary stabilization of 1-D hyperbolic systems

Auteur: Jean-Michel Coron[1]

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Hyperbolic systems in one-space dimension appear in various real-life applications (navigable rivers and irrigation channels, heat exchangers, chemical reactors, gas pipes, road traffic, chromatography, ...). This presentation will focus on the stabilization of these systems by means of boundary control. Stabilizing feedback laws will be constructed. This includes explicit feedback laws that have been implemented for the regulation of the rivers La Sambre and La Meuse. The presentation will also cover robustness issues, the case where source terms exist and the case where optimal time stabilisation is considered.

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Undecidably semilocalizable metric measure spaces and Radon-Nikodymification of arbitrary measure spaces

Auteur: Thierry De Pauw[1]

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The questions raised here grew from the desire to give an integral representation for members of the dual of BV, the Banach space of functions of bounded variation. This potentially has application to the calculus of variations since BV dual contains subgradients of energy functionals to be minimized. The question quickly links to that of identifying the dual of $L^1(\mathbb{R}^m, \mathcal{A}, \mathcal{H}^{m-1})$ where $\mathcal{H}^{m-1}$ is the Hausdorff measure. Whether the corresponding canonical map $\Upsilon : L^\infty \to (L^1)^*$ is injective or not depends upon the σ-algebra $\mathcal{A}$. For $\mathcal{A}$ being the σ-algebra of measurable sets in the sense of Caratheodory, the surjectivity of Υ is undecidable in ZFC. This calls for trying to associate with every measure space (X, Σ, μ), in a universal way, a new measure space $(\hat{X}, \hat{\Sigma}, \hat{mu})$ with respect to which the Radon-Nikodym theorem holds – alternatively such that the corresponding $\hat{\Upsilon}$ is an isometric isomorphism – and $L^1(X, \Sigma, \mu) \cong L^1(\hat{X}, \hat{\Sigma}, \hat{mu})$. I will explain how this is better stated in a specific category whose objects are "measurable spaces with negligibles". In that context, the existence of the universal "Radon-Nikodymification" is obtained via several applications of Zorn's Lemma and, therefore, is not much of practical use in general. In a particular case that pertains to BV dual, specifically when μ is an integral geometric measure (instead of Hausdorff measure), I will show that $\hat{X}$ can be described explicitly as a fibered space of $\mathcal{R}^m$ whose fiber above x consists of germs of rectifiable sets through x. Part of these results have been obtained in collaboration with Philippe Bouafia.

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The loop group method for harmonic maps with applications to Willmore surfaces

Auteur: Josef F. Dorfmeister[1]

[1] *Technical University of Munich*

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The first part of the talk gives a review of the loop group method for harmonic maps from Riemann surfaces to (semisimple) symmetric spaces. Applications of this will be illustrated by pointing out the relation between certain types of harmonic maps and certain classes of surfaces. In the second part we will describe in some detail the discussion of Willmore surfaces in spheres along the lines of the first part. This will include the occurrence of some unusual features. Finally, time permitting, we will talk about a harmonic map for Willmore surfaces, found by Frederic Helein, which gives an additional avenue for the loop group approach.

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Direct connections on jet groupoids

Auteur: Alessandra Frabetti[1]

[1] *Université Lyon 1*

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Gauge fields are the local expression of a principal connection on a principal bundle, and therefore encode the infinitesimal data of a parallel transport between the fibres along curves on the base manifold. One may wonder if there is a field interpretation for parallel transport. Reading principal connections as infinitesimal connections on the associated Atiyah Lie algebroid, this question can be answered by usual integration and gives rise to direct connections on Lie groupoids [A. Kock 1989, N. Teleman 2004, J. Kubarski and N. Teleman 2006]. In this talk we review the basic facts about

direct connections on Lie groupoids, together with some interesting examples due to Teleman, and study their jet prolongations on jet groupoids. The talk is based on a work in progress with S. Azzali, Y. Boutaib and S. Paycha.

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A Weierstrass type representation of Constrained Willmore surfaces.

Auteur: Idrisse Khemar[1]

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In a first part, we present a joint work with Josef Dorfmeister : a Weierstrass type representation for constrained Willmore surfaces in spheres. Using appropriates (moving) frames and a appropriate Lie algebra decomposition of so(1,n+3), we translate the PDE of constrained Willmore surfaces into the Lie algebra setting : namely we rewrite it as a Maurer-Cartan equation of an extended Maurer Cartan form of the frame associated to the surface. Then using some "Bruhat" decomposition of SO(1,3), we consider appropriate versions of the Iwasawa and the Birkhoff decompositions of the loop group. This allows us to construct a DPW algorithm for constrained Willmore surfaces and hence to obtain a Weierstrass type representation for these surfaces in terms of holomorphic (or meromorphic) potentials. In a second part, we characterize the conformal Gauss maps of constrained Willmore surfaces (personal work) : in particular, we generalize a theorem of Dorfmeister and Wang for Willmore surfaces to constrained Willmore surfaces.

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Q-Curvature and the Positive Mass Theorem

Auteur: Paul Laurain[1]

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After a brief review of the classical Positive Mass Theorem and a short introduction to Q-curvature, I will present a theorem on the positive mass for Q-curvature and discuss some applications.

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About Koszul-Tate resolutions

Auteur: Camille Laurent-Gengoux[1]

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I will present several results and open problems about Koszul-Tate resolutions. Several topics will be addressed : how symmetries "closing" only on a singular subset naturally give a Z-graded Q-manifold. with Koszul-Tate negative part, how to get constructive Koszul-Tate resolutions. Then I

will detail several open questions about affine varieties. Joints works with Hancharuk, Kotov, Salnikov, Strobl.

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Periodic Darboux transforms

Auteur: Katrin Leschke[1]

[1] *University of Leicester*

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In classical differential geometry, geometric transformations have been used to create new curves and surfaces from simple ones: the aim is to solve the underlying defining compatibility equations of curve or surface classes by finding solutions to a simpler system of differential equations arising from the transforms. Classically, the main concern was a local theory. In modern theory, global questions have led to a renewed interest in classical transformations. For example, in the case of a torus, the investigation of closing conditions for Darboux transforms naturally leads to the notion of the spectral curve of the torus. In this talk we discuss closing conditions for smooth and discrete polarised curves, isothermic surfaces and CMC surfaces. In particular, we obtain new explicit periodic discrete polarised curves, new discrete isothermic tori and new explicit smooth CMC cylinder.

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New variations on old themes

Auteur: Thierry Lévy[1]

[1] *Sorbonne Université*

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In a first part, I will report on an ongoing collaboration with Adrien Kassel (CNRS, ENS Lyon) where we elaborate on a classical theorem of G. Kirchhoff (1847). In a second part, I will present recent developments due to Frédéric Hélein on an early work of W.A. Mozart (1762).

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Morse Index Stability of Willmore Immersions

Auteur: Alexis Michelat[1]

[1] *EPFL*

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The Morse index of a critical point of a Lagrangian L is the dimension of the maximal vector space on which the second derivative D2L is negative-definite. In the classical theory of Hilbert spaces, one shows that the Morse index is lower semi-continuous, while the sum of the Morse index and nullity (the dimension of the Kernel of the differential operator associated to the second derivative) is upper semi-continuous. In a recent work (arXiv:2212.03124), Francesca Da Lio, Matilde Gianocca, and Tristian Rivière (ETH Zürich) developed a new method to show upper semi-continuity results in

geometric analysis—that they applied to conformally invariant Lagrangians in dimension 2 (which include harmonic maps). The proof relies on a fine analysis of the second derivative in neck regions —that link the macroscopic surface to its "bubbles"—and a pointwise estimate of the sequence of critical points in the neck regions. In this talk, we will explain how to apply this method to the Willmore energy, a conformally invariant Lagrangian associated to immersions of a surface into Euclidean spaces. Critical points of the Willmore energy—or Willmore immersions—satisfy a non-linear fourth-order elliptic differential equation, and this extension will give rise to several new technical difficulties. If time allows, we will try to show the universal character of this method, that could address (amongst others) the Morse index stability for Ginzburg-Landau energies in dimension 2, bi-harmonic maps in dimension 4, the Yang-Mills functional in dimension 4, and also apply to min-max problems.

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Bouncing Jacobi Fields and the Allen-Cahn equation on surfaces

Auteur: Frank Pacard[1]

[1] *Ecole polytechnique*

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There is a strong parallel between the theory of minimal hypersurfaces and the solutions of the double-well phase transition Allen-Cahn equation on a manifold. In this talk, I will report some recent result on the relations between geodesics on surfaces and solutions of the Allen-Cahn equation with uniform bounds on the Morse Index and energy, as the phase transition parameter tends to 0. Our results show that the situation in 2-dimensions is strikingly different from the situation in dimension 3 and higher. This is a joint work with Juncheng Wei and Yong Liu.

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Hilbert-Einstein Lagrangian on a Generalised Frame Bundle

Auteur: Jérémie Pierard de Maujouy[1]

[1] *UCLouvain*

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Einstein's equation can be obtained as the Euler-Lagrange equation associated to the Hilbert-Einstein Lagrangian, which is essentially the scalar curvature. The curvature tensor, and therefore Einstein's equation, can be formulated and studied on the frame bundle of spacetime. We will introduce a Lagrangian defined on a 10-manifold such that the solutions to the Euler-Lagrange equations equip the manifold with a structure which is almost that of the frame bundle of an Einstein manifold. This will lead us to introduce a structure which generalises that of a frame bundle provided with a principal connection.

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Non-perturbative renormalisation group from algebraic quantum field theory

Auteur: Kasia Rejzner[1]

[1] *University of York*

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In this talk I will present recent results obtained in collaboration with Brunetti, Duetsch and Fredenhagen, concerning the construction of a net of C*-algebras for interacting quantum field theories. Using these construction one can discuss features like the time-slice axiom, action of the appropriate version of the renormalisation group and anomalous Nother theorem.

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Area Variations under pointwize Lagrangian and Legendrian constraints

Auteur: Tristan Riviere[1]

[1] *ETH*

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Multiplicative kernels: from Bessel to Kontsevich, Buchstaber and Calabi-Yau

Auteur: Volodya Roubtsov[1]

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We discuss few very recent results of a work in progress (in collaboration with I. Gaiur and D. Van Straten and with V. Buchstaber and I. Gaiur) about interesting properties of multiplication Bessel kernels, which includes well-known Clausen and Sonin-Gegenbauer formulae of XIX century, special examples of Kontsevich discriminant loci polynomials, raised as addition laws for special two-valued formal groups (Buchshtaber-Novikov-Veselov) and period functions for some CY and Landau–Ginzburg models.

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Darboux theorems for volume-valued symplectic forms

Auteur: Leonid Ryvkin[1]

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We will discuss a Darboux type theorem for a certain class of multisymplectic (n+2)-form in a (n+2m)-dimensional manifold, generalizing the work for 3-forms in 5-dimensional space. In addition to closedness of the form, the involutivity of a certain distribution is a necessary condiition for flatness. When the distribution is zero or everything we recover the classical Moser and Darboux theorems for volume resp. symplectic forms. This is joint work with Aliaksandr Hancharuk and Laura Leski.

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Formality theorem for differential graded manifolds

Auteur: Mathieu Stiénon[1]

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The Atiyah class of a dg manifold $(\mathcal{M}, Q)$ is the obstruction to the existence of an affine connection on the graded manifold $\mathcal{M}$ that is compatible with the homological vector field Q. The Todd class of dg manifolds extends both the classical Todd class of complex manifolds and the Duflo element of Lie theory. Using Kontsevich's famous formality theorem, Liao, Xu and I established a formality theorem for smooth dg manifolds: given any finite-dimensional dg manifold $(\mathcal{M}, Q)$, there exists an L_∞ quasi-isomorphism of differential graded Lie algebras from the space of polyvector fields on $\mathcal{M}$ endowed with the Schouten bracket $[-,-]$ and the differential $[Q,-]$ to the space of poly-differential operators on $\mathcal{M}$ endowed with the Gerstenhaber bracket $[-,-]$ and the differential $[m+Q,-]$, whose first Taylor coefficient (1) is equal to the composition of the action of the square root of the Todd class of the dg manifold $(\mathcal{M}, Q)$ with the Hochschild–Kostant–Rosenberg map and (2) preserves the associative algebra structures on the level of cohomology. As an application, we proved the Kontsevich–Shoikhet conjecture: a Kontsevich–Duflo type theorem holds for all finite-dimensional smooth dg manifolds. This last result shows that, when understood in the unifying framework of dg manifolds, the classical Duflo theorem of Lie theory and the Kontsevich–Duflo theorem for complex manifolds are really just one and the same phenomenon.

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Regularity for solutions of H-systems and n-harmonic maps with n/2 square integrable derivatives

Auteur: Pawel Strzelecki[1]

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Over 30 years ago Frédéric Hélein proved that all harmonic maps from surfaces into compact Riemannian manifolds are smooth. Despite the existence of several partial results, for $n > 2$ the counterpart of this theorem is wide open. In a recent work with two coauthors, Michał Miśkiewicz and Bogdan Petraszczuk, we prove regularity of n-harmonic maps into compact Riemannian manifolds and weak solutions to H-systems in dimension n, under an extra assumption: that $n/2$-th derivatives of the solution are square integrable. The tools used in the proof involve, as one might guess, Hardy spaces and BMO, and the Rivière–Uhlenbeck decomposition (with estimates in Morrey spaces). A particularly prominent role is played by the Coifman–Rochberg–Weiss commutator theorem.

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Pinching theorems via Ricci flow

Auteur: Peter Topping[1]

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There is a long tradition in Differential Geometry of results that deduce topological consequences from pointwise positive curvature hypotheses. In this talk we consider the consequences of pinched curvature of various types and explain how recent developments in Ricci flow have proved to be decisive in establishing results along these lines.
Joint work with ManChun Lee (CUHK)

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Tight Best-Lipschitz Maps from Surfaces to Surfaces

Auteur: Karen Uhlenbeck[1]

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In honor of the contributions of Frederic Helein to analysis, I pose several new problems rather than present solutions. George Daskalopoulos and I have been studying best Lipschitz maps from surfaces to surfaces. While this is motivated by Thurston's distance function in Teichmuller space, it has connections with older ideas. I will give a bit of the history about Lipschitz extensions, define the notion of tight, remind the listeners about infinity harmonic functions, and describe our approximation scheme for infinity harmonic mappings. The goal is to motivate several interesting new and I believe hard questions in analysis.

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On the understanding of the Jacobian determinant

Auteur: Dong Ye[1]

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I will talk about the problem to prescribe the Jacobian determinant or the volume form, I will present some old and more recent results on this problem, and mention the link with the density problem in Sobolev spaces.

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