

Dirac operators for the BGG category \mathcal{O}

Thursday, December 8, 2022 3:00 PM (50 minutes)

Dirac operators were used in the context of Representation Theory by Parthasarathy in 1972, as invariant first order differential operators acting on sections of homogeneous vector bundles over symmetric spaces G/K in order to obtain realizations of the discrete series representations of G .

In a series of lectures in 1997, Vogan introduced an algebraic analogue of Parthasarathy's Dirac operator. By using this operator, he defined the so-called Dirac cohomology of (\mathfrak{g}, K) -modules X and conjectured a relation between the Dirac cohomology of X and its infinitesimal character, proved by Huang and Pandžić in 2001. Since then, Dirac cohomology has been computed for various families of modules, including highest weight modules, $A_{\mathfrak{q}}(\lambda)$ modules, generalized Enright-Varadarajan modules, unipotent representations, etc.

In this talk, we will present some results concerning Dirac operators for modules belonging to the standard BGG category \mathcal{O} of a complex semisimple Lie algebra \mathfrak{g} . This category consists of the finitely generated, locally \mathfrak{n} -finite weight modules of \mathfrak{g} and seems to be the "correct" module category to study questions raised by Verma concerning composition series and embeddings of Verma modules, and Jantzen concerning his so-called translation functors.

Presenter: AFENTOULIDIS, Spyros