

# Dirac operators for the BGG category $\mathcal{O}$

*jeudi 8 décembre 2022 15:00 (50 minutes)*

Dirac operators were used in the context of Representation Theory by Parthasarathy in 1972, as invariant first order differential operators acting on sections of homogeneous vector bundles over symmetric spaces  $G/K$  in order to obtain realizations of the discrete series representations of  $G$ .

In a series of lectures in 1997, Vogan introduced an algebraic analogue of Parthasarathy's Dirac operator. By using this operator, he defined the so-called Dirac cohomology of  $(\mathfrak{g}, K)$ -modules  $X$  and conjectured a relation between the Dirac cohomology of  $X$  and its infinitesimal character, proved by Huang and Pandžić in 2001. Since then, Dirac cohomology has been computed for various families of modules, including highest weight modules,  $A_{\mathfrak{q}}(\lambda)$  modules, generalized Enright-Varadarajan modules, unipotent representations, etc.

In this talk, we will present some results concerning Dirac operators for modules belonging to the standard BGG category  $\mathcal{O}$  of a complex semisimple Lie algebra  $\mathfrak{g}$ . This category consists of the finitely generated, locally  $\mathfrak{n}$ -finite weight modules of  $\mathfrak{g}$  and seems to be the "correct" module category to study questions raised by Verma concerning composition series and embeddings of Verma modules, and Jantzen concerning his so-called translation functors.

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