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Emanuel Milman: Multi-Bubble Isoperimetric Problems - Old and New

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The classical isoperimetric inequality in Euclidean space R^n states that among all sets of prescribed volume, the Euclidean ball minimizes surface area. One may similarly consider isoperimetric problems for more general metric-measure spaces, such as on the n-sphere S^n and on n-dimensional Gaussian space G^n (i.e. R^n endowed with the standard Gaussian measure). Furthermore, one may consider the "multi-bubble"isoperimetric problem, in which one prescribes the volume of $p \geq 2$ bubbles (possibly disconnected) and minimizes their total surface area –as any mutual interface will only be counted once, the bubbles are now incentivized to clump together. The classical case, referred to as the single-bubble isoperimetric problem, corresponds to p=1; the case p=2 is called the double-bubble problem, and so on.

In 2000, Hutchings, Morgan, Ritoré and Ros resolved the double-bubble conjecture in Euclidean space R^3 (and this was subsequently resolved in R^n as well) –the boundary of a minimizing double-bubble is given by three spherical caps meeting at 120°-degree angles. A more general conjecture of J. Sullivan from the 1990's asserts that when $p \leq n+1$, the optimal multi-bubble in R^n (as well as in S^n) is obtained by taking the Voronoi cells of p+1 equidistant points in S^n and applying appropriate stereographic projections to R^n (and backwards).

In 2018, together with Joe Neeman, we resolved the analogous multi-bubble conjecture for $p \leq n$ bubbles in Gaussian space G^n–the unique partition which minimizes the total Gaussian surface area is given by the Voronoi cells of (appropriately translated) p+1 equidistant points. In the present talk, we describe our recent progress with Neeman on the multi-bubble problem on R^n and S^n. In particular, we show that minimizing bubbles in R^n and S^n are always spherical when $p \leq n$, and we resolve the latter conjectures when in addition $p \leq 5$ (e.g. the triple-bubble conjectures when $n \geq 3$ and the quadruple-bubble conjectures when $n \geq 4$).

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