Stability and applications to birational and hyperkaehler geometry

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Résumé : This lecture series will be an introduction to stability conditions on derived categories, wall-crossing, and its applications to birational geometry of moduli spaces of sheaves. I will assume a passing familiarity with derived categories.

— Introduction to stability conditions. I will start with a gentle review of aspects of derived categories. Then an informal introduction to Bridgeland's notion of stability conditions on derived categories [2, 5, 6]. I will then proceed to explain the concept of wall-crossing, both in theory, and in examples [1, 2, 4, 6].

— Wall-crossing and birational geometry. Every moduli space of Bridgeland-stable objects comes equipped with a canonically defined nef line bundle. This systematically explains the connection between wall-crossing and birational geometry of moduli spaces. I will explain and illustrate the underlying construction [7].

— Applications : Moduli spaces of sheaves on K3 surfaces. I will explain how one can use the theory explained in the previous talk in order to systematically study the birational geometry of moduli spaces of sheaves, focussing on K3 surfaces [1, 8].

References :

1. D. Arcara, A. Bertram, I. Coskun, J. Huizenga, "The minimal model program for the Hilbert scheme of points on P^2 and Bridgeland stability", Adv. Math., 235 :580-626, 2013. (arXiv :math/1203.0316) ;

2. T. Bridgeland, "Stability condition on triangulated categories", Annals of Math. 166 no. 2, 317-345 (2007) (arXiv :math/0212237) ;

3. T. Bridgeland, "Spaces of stability conditions", Algebraic geometry-Seattle 2005. Part 1, 1-21, Proc. Sympos. Pure Math., 80, AMS. (arXiv :math/0611510) ;

4. T. Bridgeland, "Stability conditions on K3 surfaces", Duke Math. J. 141, no. 2, 241-291 (2008) (arXiv :math/0307164) ;

5. A. Caldararu, "Derived categories of sheaves : a skimming", In Snowbird lectures in algebraic geometry, volume 388 of Contemp. Math., pages 43-75. AMS (arXiv :math/0501094) ;

6. A. Bayer, "A tour to stability conditions on derived categories", Informal notes available on my homepage ;

7. A. Bayer, E. Macri, "Projectivity and birational geometry of Bridgeland moduli spaces", (arXiv :1203.4613) ;

8. A. Bayer, E. Macri, "MMP for moduli of sheaves on K3s via wall-crossing : nef and movable cones, Lagrangian fibrations".