Journée de rentrée 2023 du GT CalVa

Rapport sur les contributions

https://indico.math.cnrs.fr/e/8753

ID de Contribution: **5** Type: **Non spécifié**

Recent results of stability in functional inequalities

lundi 16 janvier 2023 10:00 (1 heure)

The lecture is devoted to a review of some stability results in simple interpolation inequalities, typically Gagliardo-Nirenberg-Sobolev inequalities. When optimal constants are known and optimal functions are characterized, it is a natural question to ask whether the deficit, that is, the difference of the two sides of the inequality, controls a distance to the set of optimal functions. After the work of Bianchi and Egnell on Sobolev inequalities, more than 30 years ago, many authors developped similar results, where the analysis of the linearized problem and compactness methods play an essential role. A major drawback is that explicit estimates usually cannot be obtained by such methods. In the recent years, the efforts have been focused on the quantitative properties, for instance, the optimal power of the distance controlled by the deficit. The results presented here go one step beyond this and are intended to give explicit estimates on the stability constants in Bianchi-Egnell type estimates by various methods.

Orateur: DOLBEAULT, Jean (Ceremade, Université Paris-Dauphine)

ID de Contribution: **6** Type: **Non spécifié**

Minimizing non local functionals on measures. Relaxation and asymptotics.

lundi 16 janvier 2023 11:00 (1 heure)

Optimization problems on probability measures in $\mathbb{R}^d$ are considered where the cost functional involves multi-marginal optimal transport. In a model of N interacting particles, the interaction cost is repulsive and described by a two-point function $c(x,y) = \ell(|x-y|)$ where $\ell : \mathbb{R}_+ \to [0,\infty]$ is decreasing to zero at infinity. Due to a possible loss of mass at infinity, non existence may occur and relaxing the initial problem over sub-probabilities becomes necessary. In this talk we will describe the relaxed functional to be minimized as well as its $\Gamma-$limit as $N \to \infty$. Then we study the limit optimization problem when a continuous external potential is applied. Conditions are given with explicit examples under which minimizers are probabilities or have a mass < 1 . In a last part we consider the case of a infinitesimal range interaction cost $\ell_N(r) = \ell(r/e)$ ($e \ll 1$) with the aim of determining the mean-field limit energy as $e \to 0$ of a very large number N_e of particles confined in a given compact subset of $\mathbb{R}^d$.

Orateur: BOUCHITTÉ, Guy (IMATH, Université de Toulon)

ID de Contribution: **7** Type: **Non spécifié**

On a nonlinear Schrödinger equation: uniqueness, non-degeneracy and applications to energy minimization

lundi 16 janvier 2023 13:30 (1 heure)

In this talk, I will first state a general result about the uniqueness and the non-degeneracy of positive radial solutions to some semi-linear elliptic equations $-\Delta u = g(u)$. Then I will consider the case of the double power non-linearity $g(u) = u^q - u^p - \mu u$ for $p > q > 1$ and $\mu > 0$. In this case, the non-degeneracy of the unique solution u_μ allows us to derive its behavior in certain regimes of the parameter μ. This implies the uniqueness of energy minimizers at fixed mass. Joint work with Mathieu Lewin.

Orateur: ROTA NODARI, Simona (Laboratoire J.A. Dieudonné, Université Côte d'Azur)

ID de Contribution: **8** Type: **Non spécifié**

Numerical approximation of mean curvature flows using neural networks

lundi 16 janvier 2023 15:00 (1 heure)

The mean curvature flow plays an important role in many applications in physics or numerical engineering, for example in image processing or material science. The talk will focus on new neural network-based numerical methods for approximating the mean curvature flow of general interfaces, both oriented and non-oriented. To learn the correct evolution law, our networks are trained on implicit representations of exact interface evolutions. The structure of the networks is very simple and is inspired by some splitting schemes used for the discretization of the Allen-Cahn equation, but while the latter only allows to approximate the mean curvature flow of oriented domain boundaries, our networks can also adapt to the non-orientable case. And although trained only on regular flows, they correctly handle different types of singularities. Moreover, they can be easily coupled to various constraints. The talk will show several applications that illustrate the interest of the approach: mean curvature flow with volume constraint, multiphase mean curvature flow, Steiner tree approximation, minimal surface approximation. The extension to the anisotropic mean curvature flow and the identification of anisotropies will also be discussed. This is a joint work with Elie Bretin, Roland Denis and Garry Terii.

Orateur: MASNOU, Simon (ICJ, Université Claude Bernard Lyon 1)

ID de Contribution: **9** Type: **Non spécifié**

Boundary stabilization of cross-diffusion systems in moving domains

lundi 16 janvier 2023 16:00 (1 heure)

(joint work with Jean Cauvin-Vila and Amaury Hayat)

This work is motivated by a collaboration with the French Photovoltaic Institute. The aim of the project is to propose a model in order to simulate and optimally control the fabrication process of thin film solar cells. The production of the thin film inside of which occur the photovoltaic phenomena accounting for the efficiency of the whole solar cell is done via a Physical Vapor Deposition (PVD) process. More precisely, a substrate wafer is introduced in a hot chamber where the different chemical species composing the film are injected under a gaseous form. Molecules deposit on the substrate surface, so that a thin film layer grows by epitaxy. In addition, the different components diffuse inside the bulk of the film, so that the local volumic fractions of each chemical species evolve through time. The efficiency of the final solar cell crucially depends on the final chemical composition of the film, which is freezed once the wafer is taken out of the chamber. A major challenge consists in optimizing the fluxes of the different atoms injected inside the chamber during the process for the final local volumic fractions in the layer to be as close as possible to target profiles. Two different phenomena have to be taken into account in order to correctly model the evolution of the composition of the thin film: 1) the cross-diffusion phenomena between the various components occuring inside the bulk; 2) the evolution of the surface. As a consequence, the underlying model reads as a cross-diffusion system defined on a moving boundary domain. The complete optimal control problem of the fluxes injected in the hot chamber is currently out-of-reach in terms of mathematical analysis. The aim of this talk is to theoretically investigate a simpler problem, which is the boundary stabilization of the model used to simulate the PVD process. We show first exponential stabilization and then finite-time stabilization in arbitrary small time of the linearized system around uniform equilibria, provided the underlying cross-diffusion system has an entropic structure with a symmetric mobility matrix. This stabilization is achieved with respect to both the volumic fractions of the different chemical species composing the thin film and the thickness of the latter. The feedback control is derived using the backstepping technique, adapted to the context of a time-dependent domain. In particular, the norm of the backward backstepping transform is carefully estimated with respect to time.

Orateur: EHRLACHER, Virginie (Cermics, ENPC)