ID de Contribution: **34** Type: **Non spécifié**

The One-sided Cycle Shuffles in the Symmetric Group Algebra (Zoom)

mardi 29 novembre 2022 10:15 (45 minutes)

We study a new family of elements in the group ring of a symmetric group –or, equivalently, a class of ways to shuffle a deck of cards.
Fix a positive integer n. Consider the symmetric group S_n. For each 1 n, we define an element

$t := cyc + cyc_{,+1} + cyc_{,+1,+2} + \cdot \cdot \cdot + cyc_{,+1,...,n}$

of the group ring $\mathbb{R}[S_n]$, where $cyc_{i_1,i_2,...,i_k}$ denotes the cycle that rotates through the given elements $i_1, i_2, ..., i_k$. We refer to these n elements $t_1, t_2, ..., t_k$ as the $\underline{somewhere-to-belowshuffles}$, since the standard interpretation of elements of $\mathbb{R}[S_n]$ in terms of card shuffling allows us to view them as shuffling operators. Note that t_1 is the well-known $\underline{top-to-randomshuffle}$ studied by Diaconis, Fill, Pitman and others, whereas $t_n = id$.
Similar families of elements of $\mathbb{R}[S_n]$ include the Young-Jucys-Murphy elements, the Reiner-Saliola-Welker elements, and the Diaconis-Fill-Pitman elements. Unlike the latter three families, the somewhere-to-below shuffles $t_1, t_2, ..., t_n$ do not commute. However, they come close to commuting: there is a basis of $\mathbb{R}[S_n]$ on which they all act as upper-triangular matrices; thus, they generate an algebra whose semisimple quotient is commutative (which entails, in particular, that their commutators are nilpotent).
This basis can in fact be constructed combinatorially, and bears several unexpected connections, most strikingly to the Fibonacci sequence. One of the consequences is that any $\mathbb{R}$-linear combination $_1t_1 + {}_2t_2 + \cdot \cdot \cdot + {}_nt_n$ (with $_1, {}_2, ..., {}_n$ $\mathbb{R}$) can be triangularized and its eigenvalues explicitly computed (along with their multiplicities); the number of distinct eigenvalues is at most the Fibonacci number $\int_{n+1}$. If all these $\int_{n+1}$ eigenvalues are indeed distinct, then the matrix is diagonalizable.
While we have been working over $\mathbb{R}$ for illustrative purposes, all our proofs hold over any commutative ring (or, for the diagonalizability claim, over any field). Several open questions remain (joint work with Nadia Lafreniere).

Orateur: Prof. GRINBERG, Darij (Drexel University)