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Hausdorff Moment Problems for Combinatorial Numbers: Heuristics via Meijer G-functions

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We report on further investigations of combinatorial sequences in form of integral ratios of factorials. We conceive these integers as Hausdorff power moments for weights $W(x)$, concentrated on the support $x\ (0, R)$, and we solve this moment problem by furnishing the exact expressions for $W(x)'s$. In many instances, we can formally prove that the sequences are positive definite. We considered a large set of families of such sequences including formulas of Tutte et al. for enumerations of planar maps, several generalizations of Catalan numbers such as Fuss-Catalan and Raney numbers, the constellation numbers, and the ratios of multiple factorials, such as the iconic Kontsevich ($\frac{(6n)!n!}{(3n)![(2n)!]^2}$) and Chebyshev ($\frac{(30n)!n!}{(6n)!(10n)!(15n)!}$) sequences. Furthermore, we provide the exact solutions for all three parametrized families of Bober ratios (2009) of factorials, as well as for the ”sporadic”ratios, for all of which the ordinary generating functions (ogf) are algebraic. Finally, in the same spirit, we studied the sequences recently constructed by Rodriquez Villegas (2019-2022), including ($\frac{(63n)!(8n)!(2n)!}{n!(4n)!(16n)!(21n)!(31n)}$). In all the cases listed above, we have identified a precisely defined and persistent pattern relating the Meijer G-encodings of appropriate ogf $G(z)$ and of $W(x)$. In fact, it appears that in the language of Meijer G-functions, the solutions $W(x)$ are practically automatically obtained by reshuffling of data characterizing the ogf $G(z)$ only, i.e. the parameter lists and its radius of convergence R^{-1}. We attempt to categorize these observations and try to find the criteria for moments that would allow for such an automatization. It is also intriguing that the counterexamples can be found, which clearly point to the limits of this procedure. Finding the precise criteria for moments that would permit for such a speedy method, is still a challenging open problem.

Collaboration at various stages of this work with N. Behr, G. H. E. Duchamp, K. Gorska, M. Kontsevich, and G. Koshevoy.

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