Tai-Peng TSAI (University of British Columbia)

Forward Self-Similar and Discretely Self-Similar Solutions of the 3D incompressible Navier-Stokes Equations

For 3D incompressible Navier-Stokes Equations in the whole space, the existence of forward self-similar solutions with *large* data was only shown recently by Jia and Sverak. They are of interest since they may not be unique, corresponding to stationary or Hopf bifurcations for the associated Leray equations. In this talk I will present 3 constructions, all based on a priori bounds but obtained in different ways.

The first is that of Jia and Sverak based on Holder estimates near initial time for local Leray solutions of Lemarie-Rieusset. This approach was adapted by myself to construct discretely self-similar (DSS) solutions either with DSS factor sufficiently close to 1 or with axisymmetric data. These solutions are necessarily regular. This method is not applicable in the half space due to lack of the LR theory.

The second construction is by Korobkov and myself, which also works in the half space. In the half space the local Leray solution theory is not known, and we get an a priori H^1 bound by using Leray's classical contradiction argument and reducing the problem to a Liouville problem of Euler equations in the half space. This construction does not work for DSS solutions.

The third construction is by Bradshaw and myself. It is a weak solution theory, based on a new explicit a priori bound for the Leray equations. It works for SS and DSS solutions with any weak L^3 data and no restriction on the DSS factor. It is also valid in the half space. Such solutions may not be regular and are good candidates for the failure of eventual regularity.

Finally I will show that the third construction also gives Rotated SS/DSS solutions for general weak L^3 data. The nonexistence problem of *backward" rotated self-similar solutions in weak L^3 was proposed by Grisha Perelman and is still open. I will present the relevant backward problems briefly.