A.N. Shiryaev and Contemporary Probability Theory

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Recueil des résumés

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Optimization problem for a portfolio with an illiquid asset: Lie group analysis

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Management of a portfolio that includes an illiquid asset is an important problem of modern mathematical finance. One of the ways to model illiquidity among others is to build an optimization problem and assume that one of the assets in a portfolio can not be sold until a certain finite, infinite or random moment of time. This approach arises a certain amount of models that are actively studied at the moment.
Working in the Merton's optimal consumption framework with continuous time we consider an optimization problem for a portfolio with an illiquid, a risky and a risk-free asset. Our goal in this paper is to carry out a complete Lie group analysis of PDEs describing value function and investment and consumption strategies for an portfolio with an illiquid asset that is sold in a random moment of time with a prescribed liquidation time distribution. Study of
optimization problems with an illiquid asset leads to three dimensional non-linear Hamilton-Jacobi-Bellman (HJB) equations. Such equations are not only tedious for analytical methods but are also quite challenging form a numeric point of view. To reduce the three-dimensional problem to a two-dimensional one or even to an ODE one uses some substitutions, yet the methods used to find such substitutions are rarely discussed by the authors.
We find the admitted Lie algebra for a certain class of liquidation time distributions in cases of HARA and log utility functions and formulated theorems
for these cases. We use them to obtain corresponding reductions. Several of these substitutions were used in other papers before and other ones are new to our knowledge. This method gives us the possibility to provide a complete set of non-equivalent substitutions and reduced equations.

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Inversion, duality and h-processes of self-similar Markov processes

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We show that any $\mathbb{R}^d$-valued self-similar Markov process X with index $\alpha > 0$ absorbed at 0, can be represented as a path transformation of some Markov additive process (MAP) (θ, ξ) in $S_{d-1} \times \mathbb{R}$.
This result extends the well known Lamperti transformation. Then we prove that the same transformation of the dual MAP in the weak sense of (θ, ξ) is itself in weak duality with X, with respect to the
measure $\pi(x/\|x\|)\|x\|^{\alpha-d}dx$, if and only if (θ, ξ) is reversible with respect to the measure $\pi(s)ds$, where ds is
the Lebesgue measure on S_{d-1}. Besides, the dual process $\widehat{X}$ has the same law as the inversion $(X_{\gamma_t}/\|X_{\gamma_t}\|^2, t \geq 0)$ of X, where γ_t is the inverse of $t \mapsto \int_0^t \|X\|_s^{-2\alpha}\, ds$.

As an application, we prove that in some instances, the Kelvin transform of X can be obtained as an h-transform of some functional of X.

This is a joint work with Larbi Alili, Piotr Graczyk and Tomasz Zak.

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Two price valuation theory

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In classical economic theory the law of one price prevails and market participants trade freely in both directions at the same price. This approach is appropriate for highly liquid markets. In the absence of perfect liquidity, the law of one price has to be replaced by a two price economy where market participants continue to trade freely with the market but the terms of trade now depend on the direction of the trade.
We give an introduction to this new approach. The two prices are termed bid and ask or lower and upper price but they should not be confused with the literature relating bid-ask spreads to transaction costs or other frictions involved in modeling financial markets. The two prices are determined in a non marketclearing equilibrium with a view to make loss exposures acceptable. Acceptability is defined via a positive expectation under a family of test measures or scenarios. As a result the bid price is the infimum of test valuations whereas the ask price is the supremum of such valuations. The two prices are related to nonlinear expectation operators. We consider examples where the uncertainty is given by purely discontinuous Lévy processes. Various aspects such as liquidity measurement and portfolio theory are discussed. Finally we present a defaultable asset price model (DAM) in the context of the two price valuation.

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On the Martingale Property of Local Martingales: When a Local Martingale is a True Martingale?

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For many problems in the theory of stochastic processes and its applications it is of great importance to know effective
criteria ensuring that a given local martingale is a true martingale or even a uniformly integrable martingale. This question is closely related to absolute continuity of probability measures and change

of measure and has been the subject of research over many decades. Starting from an arbitrary non-negative local martingale, the aim of the present talk is to give necessary and sufficient conditions in terms of another but intrinsic probability which is locally equivalent to the given one. The results and the verifiability of the conditions will be illustrated by several applications. First we shall consider a geometric Ornstein-Uhlenbeck financial market. Then we pass on to general stochastic exponentials of continuous local martingales and it will be demonstrated that Novikov's and Kazamaki's conditions are simple consequences of ours. Special examples are discussed for stochastic exponentials of solutions
of one-dimensional SDEs without drift in which case purely analytical criteria in terms of the diffusion coefficient are obtained. In particular, if these criteria are not satisfied, then the corresponding process is a strict local martingale which is also referred to as a bubble in the mathematical finance literature.

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Existence and uniqueness of viscosity solutions for second order integro-differential equations without monotonicity condition

Auteur: Said Hamadène[1]

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In this talk, we discuss a new existence and uniqueness result of a continuous viscosity solution for integro-partial differential equation (IPDE in short).
The novelty is that we relax the so-called monotonicity assumption on the driver which is classically assumed in the literature of viscosity solution of equation with a non local term. Our method is based on the link of those IPDEs with backward stochastic differential equations (BSDEs in short) with jumps for which we already know that the solution exists and is unique.

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Estimation of volatility in presence of high activity jumps, noise and irregular sampling

Auteur: Jean Jacod[1]

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We consider an Itô semimartingale which is observed along a discrete time grid, within a fixed time interval. The observations are contaminated by noise, and the semimartingale has jumps with a degree of activity bigger than 1. Our aim is to revisit the estimation of the integrated volatility in such a setting: we use a mixture of the pre-averaging method (to eliminate noise) and of the empirical characteristic function method, which has been shown to be efficient (after proper de-biasing) even when the jump activity is bigger than 1, in contrast with most other methods.

This is a joint work with Viktor Todorov.

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On one application of the Cherny-Shiryaev criterion of stochastic integrability

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The Cherny-Shiryaev criterion provides a description of predictable processes which are integrable with
respect to a vector-valued semimartingale in terms of its local characteristics.
We provide an example how this result can be used in the problem of existence of local martingale numéraire
in a model of financial market which has no asymptotic arbitrage opportunities of the first kind.

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Mixed Gaussian processes: a filtering approach

Auteur: Marina Kleptsyna[1]

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We present a new approach to the analysis of mixed processes: for $t \in [0, T]$

$$X_t = B_t + G_t,$$

where B_t is a Brownian motion and G_t is an independent centered Gaussian process.
We obtain a new canonical innovation representation of X, using linear filtering theory.

When the kernel

$$K(s,t) = \frac{\partial^2}{\partial s \partial t} EG_t G_s, \quad s \neq t$$

has a weak singularity on the diagonal, our results generalize the classical innovation formulas beyond the square integrable setting.
For kernels with stronger singularity, our approach is applicable to processes with additional "fractional" structure,
including the mixed fractional Brownian motion from mathematical finance.
We show how previously known measure equivalence relations and semimartingale properties follow from our canonical representation in a unified way, and complement them with new formulas for Radon-Nikodym densities.

3

Probabilistic interpretation for Fully nonlinear Stochastic PDEs

Auteur: Anis Matoussi[1]

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We present an overview on different classes of nonlinear stochastic partial differential equations (SPDEs in short). In particular, we focus on providing a probabilistic representation of solution of Fully nonlinear SPDEs (stochastic Viscosity solutions) by means of solution of the associated class of Second order BSDEs. This presentation includes the numerical study of quasilinear and semi-linear SPDEs (the time discretization error and numerical tests) and some applications in pathwise stochastic control problems arising in finance.

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Valuation, put-call parity and bubbles

Auteur: Martin Schweizer[1]

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We discuss a systematic approach to the valuation of general European contingent claims in general continuous-time financial markets. We want to provide bounds on economically reasonable valuations that do not depend too much on precise assumptions on the underlying primary assets. This allows us to provide a general result on put-call parity and to give an explanation for some pricing anomalies observed in the literature.
This is based on joint work with Martin Herdegen (ETH Zurich).

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Probability Distributions: Characterizations by Their Moments

Auteur: Jordan Stoyanov[1]

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The talk is on selected new results on uniqueness and non-uniqueness of distributions in terms of their moments. The results cover distributions of random variables, random vectors and stochastic processes, including solutions of SDEs. A couple of open questions will be outlined.

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Moral hazard and second order backward SDEs

Auteur: Nizar Touzi[1]

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We consider a general formulation of the Principal-Agent problem with a lump-sum payment on a finite horizon. Our main result is a reduction of this problem to a standard stochastic control problem, so that the principal's problem is solved by the standard tools of control theory. Our proofs rely on the Backward Stochastic Differential Equations approach to non-Markovian stochastic control, and more specifically, on the recent extensions to the second order case.

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On exponential functionals for processes with independent increments

Auteur: Lioudmila Vostrikova[1]

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We study the exponential functionals of
the processes with independent increments which are integrable semi-martingales with
absolutely continuous characteristics. We give necessary and sufficient conditions for existence of Laplace exponent, and also the
sufficient conditions of finiteness of the moments of exponential functionals. We derive
a recurrent integral equation for its Mellin transform and recurrent
formulas for the moments. In particular, we obtain the results for Levy subordinators given in the paper of Bertoin, Yor (2005).

This is joint work with Paavo Salminen (Turku University, Finland).