

Khintchine's inequality: old and new



Labex MME-DII

Modèles Mathématiques et Économiques de la
Dynamique, de l'Incertain et des Interactions

lundi 26 juin 2023 - vendredi 30 juin 2023

IHP

Programme Scientifique

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\bf \mbox{On the Khintchine inequality and its relatives}
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Approximately a century ago Alexandre Khintchine formulated the law of the iterated logarithm. One of the tools developed on the way to proving it was an important moment comparison inequality for Rademacher sums, which became known as the Khintchine inequality. In a slightly more modern phrasing it says that for every $p > q > 0$ there exists a constant $C_{p,q} > 0$ such that

$$\| \sum_{i=1}^n \epsilon_i S_i^p \|_{L^p} \leq C_{p,q} \cdot \| \sum_{i=1}^n \epsilon_i S_i^q \|_{L^q}$$

for every linear combination of independent symmetric $\{-1,1\}$ -valued random variables.

Rademacher sums, as well as other low complexity functions of the independent symmetric bits, such as polynomials of low degree or norms of linear combinations with vector coefficients instead of the real ones, have been objects of intense studies for many decades. They found use not only in probability theory, but also in areas as diverse as functional analysis, harmonic analysis, convex geometry, combinatorics, and theoretical computer science. Their quite elementary structure makes them a natural starting point in dealing with more complicated expressions. By centering, symmetrization and conditioning techniques, many mathematical problems can be, partially or even completely, reduced to their i.i.d. symmetric ± 1 counterparts.

We plan to present a number of proofs of the Khintchine inequality and its variants, such as the Kahane-Khintchine inequality or moment comparison bounds for sums of independent random variables and Rademacher chaoses. Many related results will be discussed, including very classical ones, such as the Rosenthal inequality and Kwapien's theorem. We will also discuss moment comparison inequalities appearing in the problem of finding extremal (in terms of volume) sections and projections of balls in L_p^n norms. However, the main effort will be to demonstrate a variety of methods, notions, approaches, and connections between seemingly distant branches of mathematics, while keeping the exposition accessible to graduate students.

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	\mbox{Monday 26}	\mbox{Tuesday 27}	\mbox{Wednesday 28}	\mbox{Thursday 29}	\mbox{Friday 30}
	10h-12h	Nayar	Nayar	Nayar	Nayar
	13h45-15h45	Oleszkiewicz	Oleszkiewicz	Oleszkiewicz	Oleszkiewicz

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