The stochastic mass conserved Allen-Cahn equation with nonlinear diffusion

We study the initial boundary value problem for the stochastic nonlocal Allen-Cahn equation on an open bounded set D of $\mathbb{R}^n$ with a smooth boundary ∂D:

$$(P) \quad \begin{cases} \partial_t u = div(A(\nabla u)) + f(u) - \dfrac{1}{|D|}\displaystyle\int_D f(u) + \dot{W}(x,t), & x \in D, \quad t \geq 0 \\ A(\nabla u).n = 0, & \text{on} \quad \partial D \times \mathbb{R}^+ \\ u(x,0) = u_0(x), & x \in D \end{cases}$$

- The operator A is Lipschitz continuous from $\mathbb{R}^n$ to $\mathbb{R}^n$, A is coercive and $f(u) = u - u^3$;
- $W(x,t) = \sum_{k=1}^{\infty} \sqrt{\lambda_k}\beta_k(t)e_k(x)$ is a Q-Brownian motion.

The deterministic problem in the case of linear diffusion was introduced by Rubinstein and Sternberg as a model for the phase separation in a binary mixture, and the well-posedness and the stabilization of the solution for large times were proved by Boussaïd, Hilhorst and Nguyen.
A singular limit of the stochastic Problem (P) with a small parameter and linear diffusion has been studied by [1] to model the motion of a droplet. In this talk, we prove the existence and uniqueness of the weak solution of Problem (P); this had remained as an open problem even in the linear diffusion case studied by [1].

The first step is to perform the change of unknown function $v(t) = u(t) - W_A(t)$ where W_A is the solution of the corresponding stochastic heat equation with nonlinear diffusion.
We apply a Galerkin method, and search for suitable a priori estimates. We deduce that the approximate solution v_m weakly converges along a subsequence to a limit $\bar{v}$ as $m \to \infty$. The main problem is then to identify the limit of the terms $div(A(\nabla(v_m + W_A)))$ and $f(v_m + W_A)$ as $m \to \infty$, which we do by means of the monotonicity method [2]. We also prove the uniqueness of the weak solution. This is joint work with D. Hilhorst and K. Lee.

Références

[1] D.C. Antonopoulou, P.W. Bates, D. Blömker and G.D. Karali, *Motion of a droplet for the stochastic mass-conserving Allen-Cahn equation*, SIAM J. Math. Anal. 48, 670-708, 2016.

[2] N.V. Krylov and B.L. Rozovskii, *Stochastic evolution equations.* Stochastic differential equations: theory and applications, Interdiscip. Math. Sci., 2, World Sci. Publ., Hackensack, NJ, 1-69, 2007.