Persistence Homology in Symplectic and Contact Topology

Rapport sur les contributions

https://indico.math.cnrs.fr/e/8536

ID de Contribution: **1** Type: **Non spécifié**

Interlevel persistence in Floer theory

lundi 12 juin 2023 09:30 (1 heure)

The usual filtered Floer homology groups are formal analogues of the homologies of the sublevel sets of a Morse function on a manifold. In the Morse setting,
by instead considering interlevel sets (preimages of general intervals) one obtains an algebraic structure that is classified by a barcode that refines the usual
sublevel persistence barcode. I will describe an algebraic formalism that allows one to adapt this to Floer-theoretic settings. In this case of Hamiltonian Floer
theory this gives rise to a pairing between distinguished spectral invariants of a Hamiltonian flow and of its inverse that satisfies stability and duality theorems.

Orateur: USHER, Michael

ID de Contribution: **2** Type: **Non spécifié**

Coarse topology, oscillations and persistence

lundi 12 juin 2023 11:00 (1 heure)

Studying topology of the zero set of a polynomial map is a classical topic in algebraic geometry. One may attempt to extend this study to less regular objects, such as linear combinations of Laplace-Beltrami eigenfunctions or entire maps in several complex variables. A phenomenon which ultimately breaks the analogy is the existence of highly oscillatory behaviour on small scales. We will explain how, using persistence modules and barcodes, it is possible to systematically discard small oscillations and prove coarse analogues of certain classical theorems, such as Bézout's theorem, in the above mentioned setups. The talk is based on joint works with L. Buhovsky, J. Payette, I. Polterovich, L. Polterovich and E. Shelukhin.

Orateur: STOJISALVJEVIC, Vukasin

ID de Contribution: **3** Type: **Non spécifié**

Elementary SFT Spectral Invariants And The Strong Closing Property

vendredi 16 juin 2023 11:00 (1 heure)

In this talk, I will discuss ESFT spectral gaps: a new, general class of spectral invariants for large class of stable Hamiltonian manifolds and their cobordisms, in any dimension. They are built using a min-max construction applied to J-holomorphic curves in symplectic field theory. I will explain their formal properties and provide a holomorphic curve "closing criterion" for the ESFT gaps that implies a strong version of the smooth closing lemma. I will explain how to prove this criterion for flows that are "Hofer near periodic" and I will state some open problems.
This is joint work with Shira Tanny (IAS).

Orateur: CHAIDEZ, Julian

ID de Contribution: **4** Type: **Non spécifié**

Symplectic embeddings into convex toric domains (via Zoom)

lundi 12 juin 2023 16:00 (1 heure)

This is a report on joint work with Cristofaro-Gardiner and Magill, that investigates symplectic embeddings into toric domains with irrational convex boundary.

Orateur: MCDUFF, Dusa

ID de Contribution: **5** Type: **Non spécifié**

A conjectural chain model for positive S^1-equivariant symplectic homology of star-shaped toric domains in $\mathbb{C}^2$ (via Zoom)

mardi 13 juin 2023 09:30 (1 heure)

For any star-shaped toric domain in $\mathbb{C}^2$, we define a filtered chain complex which conjecturally computes positive S^1-equivariant symplectic homology of the domain.
Assuming this conjecture, we show that the sequence $(c_k^{\mathrm{GH}}(X)/k)_k$ has a limit as k goes to ∞, where c_k^{GH} denotes the k-th Gutt-Hutchings capacity.

Orateur: IRIE, Kei

ID de Contribution: **6** Type: **Non spécifié**

Contact Hamiltonian Floer homology and its applications

mardi 13 juin 2023 11:00 (1 heure)

In this talk, we will discuss a Floer-theoretic approach to studying Hamiltonian dynamics on contact manifolds, called contact Hamiltonian Floer homology. On the one hand, it provides a characterization of the rigidity of positive loops in the contactomorphism group which is a peculiar object often investigated in contact geometry. Mysteriously, our characterization highly relates to certain exotic behavior of the Floer continuation map in this setting. On the other hand, from an algebraic perspective, it admits a persistence module type upgrade (we call a gapped module), where one can read off numerical data from the associated barcode. These data help to define contact spectral invariant, contact boundary depth, etc., which also initiates the discovery of novel rigidity phenomenon of subsets in a contact manifold. This talk is based on joint work with Igor Uljarević.

Orateur: ZHANG, Jun

ID de Contribution: **7** Type: **Non spécifié**

Sesquicuspidal curves

mardi 13 juin 2023 14:30 (1 heure)

A classic question in algebraic geometry asks what are the possible singularities for a plane curve of a given degree and genus. This turns out to be closely connected with the theory of (stabilized) symplectic embeddings of ellipsoids. In this talk I will describe a construction (joint with D. McDuff) of new families of rational plane curves with desirable singularities. Key ingredients include a generalization of Orevkov's birational transformation and scattering diagrams for tropical curves.

Orateur: SIEGEL, Kyler

ID de Contribution: **8** Type: **Non spécifié**

The Toda lattice, billiards and the Viterbo conjecture (via Zoom)

mardi 13 juin 2023 16:00 (1 heure)

The Toda lattice is one of the earliest examples of non-linear completely integrable systems. Under a large deformation, the Hamiltonian flow can be seen to converge to a billiard flow in a simplex. In the 1970s, action-angle coordinates were computed for the standard system using a non-canonical transformation and some spectral theory. In this talk, I will explain how to adapt these coordinates to the situation of a large deformation and how this leads to new examples of symplectomorphisms of Lagrangian products with toric domains. In particular, we find a sequence of Lagrangian products whose symplectic systolic ratio is one and we prove that they are symplectic balls. This is joint work with Y. Ostrover and D. Sepe.

Orateur: RAMOS, Vinicius

ID de Contribution: **9** Type: **Non spécifié**

Spectral norm and conformally symplectic maps: two applications

mercredi 14 juin 2023 09:30 (1 heure)

I'll present two applications of the conformal invariance of the spectral norm: 1) a generalization of the Birkhoff attractor to higher dimension, 2) a weak version of the Nearby Lagrangian Conjecture. The completion of the space of Lagrangians with respect to the spectral norm plays a role in both cases.
This is joint work with Marie-Claude Arnaud and Claude Viterbo.

Orateur: HUMILIÈRE, Vincent

ID de Contribution: **10** Type: **Non spécifié**

On the Hofer-rigidity of iterations and continuous Hamiltonian torsion

mercredi 14 juin 2023 11:00 (1 heure)

We discuss a new rigidity phenomenon for iterations of Hamiltonian maps in the Hofer metric and in the C^0 metric. Joint work in progress with Nicholas Wilkins

Orateur: SHELUKHIN, Egor

ID de Contribution: **11** Type: **Non spécifié**

Lagrangian link quasimorphisms and the non-simplicity of Hameomorphism group of surfaces

jeudi 15 juin 2023 09:30 (1 heure)

In this talk, we will explain the construction of a sequence of homogeneous quasi-morphisms of the area-preserving homeomorphism group of the sphere using Lagrangian Floer theory for links. This sequence of quasi-morphisms has asymptotically vanishing defects, so it is asymptotically a homomorphism. It enables us to show that the Hameomorphism group is not the smallest normal subgroup of the area-preserving homeomorphism group.
If time permits, we will explain how to generalize it to all positive genus surfaces even though we no longer have quasi-morphisms.
The case of the sphere is joint work with Daniel Cristofaro-Gardiner, Vincent Humilière, Sobhan Seyfaddini, and Ivan Smith. The case of positive genus surfaces is joint work with Ibrahim Trifa.

Orateur: MAK, Cheuk Yu

ID de Contribution: **12** Type: **Non spécifié**

Symplectic Barriers

jeudi 15 juin 2023 11:00 (1 heure)

In his seminal 2001 paper, Biran introduced the concept of Lagrangian Barriers, a symplectic rigidity phenomenon coming from obligatory intersections with Lagrangian submanifolds which doesn't come from mere topology.
One simple example is that when removing a Lagrangian plane from the four dimensional ball, the symplectic capacity shrinks down to half of the original capacity of the ball.
Considering this example for a Lagrangian barrier, we ask how would the capacity change if one removes a plane which is not necessarily Lagrangian, or several such planes.
We show that indeed there are non-trivial obstructions which give the first example (as far as we know) of Symplectic Barriers: obligatory intersections of symplectic embeddings with symplectic submanifolds.
This is a joint work with Richard Hind and Yaron Ostrover.

Orateur: HAIM-KISLEV, Pazit

ID de Contribution: **13** Type: **Non spécifié**

On the strong Arnold chord conjecture for convex contact forms

jeudi 15 juin 2023 14:30 (1 heure)

Arnold conjectured that every closed Legendrian submanifold in the standard contact sphere S^{2n-1} with a contact form has a Reeb chord. This was confirmed by K. Mohnke in 2001. In fact, Arnold originally conjectured that a Reeb chord with distinct endpoints exists. I will give a proof of this strong version of the conjecture for convex contact forms, namely contact forms on S^{2n-1} induced by convex embeddings into $\mathbb{R}^{2n}$. I will also present a counterexample to the conjecture for nonconvex contact forms due to M. Hutchings.

Orateur: KANG, Jungsoo

ID de Contribution: **14** Type: **Non spécifié**

Completed signed barcodes and zeta functions for open domains

jeudi 15 juin 2023 16:00 (1 heure)

We describe some invariants of open domains related to bar codes coming from ECH and contact homology. These can be used for example to show that some open domains are not toric, and to distinguish some open toric domains which are difficult to distinguish otherwise.

Orateur: HUTCHINGS, Michael

ID de Contribution: **15** Type: **Non spécifié**

The local strong Viterbo conjecture

vendredi 16 juin 2023 09:30 (1 heure)

I will report on joint work in progress with Abbondandolo and Benedetti confirming the strong Viterbo conjecture for convex domains close to the ball in arbitrary dimension. The proof involves a quasi-invariant normal form for contact forms close to a Zoll one. I will explain how this normal form can be interpreted in terms of geodesics for symplectic Banach-Mazur type distances.

Orateur: EDTMAIR, Oliver

ID de Contribution: **16** Type: **Non spécifié**

Sheaves for the completion of the space of Lagrangians

lundi 12 juin 2023 14:30 (1 heure)

For a manifold M it is known that we can associate a sheaf on $M \times \mathbb{R}$ to any exact Lagrangian submanifold of T^*M. The space of exact Lagrangians carries the Viterbo's metric and, following Humili\'ere, we can consider its completion. We will see that the correspondence between sheaves and symplectic geometry extends to this framework and, moreover, natural notions defined for the objects on both sides coincide: to any element in the completion of the space of Lagrangians we can associate a sheaf; the notions of γ-support and microsupport coincide and the microsupport of any sheaf if γ-coisotropic.

These are joint works with Viterbo and Asano, Humilière, Ike, Viterbo.

Orateur: GUILLERMOU, Stéphane

ID de Contribution: **17** Type: **Non spécifié**

Inverse reduction inequalities

vendredi 16 juin 2023 14:00 (1 heure)

Classically symplectic reduction yields an estimate from below for the spectral distance between Lagrangians (for example in a cotangent bundle) by the spectral distance between reductions.
We shall show here how using a result by Kiselev-Shelukhin using some enriched structure on barcodes, one can obtain upper bounds on the spectral distance from above and give a number of applications and conjectures.

Orateur: VITERBO, Claude

ID de Contribution: **18** Type: **Non spécifié**

Welcome

lundi 12 juin 2023 09:00 (30 minutes)

ID de Contribution: **19** Type: **Non spécifié**

Guided visit of the town of Albi

mercredi 14 juin 2023 15:30 (2 heures)

ID de Contribution: **20** Type: **Non spécifié**

Cocktail at the Hôtel de Ville

mercredi 14 juin 2023 18:00 (1h 30m)