

Spectral theory for random Poincaré maps

Manon Baudel

We consider stochastic differential equations, obtained by adding weak Gaussian white noise to ordinary differential equations admitting N asymptotically stable periodic orbits. To quantify the rare transitions between periodic orbits, we construct a discrete-time, continuous-space Markov chain, called a random Poincaré map. We show that this process admits exactly N eigenvalues which are exponentially close to 1, and provide expressions for these eigenvalues and eigenfunctions in terms of committor functions of neighbourhoods of periodic orbits. The eigenvalues and eigenfunctions are well-approximated by principal eigenvalues and quasistationary distributions of processes killed upon hitting some of these neighbourhoods. The proofs rely on Feynman–Kac-type representation formulas for eigenfunctions, Doob’s h -transform, spectral theory of compact operators, and a detailed balance property satisfied by committor functions. Joint work with Nils Berglund (Orléans).