Lecture outline "Driven Diffusive Systems"

CIRM school Dynamiques stochastiques hors d'équilibre, 3-7 April 2017

G.M. Schütz

Institute of Complex Systems II, Forschungszentrum Jülich, 52425 Jülich, Germany
and
Interdisziplinäres Zentrum für Komplexe Systeme, Universität Bonn, Brühler Str. 7, 53119 Bonn, Germany

Abstract

These three lectures give an overview over some fundamental properties of one-dimensional driven diffusive systems far from thermal equilibrium which can be studied with exclusion processes by investigating their stationary properties and fluctuations of the density and of currents. We point out that writing the generator of these processes as a matrix is a convenient tool to obtain detailed rigorous results (Lecture 1). This approach is employed in Lecture 2 to elucidate the relation between reversibility, symmetries and duality, with applications to fluctuation theorems for the current and to the dynamical structure function of the symmetric simple exclusion process. Dynamical universality classes in multi-component systems are treated in Lecture 3. It is not the aim to present technical details of proofs, but to expose the underlying ideas which are symmetries, scaling arguments and the notion of dynamical universality.

1 Exclusion Processes: Motivation, Definition and Techniques

- Models of non-equilibrium behaviour: The simple exclusion process and other lattice gas models on a finite lattice (10')
- Phenomenology: Large-scale behaviour, phase transitions and dynamical scaling (15')
- The generator in the so-called "quantum Hamiltonian" formulation (35')

Literature and further reading:

- Schütz, G.M.: Exactly solvable models for many-body systems far from equilibrium. In: Domb, C., Lebowitz, J. (eds.) Phase Transitions and Critical Phenomena, Vol. 19, pp. 1-251. Academic Press, London (2001)

- Liggett, T.M., Interacting particle systems, Springer, Berlin, (1985); Stochastic Interacting Systems: Contact, Voter and Exclusion Processes, Springer, Berlin (1999).

- Lloyd, P., Sudbury, A., and Donnelly, P., Quantum operators in classical probability theory: I. "Quantum spin" techniques and the exclusion model of diffusion, Stoch. Processes Appl. **61**(2), 205–221 (1996).

2 Fluctuations of current and density

- Reversibility, Self-duality and Symmetry
- Application 1: Gallavotti-Cohen-Theorem for stochastic interacting particle systems
- Application 2: Dynamical structure function for the symmetric exclusion process

Literature and further reading:

- Schütz, G., and Sandow, S.: Non-abelian symmetries of stochastic processes: derivation of correlation functions for random vertex models and disordered interacting many-particle systems. Phys. Rev. E **49**, 2726–2744 (1994).

- Sudbury, A., and Lloyd, P., Quantum operators in classical probability theory. II: The concept of duality in interacting particle systems. Ann. Probab. 23(4), 1816–1830 (1995).

- Giardinà, C., Kurchan, J., Redig, F., and Vafayi, K.: Duality and Hidden Symmetries in Interacting Particle Systems, J. Stat. Phys. **135**, 25–55 (2009).

3 Multi-component systems

- Multi-lane exclusion processes (10')
- Linear hydrodynamics (10')
- Brief outline of nonlinear fluctuating hydrodynamics (20')

- General dynamical universality classes (20')

Literature and further reading:

- Schütz, G.M.: Critical phenomena and universal dynamics in one-dimensional driven diffusive systems with two species of particles, J. Phys. A **36**, R339 - R379 (2003)
  http://arxiv.org/pdf/cond-mat/0308450

- H. van Beijeren, Exact results for anomalous transport in one-dimensional Hamiltonian systems. Phys. Rev. Lett. **108**, 108601 (2012). http://arxiv.org/abs/1106.3298v4

- Spohn, H.: Nonlinear Fluctuating hydrodynamics for anharmonic chains. J. Stat. Phys. **154**, 1191–1227 (2014)
  http://arxiv.org/pdf/1305.6412v5

- Popkov, V., Schadschneider, A., Schmidt, J., Schütz, G.M.: Fibonacci family of dynamical universality classes PNAS **112**, 12645-12650 (2015)
  http://arxiv.org/pdf/1505.04461v2