Some thoughts on the information loss paradox

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2. S_{end} and S_{BH}
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6. Summary and discussion

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ref.) Buoninfante, Di Filippo and Mukohyama, JHEP 10 (2021) 081.
Mukohyama, *Phys.Rev.D* 58 (1998) 104023.

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ref.) Buoninfante, Di Filippo and Mukohyama, JHEP 10 (2021) 081.
Mukohyama, *Phys.Rev.D* 58 (1998) 104023.

Introduction

BH entropy

$$S_{bh} = S_{BH} = \frac{k_B c^3}{4\hbar G_N} A_H$$

- Gravity (G_N) & quantum mechanics ($\hbar$) & statistical mechanics (k_B) are involved!
- **BH entropy: S = ln(# of states)?** Can we understood it microscopically?
- We might be able to learn something about **quantum gravity** from BH entropy.
- BH entropy is also expected to be a key to understand **information loss paradox**.

BH entropy

$(c = \hbar = G_N = k_B = 1)$

- Schwarzschild BH
  energy $E_{bh} = M_{bh}$
  temperature $T_{bh} = T_{Hawking}$

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2$$

$$f(r) = 1 - \frac{r_H}{r} \qquad r_H = 2M_{bh}$$

$$T_{Hawking} = \frac{\kappa}{2\pi} = \frac{f'(r_H)}{4\pi} = \frac{1}{8\pi M_{bh}}$$

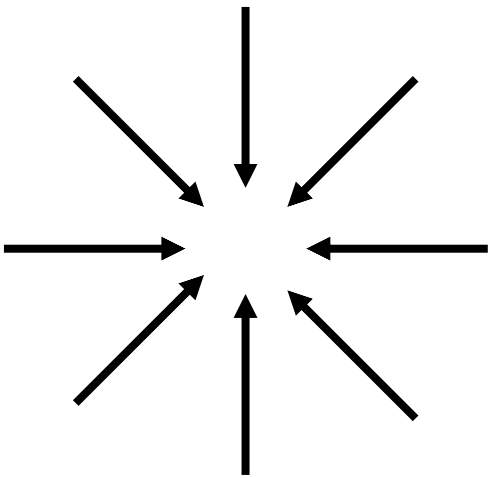
- 1st law (Bardeen-Carter-Hawking 1973)
  $T_{bh}\, dS_{bh} = dE_{bh}$
  $dS_{bh} = dE_{bh} / T_{bh} = 8\pi M_{bh} dM_{bh} = d(4\pi M_{bh}^2)$
  $S_{bh} = 4\pi M_{bh}^2 = A_H/4$

$$S_{bh} = S_{BH} = \frac{k_B c^3}{4\hbar G_N} A_H$$

- (classical) 2nd law
  $\Delta S_{bh} \geqq 0$
- (semi-classical) generalized 2nd law (GSL)
  $\Delta S_{tot} \geqq 0$, where $S_{tot} = S_{bh} + S_{matter}$

BH evaporation & information loss?

Gravitational
collapse



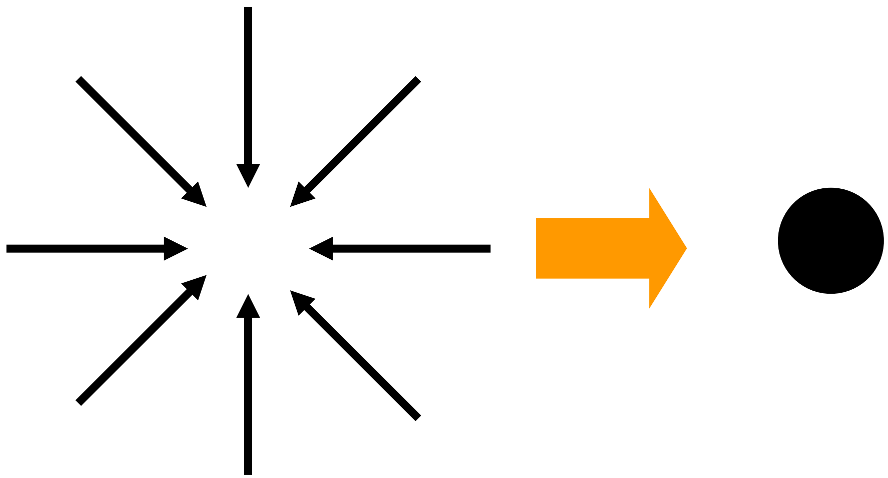
$S_{tot}=S_{matter}=0$

Pure State

BH evaporation & information loss?

Gravitational collapse

BH formation



$S_{tot}=S_{matter}=0$

Pure State

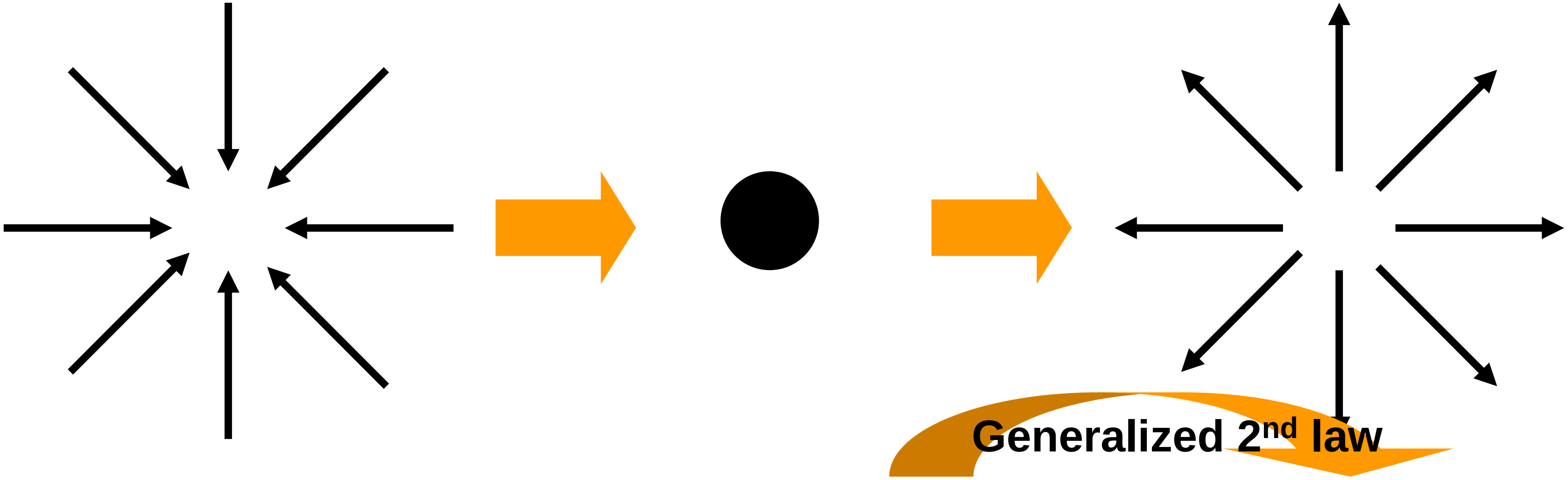
$S_{tot}=S_{bh}=A/4$

BH evaporation & information loss?

Gravitational collapse

BH formation

BH evaporation



$S_{tot}=S_{matter}=0$

Pure State

$S_{tot}=S_{bh}=A/4$

$S_{tot}=S_{matter}\geqq A/4$?

Mixed State?

Information loss? Unitarity violation?

S_{ent} and S_{BH}

Entanglement entropy

(Bombelli, et. al. 1986)

Hilbert space $F = F_1 \bar{\otimes} F_2$

i. Pure density operator $\rho = uu^\dagger$ ($|u|=1$)

$S[\rho] = -\mathrm{Tr}\rho\ln\rho = 0$

ii. Reduced density operator $\rho_2 = \mathrm{Tr}_1\rho$

$\mathrm{Tr}_2[O_2\rho_2] = \mathrm{Tr}[O_2\rho]$

iii. Entanglement entropy

$S_{ent} = -\mathrm{Tr}\rho_2\ln\rho_2 \neq 0$

$S_{ent} = -Tr\, \rho_1 \ln \rho_1 = -Tr\, \rho_2 \ln \rho_2$

$$u = \sum_{m,n} C_{mn} x_m \otimes y_n$$

Hermite　　Unitary

$$H \equiv \sqrt{C^+ C}, W \equiv CH^{-1}$$

$$C = WH$$

$$= U_1^T \begin{pmatrix} C_1 & & \\ & C_2 & \\ & & \ddots \end{pmatrix} U_2 \Leftarrow H = V^+ \begin{pmatrix} C_1 & & \\ & C_2 & \\ & & \ddots \end{pmatrix} V$$

$$\Rightarrow u = \sum_l C_l x'_l \otimes y'_l \qquad x' = U_1 x, y' = U_2 y$$

$$\rho_1 = \sum_l C_l^2 x'_l x'^+_l \qquad \rho_2 = \sum_l C_l^2 y'_l y'^+_l$$

$$\Rightarrow S_{ent} = -\sum_l C_l^2 \ln C_l^2 = -Tr \rho_1 \ln \rho_1 = -Tr \rho_2 \ln \rho_2$$

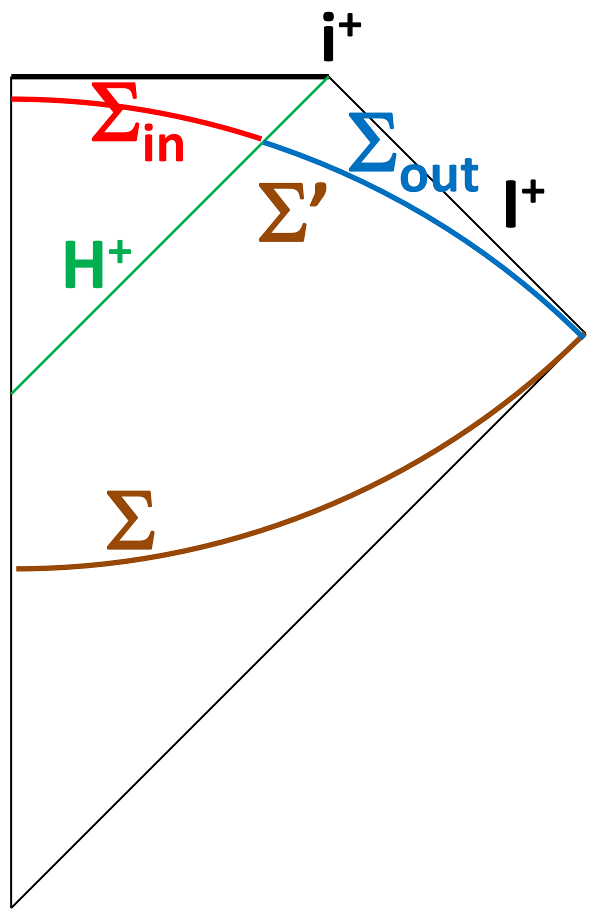
See Appendix A of Mukohyama, Seriu & KodamaPhys.Rev. D55 (1997) 7666 for the extension to infinite dimensional spaces.

S_{ent} and S_{BH}

(Bombelli, et. al. 1986)

- $S_{ent} = -\mathrm{Tr}\, \rho_1 \ln \rho_1 = -\mathrm{Tr}\, \rho_2 \ln \rho_2$ for pure ρ
  → ~~$S_{ent} \propto V_1$~~ , ~~$S_{ent} \propto V_2$~~
- It is expected that $S_{ent} \propto A_B$
- Entropy is dimension-less
  → $S_{ent} \sim A_B/a^2$
- $S_{ent} \sim A/l_{Pl}^2 \sim S_{bh}$ if $a \sim l_{Pl}$.

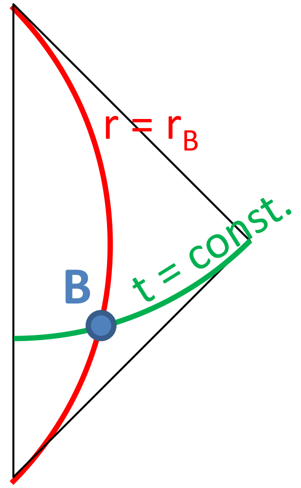
Black hole background

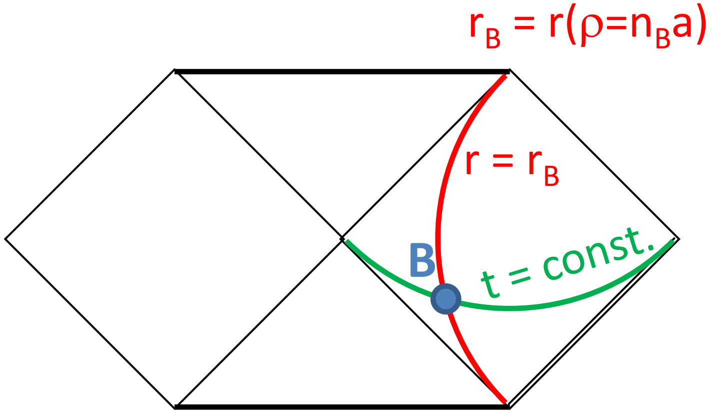


- Σ does not intersect H^+, but Σ' does intersect H^+ ($\Sigma \rightarrow \Sigma' = \Sigma_{in} + \Sigma_{out}$)
- Any observers who reach i^+ or I^+ cannot see information on Σ_{in}
- This leads to S_{ent}
- Does it agree with S_{BH}?

Simple model

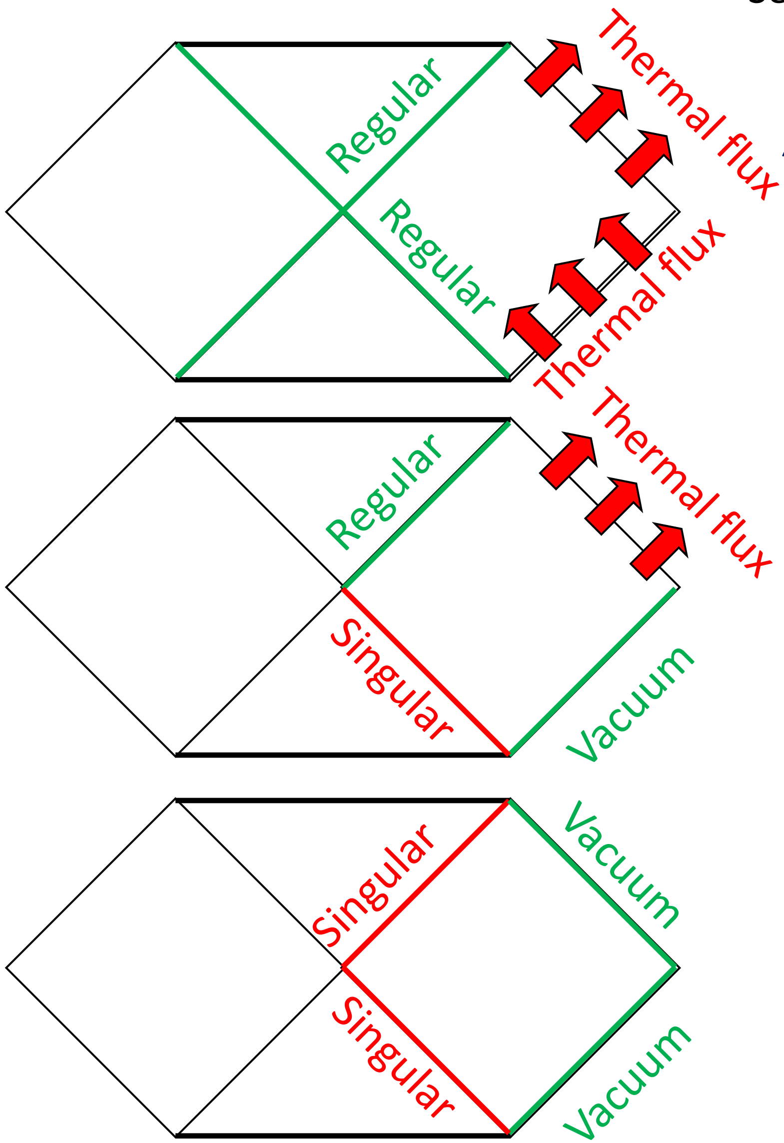
- Real, massless scalar field
  → discretize with the lattice spacing **a**
- $ds^2 = -N(\rho)^2 dt^2 + d\rho^2 + r(\rho)^2 d\Omega^2$
- u ← Boulware state (i.e. Killing vacuum)
- **B** ← $r = r_B$





3 quantum states

See e.g. Mukohyama and Israel, Phys. Rev. D 58, 104005 (1998)



Hartle-Hawking state

"Black hole in a box"

Equilibrium state for BH + QFT

Finite $T_{\mu\nu}$ on the horizon

Unruh state

BH formed by gravitational collapse

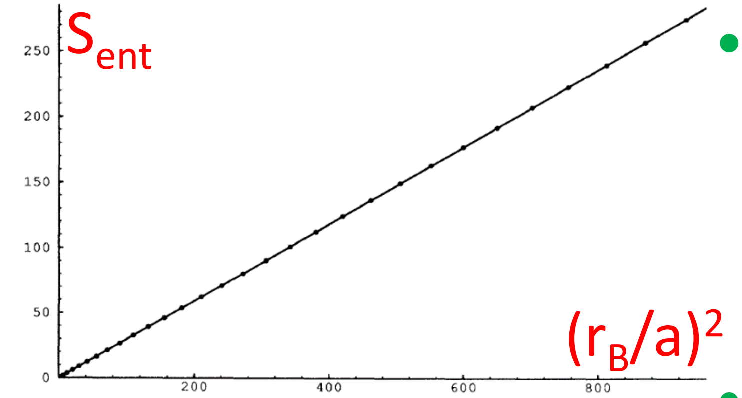
Vacuum to thermal radiation

Boulware state

Vacuum for static observers

Natural state outside a star

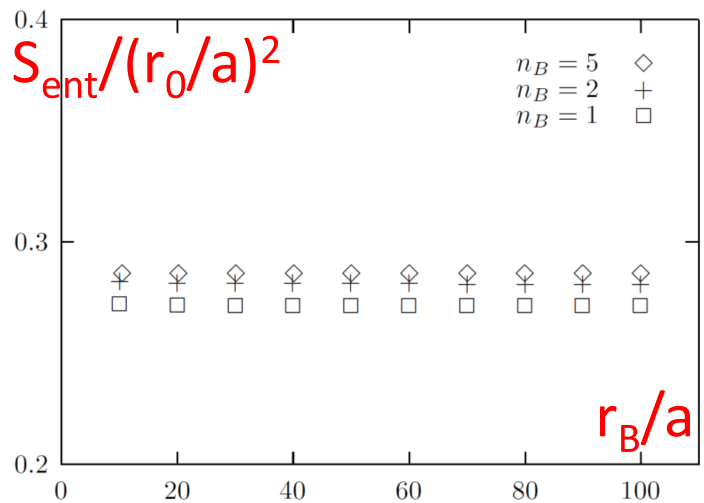
$T_{\mu\nu}$ diverges on the horizon (negative E)



- **Minkowski** (Srednick 1993)

  N = 1, r = r

  → $S_{ent} \sim 0.3(r_B/a)^2$



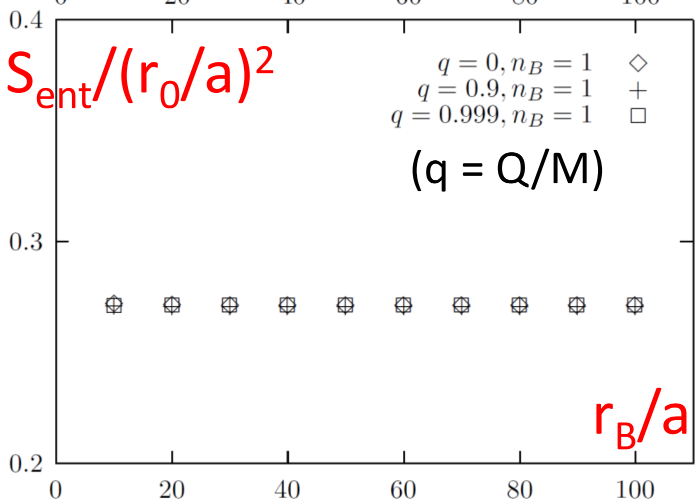
- **Schwarzshild**

  (Mukohyama, Seriu & Kodama, PRD58 (1998) 064001)

  $$N = \sqrt{1 - r_0 / r} \qquad (y = 2r / r_0 - 1)$$

  $$\rho = r_0 \left[\sqrt{y^2 - 1} + \ln\left(y + \sqrt{y^2 - 1} \right) \right] / 2$$

  → $S_{ent} \sim 0.3(r_B/a)^2$



- **Reissner-Nordstrom**

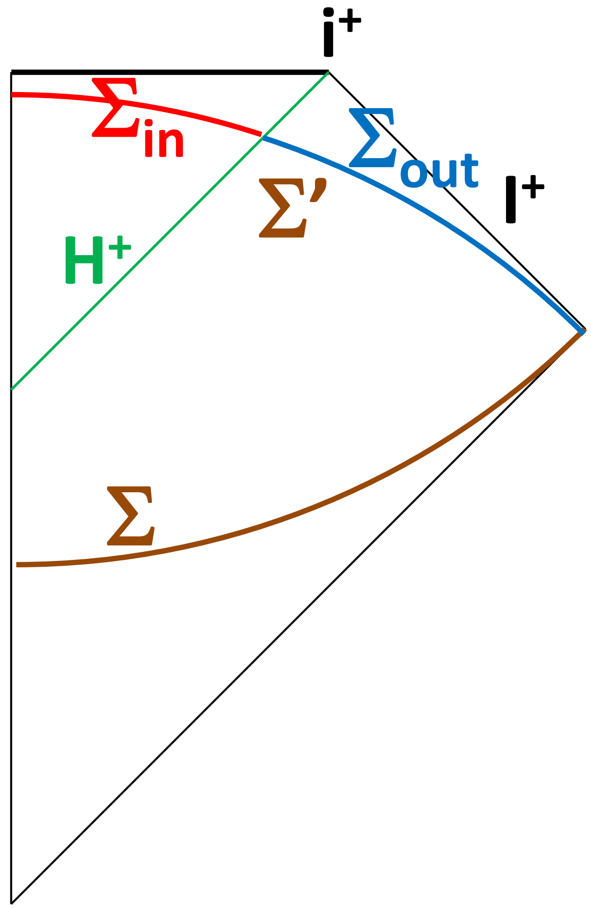
  (Mukohyama, gr-qc/9812079)

  $$N = \sqrt{1 - 2M / r + Q^2 / r^2} \quad \left(y = (r - M) / \sqrt{M^2 - Q^2} \right)$$

  $$\rho = \sqrt{(M^2 - Q^2)(y^2 - 1)} + M \ln\left(y + \sqrt{y^2 - 1} \right)$$

  → $S_{ent} \sim 0.3(r_B/a)^2$

Black hole background



- Σ does not intersect H^+, but Σ' does intersect H^+ ($\Sigma \rightarrow \Sigma' = \Sigma_{in} + \Sigma_{out}$)
- Any observers who reach i^+ or I^+ cannot see information on Σ_{in}
- This leads to S_{ent}
- $S_{ent} \sim S_{BH}$ if $a \sim l_{Pl}$

c.f. The argument can be made more precise by renormalization of G_N

Entanglement Thermodynamics

(Mukohyama, Seriu & Kodama, Phys.Rev. D55 (1997) 7666; Phys.Rev. D58 (1998) 064001)

Black-hole thermodynamics

Black-hole thermodynamics		
$S_{bh} \propto A/l_{Pl}^2$	**entropy**	$S_{ent} \propto A/a^2$
$E_{bh} \propto A^{1/2}/l_{Pl}^2$	**energy**	
$T_{bh} \propto A^{-1/2}$	**temperature**	
$dE_{bh} = T_{bh} dS_{bh}$	**1st law**	

Entanglement Thermodynamics

(Mukohyama, Seriu & Kodama, Phys.Rev. D55 (1997) 7666; Phys.Rev. D58 (1998) 064001)

Black-hole thermodynamics		Entanglement thermodynamics
$S_{bh} \propto A/l_{Pl}^2$	entropy	$S_{ent} \propto A/a^2$
$E_{bh} \propto A^{1/2}/l_{Pl}^2$	energy	$E_{ent} \propto$?
$T_{bh} \propto A^{-1/2}$	temperature	$T_{ent} \propto$?
$dE_{bh} = T_{bh}dS_{bh}$	1st law	$dE_{ent} = T_{ent}dS_{ent}$

Construction of entanglement thermodynamics

i. Calculate S_{ent}
ii. Define and calculate E_{ent}
iii. Obtain T_{ent} by requiring $dE_{ent} = T_{ent}dS_{ent}$

Entanglement Thermodynamics

(Mukohyama, Seriu & Kodama, Phys.Rev. D55 (1997) 7666; Phys.Rev. D58 (1998) 064001)

Black-hole thermodynamics		Entanglement thermodynamics
$S_{bh} \propto A/l_{Pl}^2$	entropy	$S_{ent} \propto A/a^2$
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Construction of entanglement thermodynamics

i. Calculate S_{ent}
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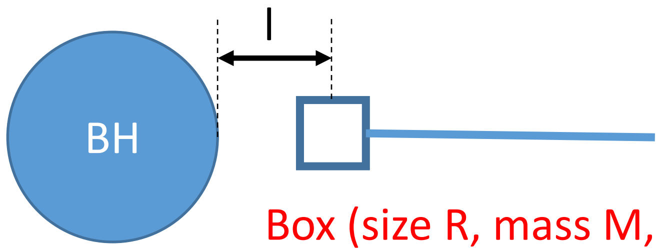
Entropy bounds

Bekenstein bound (1981)



Box (size R, mass M, entropy S)

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2$$

$$T_{bh} = \frac{\kappa}{2\pi} = \frac{f'(r_H)}{4\pi}$$

- Near horizon behavior (r : box's position)

$$f(r) \approx f'(r_H)(r - r_H) = 4\pi T_{bh}(r - r_H)$$
$$\approx (2\pi T_{bh} l)^2$$

$$\left(l = \int_{r_H}^{r} \frac{dr'}{\sqrt{f(r')}} \simeq \frac{1}{\sqrt{4\pi T_{bh}}} \int_{r_H}^{r} \frac{dr'}{\sqrt{r' - r_H}} = \sqrt{\frac{r - r_H}{\pi T_{bh}}} \right)$$

- Box's energy measured @ infinity

$$E = M\sqrt{f(r)} \simeq 2\pi M T_{bh} l$$

- 1^{st} law with $\Delta M_{bh} = E$

$$\Delta S_{bh} = \frac{\Delta M_{bh}}{T_{bh}} = \frac{E}{T_{bh}} \approx 2\pi M l$$

- Total entropy

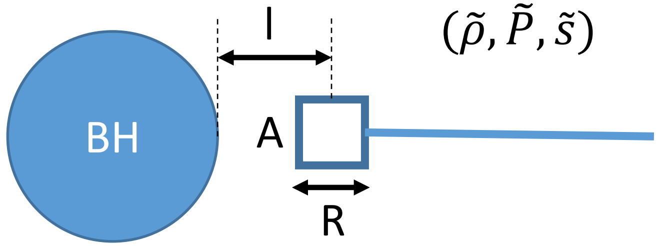
$$\Delta S_{tot} = \Delta S_{bh} - S \approx 2\pi M l - S$$

- **GSL ($\Delta S_{tot} \geqq 0$) for $^{\forall} l \geqq R$ requires**

$$\boxed{S \leq 2\pi M R}$$

Unruh-Wald argument (1982)

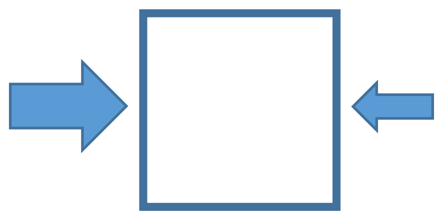
Thermal atmosphere around BH causes a buoyancy force



Box filled with a gas (ρ, P, s)

- Buoyancy force

$(A\tilde{P}\sqrt{f})_{l-R/2}$



$(A\tilde{P}\sqrt{f})_{l+R/2}$

$$f_b(l) = (A\tilde{P}\sqrt{f})_{l-R/2} - (A\tilde{P}\sqrt{f})_{l+R/2}$$

- Work done against the buoyancy force

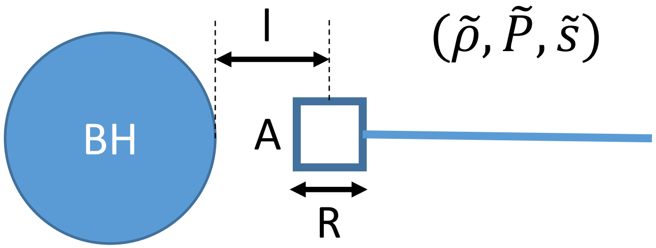
$$W_b(l) = -\int_{\infty}^{l} f_b(l')dl' = \int_{box} \tilde{P}\sqrt{f}\,dV$$

- Box's energy measured @ infinity

$$E_{box} = \int_{box} \rho\sqrt{f}\,dV$$

Unruh-Wald argument (1982)

Thermal atmosphere around BH causes a buoyancy force



Box filled with a gas (ρ, P, s)

- 1[st] law with $\Delta M_{bh} = E_{box} + W_b$

$$\Delta S_{bh} = \frac{\Delta M_{bh}}{T_{bh}} = \frac{1}{T_{bh}} \int_{box} (\rho + \tilde{P}) \sqrt{f} dV$$

- Total entropy

$$\Delta S_{tot} = \Delta S_{bh} - S = \int_{box} \left[\frac{1}{\tilde{T}} (\rho + \tilde{P}) - s \right] dV$$

$$= \int_{box} \frac{1}{\tilde{T}} [(\rho - \tilde{T}s) - (\tilde{\rho} - \tilde{T}\tilde{s})] dV \geq 0$$

$\tilde{T} \equiv \frac{T_{bh}}{\sqrt{f}}$: Tolman temperature

s : entropy density of gas

Gibbs-Duhem relation
$\tilde{\rho} = \tilde{T}\tilde{s} - \tilde{P}$

The thermal state minimizes $\rho - \tilde{T}s$

➡ **Bekenstein bound is NOT needed for the validity of GSL!**

This argument can be extended to a charged bh (Shimomura, Mukohyama, PRD61 (2000) 064020) & a rotating bh (Gao & Wald 2001).

Casini's proof of "Bekenstein bound" (2008)

- Relative entropy

$$S(\rho_1|\rho_2) \equiv Tr(\rho_1 \ln\rho_1) - Tr(\rho_1 \ln\rho_2)$$

non-negativity of relative entropy

$S(\rho_1|\rho_2) \geqq 0$, where equality holds iff $\rho_1 = \rho_2$

(proof)

$\{|a_i\rangle\}\&\{|b_i\rangle\}$: complete orthonormal sets of eigenvectors of ρ_1 & ρ_2

$$\rho_1 = \sum_i |a_i\rangle a_i \langle a_i| \qquad \rho_2 = \sum_i |b_i\rangle b_i \langle b_i|$$

$$S(\rho_1|\rho_2) = Tr(\rho_1 \ln\rho_1) - Tr(\rho_1 \ln\rho_2) + Tr\rho_2 - Tr\rho_1 = \sum_{i,j} \left|\langle a_i|b_j\rangle\right|^2 \left(a_i \ln a_i - a_i \ln b_j + b_j - a_i\right) \geq 0$$

Q.E.D.

- Setup

V : a spatial region on a Cauchy surface

-V : complementary set of V

ρ : a quantum state

ρ^0 : vacuum

$$\rho_V \equiv Tr_{-V}\rho$$

$$\rho_V^0 \equiv Tr_{-V}\rho^0$$

- **Local Hamiltonian** K (**modular Hamiltonian** in continuum theory)

$$\rho_V^0 = \frac{e^{-K}}{Tr e^{-K}}$$

e.g.) $K = 2\pi \int dxdy \int_0^\infty dz\, zH(x,y,z) = \int d^3x \frac{H(x,y,z)}{T_{Rindler}(z)}$ for V = half space

Casini's proof of "Bekenstein bound" (2008)

- "Proof"

$$0 \leq S(\rho_V|\rho_V^0) \equiv Tr(\rho_V \ln\rho_V) - Tr(\rho_V \ln\rho_V^0)$$

$\ln\rho_V^0 = -K - \ln(Tre^{-K})$

$$= Tr(\rho_V \ln\rho_V) + Tr(K\rho_V) + \ln(Tre^{-K})\,Tr\rho_V$$

$Tr\rho_V = 1 = Tr\rho_V^0$

$\ln(Tre^{-K})\,Tr\rho_V = Tr[\rho_V^0 \ln(Tre^{-K})]$, $\ln(Tre^{-K}) = -K - \ln\rho_V^0$

$$= Tr(\rho_V \ln\rho_V) - Tr(\rho_V^0 \ln\rho_V^0) + Tr(K\rho_V) - Tr(K\rho_V^0)$$

$Tr(\rho_V \ln\rho_V) = -S(\rho_V)$, $Tr(\rho_V^0 \ln\rho_V^0) = -S(\rho_V^0)$

$$\Rightarrow \quad S(\rho_V) - S(\rho_V^0) \leq Tr(K\rho_V) - Tr(K\rho_V^0)$$

$S(\rho_V) - S(\rho_V^0) = S$ "renormalized"

$Tr(K\rho_V) - Tr(K\rho_V^0) = O(1) \times MR$ for V=half space

"Q.E.D."

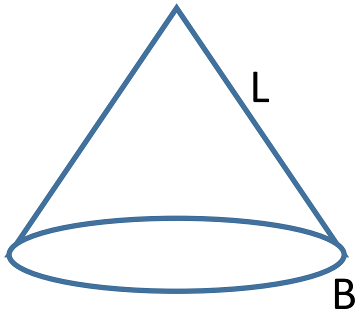
- This looks similar to Bekenstein bound $S \leq 2\pi MR$

- **The proof holds for any quantum systems and any quantum states.**
- However, the proved inequality can be interpreted as Bekenstein bound only in special cases.

Covariant entropy bound (Bousso 1999)

$$S \leq \frac{A}{4}$$

S : entropy on L
A : area of B



L (light-sheet) : a hypersuraface generated by null geodesics that are orthogonal to B and that have non-positive expansion

B : a spacelike 2-surface

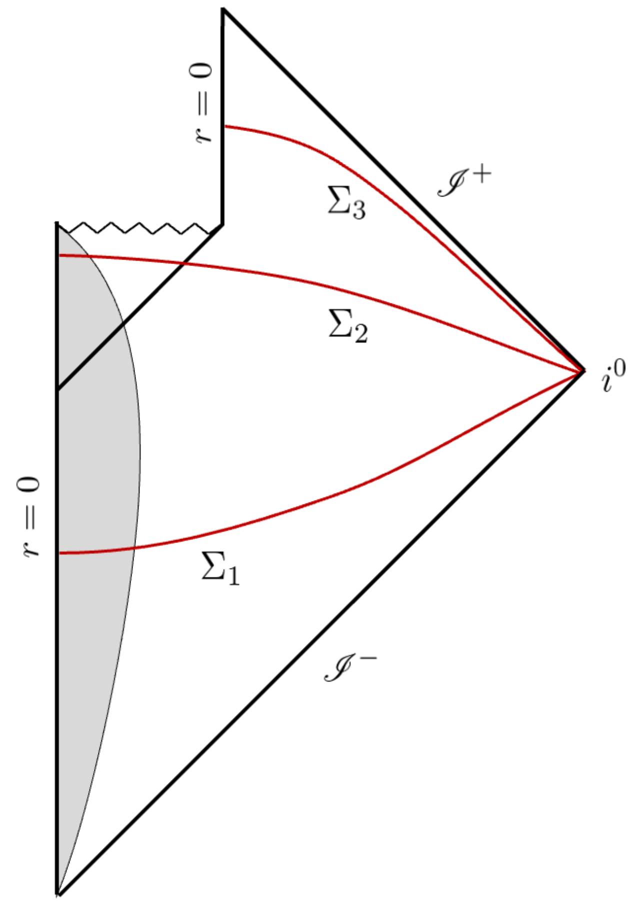
- Bekenstein bound is not covariant and it assumes constant and finite size, negligible gravity, and no negative energy.
- Bousso bound is covariant and can be applied to gravitational collapse and FLRW universes.
- Bousso bound can be “proved” under certain assumptions [Flanagan, Marolf & Wald 2000, Strominger & Thompson 2004] but can be violated in the presence of negative energy, e.g. Boulware energy.
- Can be extended to scalar-tensor theories, f(R) theories, Einstein-Gauss-Bonnet theory [Matsuda & Mukohyama, Phys.Rev.D 103 (2021) 024002]

Revisiting the paradox

Case A. Unitary Problem

A1. Quantum states evolve in a unitary way. In particular, pure states evolve into pure states.

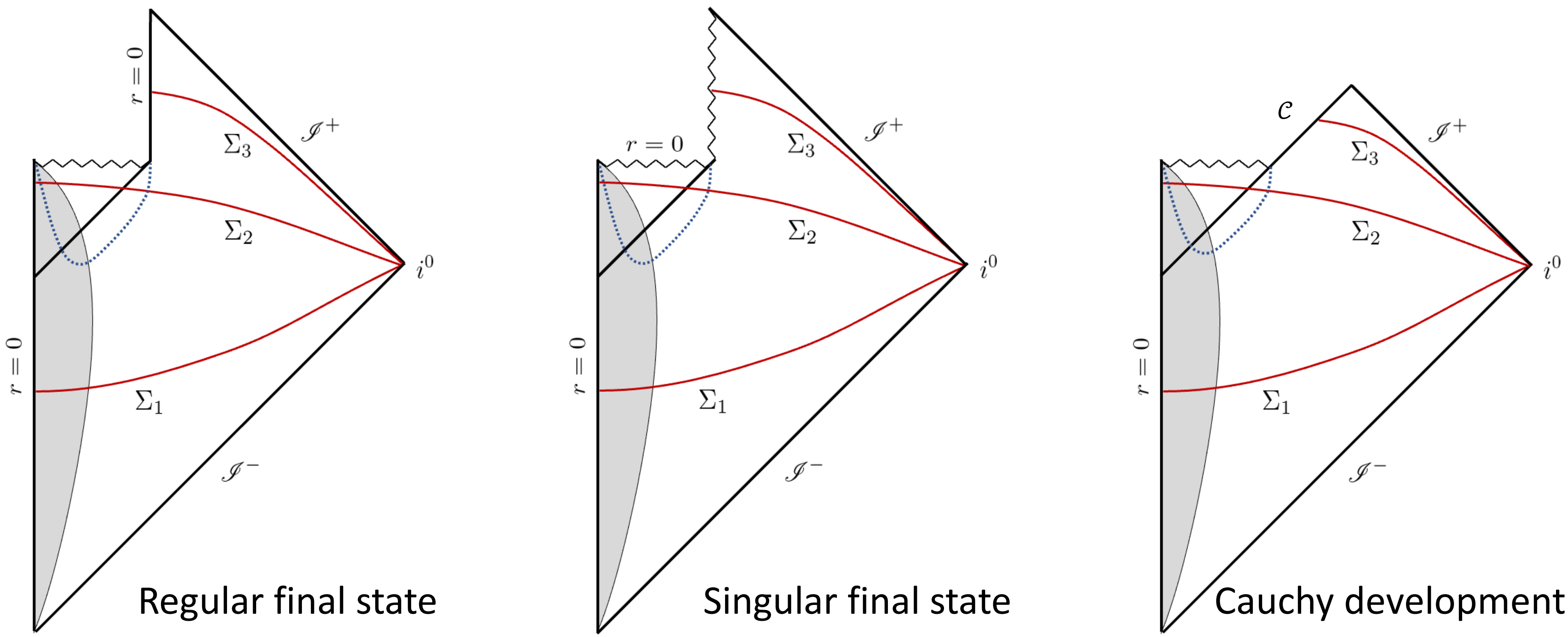
A2. Semiclassical general relativity is a valid low-energy effective field theory to describe black hole physics during the entire evaporation process: black holes evaporate completely emitting thermal radiation and end up leaving a regular spacetime.



Case A. Unitary Problem

This formulation of the information loss paradox is not particularly worrisome

- No reason why we expect semiclassical GR to be valid till the end of BH evaporation → A2 is likely to be violated
- Whether the final state is regular or singular entirely depends on unknown quantum gravity.
- In particular, semiclassical GR cannot predict anything beyond Cauchy horizon.



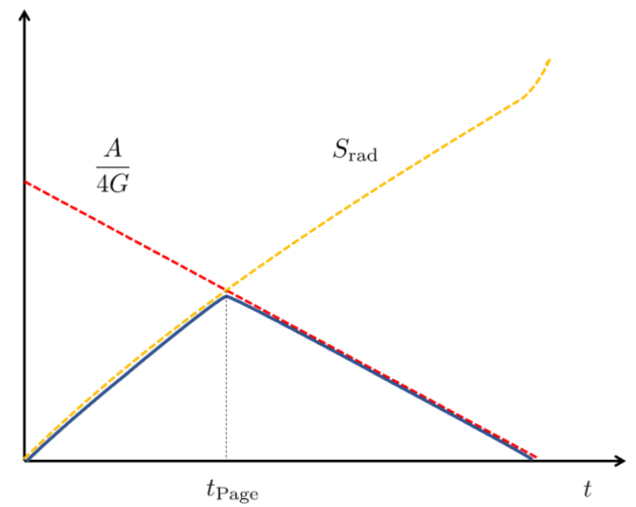
Regular final state | Singular final state | Cauchy development

Case B. Entropy Problem

B1. Quantum states evolve in a unitary way. In particular, pure states evolve into pure states.

B2. Semiclassical general relativity is a valid low-energy effective field theory to describe black hole physics far from the Planckian regime.

B3. As seen from the outside, a black hole behaves like a quantum system whose number of degrees of freedom is given by A/4G, with A being the apparent-horizon area.



Hawking's prediction
vs
Page curve

- 1st assumption unchanged (B1 = A1)
- 2nd assumption significantly weakened (B2 < A2), c.f. "nice slicing"
- 3rd assumption is often called "central dogma"

Case B. Entropy Problem

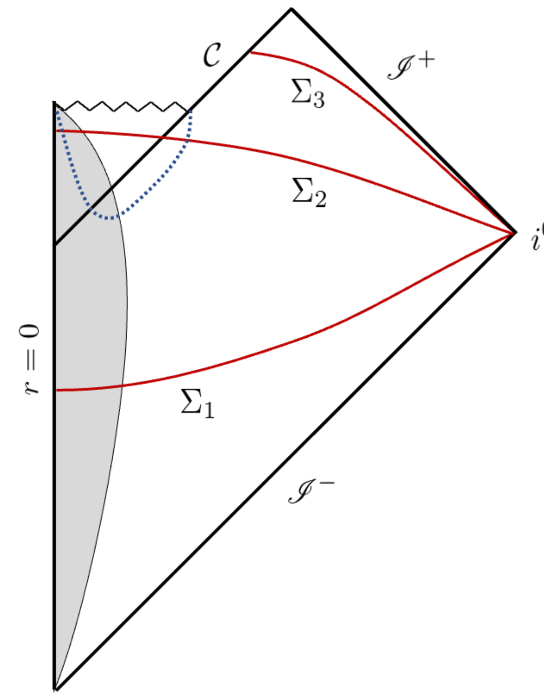
- Hawking rad from BH → $S_{rad} = S_{ent}$ increases but S_{BH} ($\geqq S_{ent}$ due to B3) decreases → semiclassical description should break down @ Page time, i.e. when S_{BH} ~ half of $S_{BH,init}$
- After Page time, B1+B2 and B1+B3 are in contradiction

Case C. No Paradox

- Dropping the 3rd assumption, i.e. "central dogma", from Case B,

C1. Quantum states evolve in a unitary way. In particular, pure states evolve into pure states.

C2. Semiclassical general relativity is a valid low-energy effective field theory to describe black hole physics far from the Planckian regime.



There is no contradiction between C1 (= B1 = A1) and C2 (= B2 < A2) since test of C1 requires information about the region beyond Cauchy horizon and C2 is compatible with any evolution beyond Cauchy horizon.

Thoughts on central dogma

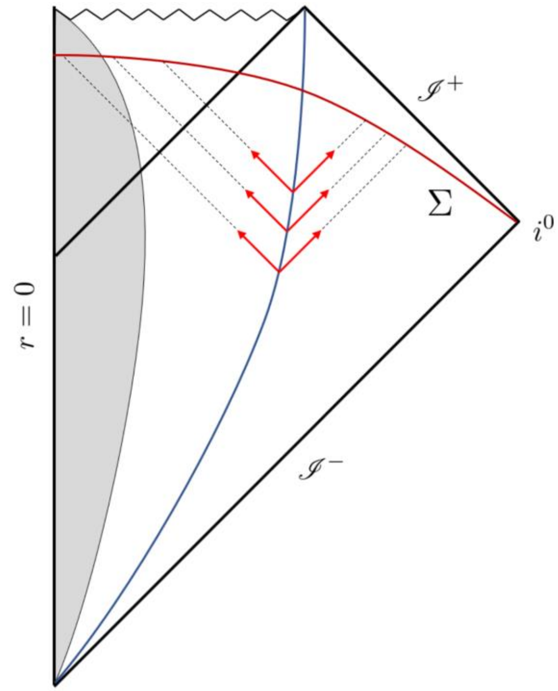
Standard motivations for central dogma B3

- S_{BH} = A/4G plays the role of thermal (maximum) entropy in BH thermodynamics.
- The D-brane state counting confirms max $S_{bh} = S_{BH} = A/4G$.
- Bekenstein bound $S \leqq 2\pi ER$ applied to a BH with R=2GM, E=M → $S_{bh} \leqq A/4G$.
- Bousso's covariant entropy bound applied to Schwarzschild BH → $S_{bh} \leqq A/4G$.
- Boundedness of BH creation rate seems to require finite number of BH states .
- Holographic principle: # of d.o.f. $\propto$ area .
- Island program in AdS/CFT → Page curve reproduced .

Thought experiment

Let us assume that B2 holds

B2. Semiclassical general relativity is a valid low-energy effective field theory to describe black hole physics far from the Planckian regime.

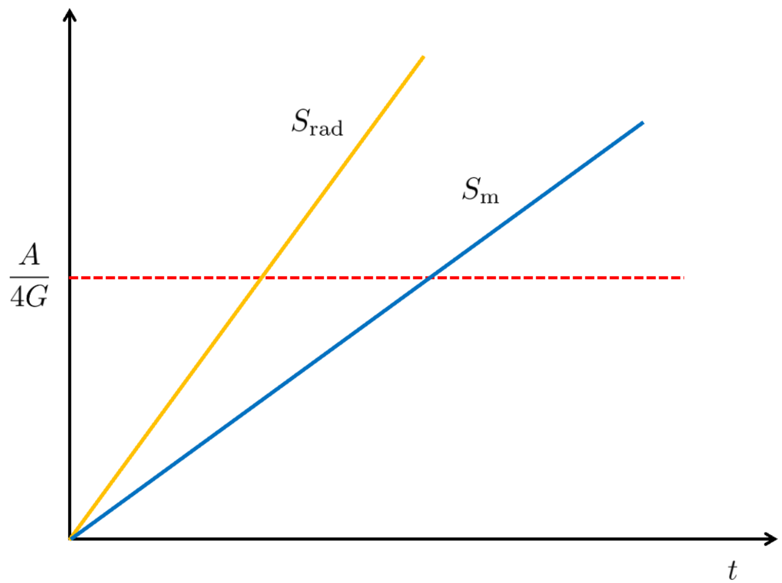


A far away observer prepares a pure state. Half of the state falls into the black hole, the other half reaches $\mathcal{J}^+$.

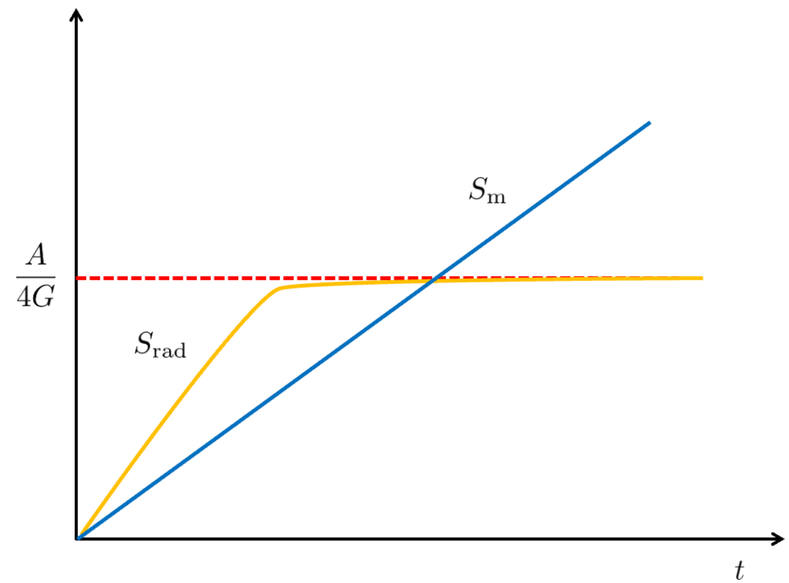
Ingoing energy flux tuned to Hawking flux.

→ The mass of the black hole stays constant.

Thought experiment



Hawking's prediction



Page curve

- Independently from the behavior of S_{rad}, the central dogma should be violated if B2 holds.
- In order to change the behavior of S_m, the black hole must "know" how someone far away prepared a pure state and sent a part of it into the black hole. Someone else may or may not decide to do similar experiments at any time at any places and in any ways. The black hole must "know" all those activities.

Q and A

Question
In the thought experiment, how to tune the matter flux with the Hawking flux?

Answer

Keep watching the evolution of the BH mass by observing the motion of a test particle @ $r \simeq \alpha\, r_s$ with $\alpha = O(1) > 1$. If the BH mass is decreasing then increase the matter flux. If the BH is increasing then decrease the matter flux. Repeat this as long as you want.

Some details

Time-scale to measure the BH mass is the Kepler time $P \sim \alpha^{3/2}\, r_s$. If there is no matter flux then the BH mass decreases by Hawking radiation within this time-scale by the amount $|\Delta M_{bh}| \sim r_s^2\, T_{bh}^4\, P \sim \alpha^{3/2}\, r_s^{-1}$. The tuning should be possible if $|\Delta M_{bh}| << M_{bh}$. This is definitely the case for a BH larger than Planckian size since $|\Delta M_{bh}| / M_{bh} \sim \alpha^{3/2} (l_{Pl}/r_s)^2 << 1$.

Contradiction between B2 and B3 (central dogma)

B2. Semiclassical general relativity is a valid low-energy effective field theory to describe black hole physics far from the Planckian regime.

B3. As seen from the outside, a black hole behaves like a quantum system whose number of degrees of freedom is given by A/4G, with A being the apparent-horizon area.

No assumption about unitarity was required!

Stronger contradiction

The assumption B2 can be split into the following two:

B2a. Black holes whose mass is larger than Planck mass emit thermal radiation according to semiclassical general relativity;

B2b. Infalling matter far from the Planckian regime obeys the laws of general relativity.

The contradiction is between B2b and the central dogma B3.
If B3 is correct then B2b must be abandoned.

The entropy problem can be formulated in terms of B2a, i.e. B1+B2a+B3. However, if we abandon GR (B2b) then why do we trust semiclassical GR (B2a)? It seems that the statement of the paradox needs refinement.

Summary and discussion

- The information loss paradox is usually stated as the incompatibility between the following assumptions:

B1. Quantum states evolve in a unitary way. In particular, pure states evolve into pure states.

B2a. Black holes whose mass is larger than Planck mass emit thermal radiation according to semiclassical general relativity;

B2b. Infalling matter far from the Planckian regime obeys the laws of general relativity.

B3. As seen from the outside, a black hole behaves like a quantum system whose number of degrees of freedom is given by A/4G, with A being the apparent-horizon area.

- However, a thought experiment shows incompatibility between B2b & B3 without requiring other assumptions.
- We are free to choose B2b or B3, but at most one.

- If we keep B3 (central dogma) then the information loss paradox is reformulated as the incompatibility between the following assumptions:

B1. Quantum states evolve in a unitary way. In particular, pure states evolve into pure states.

B2a. Black holes whose mass is larger than Planck mass emit thermal radiation according to semiclassical general relativity;

B3. As seen from the outside, a black hole behaves like a quantum system whose number of degrees of freedom is given by A/4G, with A being the apparent-horizon area.

- The price to pay is the violation of B2b, meaning that
  Infalling matter far from the Planckian regime does not obey the laws of general relativity
  and in particular that the equivalence principle is violated.

Standard motivations for central dogma B3

- $S_{BH} = A/4G$ plays the role of thermal (maximum) entropy in BH thermodynamics.

- The D-brane state counting confirms max $S_{bh} = S_{BH} = A/4G$.

- Bekenstein bound $S \leqq 2\pi ER$ applied to a BH with R=2GM, E=M → $S_{bh} \leqq A/4G$.

- Bousso's covariant entropy bound applied to Schwarzschild BH → $S_{bh} \leqq A/4G$.

- Boundedness of BH creation rate seems to require finite number of BH states .

- Holographic principle: # of d.o.f. $\propto$ area .

- Island program in AdS/CFT → Page curve reproduced .

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Does S_{BH} = A/4G count not only gravitational d.o.f. but also matter d.o.f. inside a black hole?
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Does the island program always work, beyond specific setups? Does $S_{gen}(I \cup R)$ really represent entropy of radiation measured at infinity? Do we know for sure which saddle points contribute to the path integral?

Possible scenario without central dogma B3

Quantum teleportation [Mukohyama, Phys.Rev.D 58 (1998) 104023]

- "entanglement entropy of a pure state with respect to a division of Hilbert space into two subspaces 1 and 2 is an amount of information which can be transmitted through 1 and 2"
- "information to be sent to the receiver (Bob) in the classical channel is only two integers n and m …"
- "the entanglement entropy is a quantity which cancels the black hole entropy to restore information loss … Both entropies appear and disappear together from the sea of zero entropy state"

Classical channel needs to carry only small amount of data.

HH state maximizes S_{ent} [Mukohyama, Phys.Rev. D61 (2000) 064015]

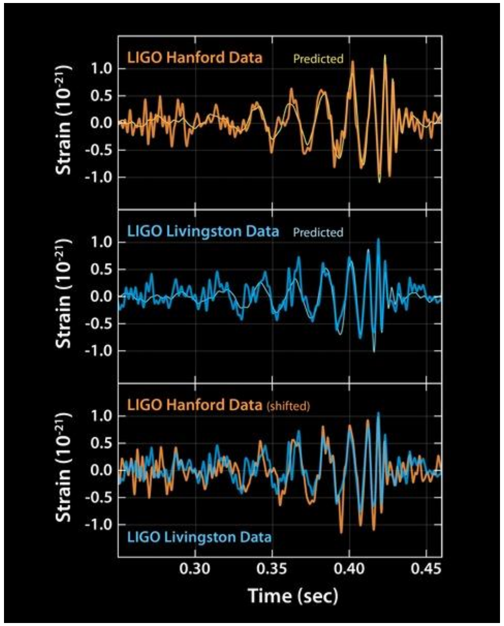
If the BH final state is unique then the classical channel is not needed.

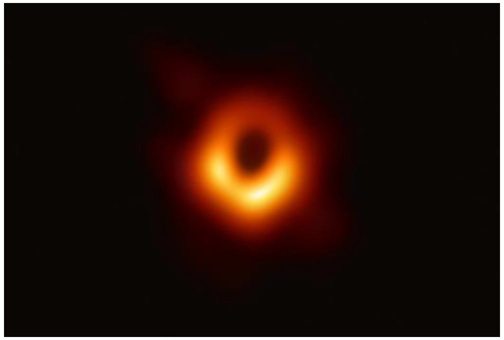
[Horowitz & Maldacena 2004]

A mixed final state with a remnant storing just the classical channel may also be yet another possibility.

If central dogma B3 is correct then…

- B2b should be violated, meaning that **infalling matter far from the Planckian regime does not obey the laws of general relativity.** In particular, the equivalence principle should be violated.
- On the other hand, gravitational waves from merger of black holes are observed. Black hole shadow is also observed. More data will come.
- We may have chances to see O(1) deviations from general relativity far from the Planckian regime, e.g. exotic compact objects (boson stars, fuzzballs, hairy BHs, …), GW echoes, etc. **New windows to quantum gravity!**
- **Really? Let's see what observations tell.**





Thank you!

Luca Buoninfante

Francesco Di Filippo

ref.) Buoninfante, Di Filippo and Mukohyama, JHEP 10 (2021) 081.
Mukohyama, *Phys.Rev.D* 58 (1998) 104023.