Quantum Criticality in Long Range Models

Nicolò Defenu

Outline

- Quantum Ising
- Rotor models
- Motivation
- Field theoretical formalism
- FRG approach
- Phase Diagram
- Critical Exponents
- BKT and Sine Gordon model
- Perspectives

Quantum Ising Chain

$$H_{\mathrm{I}} = -\sum_{ij} \frac{J_{ij}}{2} \sigma_i^z \sigma_j^z - h \sum_i \sigma_i^x$$

Pauli matrices

$$[\sigma^\mu, \sigma^\nu] = -2\, i \epsilon_{\mu\nu\rho} \sigma^\rho$$

Parity operator $P \equiv \Pi_i \sigma_i^x$

$$[P, H_I] = 0$$

Nearest Neighbor case $J_{ij} = 2J\delta_{j,i+1}$

$$J \gg h \ \Rightarrow \ |0\rangle \equiv \Pi_i |\uparrow\rangle_i \ \text{ or } \ \Pi_i |\downarrow\rangle_i \ \text{ and } \ \lim_{|i-j|\to\infty} \langle \sigma_i^z \sigma_j^z \rangle \propto N_0^2$$

$$J \ll h \ \Rightarrow \ |0\rangle \equiv \Pi_i |\rightarrow\rangle_i \ \text{ with } \ |\rightarrow\rangle \equiv |\uparrow\rangle + |\downarrow\rangle \ \text{ and } \ \lim_{|i-j|\to\infty} \langle \sigma_i^z \sigma_j^z \rangle \propto e^{-\frac{|i-j|}{\xi}}$$

Quantum Critical Point $h_c = J$

Quantum Rotor Models

$$H_{\mathrm{R}} = -\sum_{ij} \frac{J_{ij}}{2} \hat{\boldsymbol{n}}_i \cdot \hat{\boldsymbol{n}}_j + \frac{\lambda}{2} \sum_i \mathcal{L}_i^2 \quad \text{with} \quad \hat{\boldsymbol{n}}_i^2 = 1$$

$$\hat{\boldsymbol{n}} \equiv (\hat{n}_1, \cdots, \hat{n}_N) \qquad [\hat{n}_\alpha, \hat{p}_\beta] = i\delta_{\alpha\beta} \qquad \mathcal{L}^2 = \frac{1}{2} \sum_{\alpha\beta} \hat{L}_{\alpha\beta}^2$$

$$\hat{L}_{\alpha\beta} = \hat{n}_\alpha \hat{p}_\beta - \hat{n}_\beta \hat{p}_\alpha$$

Quantum Critical Point λ_c even for $J_{ij} = \dfrac{J}{|i-j|^{d+\sigma}}$

$$\lambda < \lambda_c \;\Rightarrow\; \lim_{|i-j|\to\infty} \langle \hat{\boldsymbol{n}}_i \cdot \hat{\boldsymbol{n}}_j \rangle \propto N_0^2$$

$$\lambda > \lambda_c \;\Rightarrow\; \lim_{|i-j|\to\infty} \langle \hat{\boldsymbol{n}}_i \cdot \hat{\boldsymbol{n}}_j \rangle \propto e^{-\frac{|i-j|}{\xi}}$$

Motivations

- Recent interest in the critical behavior of classical long range spin systems

- Recent realization of quantum long range systems in AMO devices

- Mapping to Heisenberg antiferromagnets for N=3
- $TlCuCl_3$ Insulator.
- Spin ladder compounds in d=1.
- High temperature superconductors.

- Mapping to lattice boson models for N=2

- Quantum critical point of the Bose-Hubbard model

- Investigate effective dimension approach

Field Theoretical Formalism

$$S[\varphi] = \int d\tau \int d^d x\{K\partial_\tau\varphi_i\partial_\tau\varphi_i - Z\varphi_i\Delta^{\frac{\sigma}{2}}\varphi - Z_2\varphi_i\Delta\varphi + U(\rho)\}$$

$$\rho = \sum_i \frac{\varphi_i^2}{2} \quad i \in \{1, N\}$$

$$\frac{\delta^2\Gamma_k}{\delta\varphi^2} \equiv G_k^{-1}$$

Critical Exponents

$$\xi \propto (\lambda - \lambda_c)^{-\nu}$$

$$\lim_{q\to 0} G^{-1}(0,q) \propto q^{2-\eta}$$

$$\Delta \propto (\lambda - \lambda_c)^{-z\nu}$$

$$\frac{\partial \log K_k}{\partial \log k} = -\eta_\omega$$

$$\frac{\partial \log Z_k}{\partial \log k} = -\delta\eta$$

$$\frac{\partial \log Z_{2,k}}{\partial \log k} = -\eta$$

$$z = \frac{2-\eta}{2-\eta_\omega}$$

Functional RG

Exact flow equation for the effective action

$$\partial_t \Gamma_k[\tilde{\varphi}] = \frac{1}{2}\mathrm{Tr}\left(\frac{\partial_t R_k}{\Gamma^{(2)} + R_k}\right)$$

$k \sim L^{-1} \sim N^{-\frac{1}{d}}$: scale $\quad k_0 \sim a^{-1} >> 1$: ultraviolet scale

$$\Gamma_{k_0}[\tilde{\varphi}] = S[\tilde{\varphi}] \underset{k_0 > k > 0}{\Longrightarrow} \Gamma_k[\tilde{\varphi}] \underset{k \equiv 0}{\Longrightarrow} \Gamma[\tilde{\varphi}]$$

$$t = \log\left(\frac{k}{k_0}\right)$$

Momentum shells

$$G_k = \left(\Gamma^{(2)} + R_k\right)^{-1}$$

Fourier space

$$R_k \equiv 0 \implies G_{k_0}(q) = \left(q^2 + m^2\right)^{-1}$$

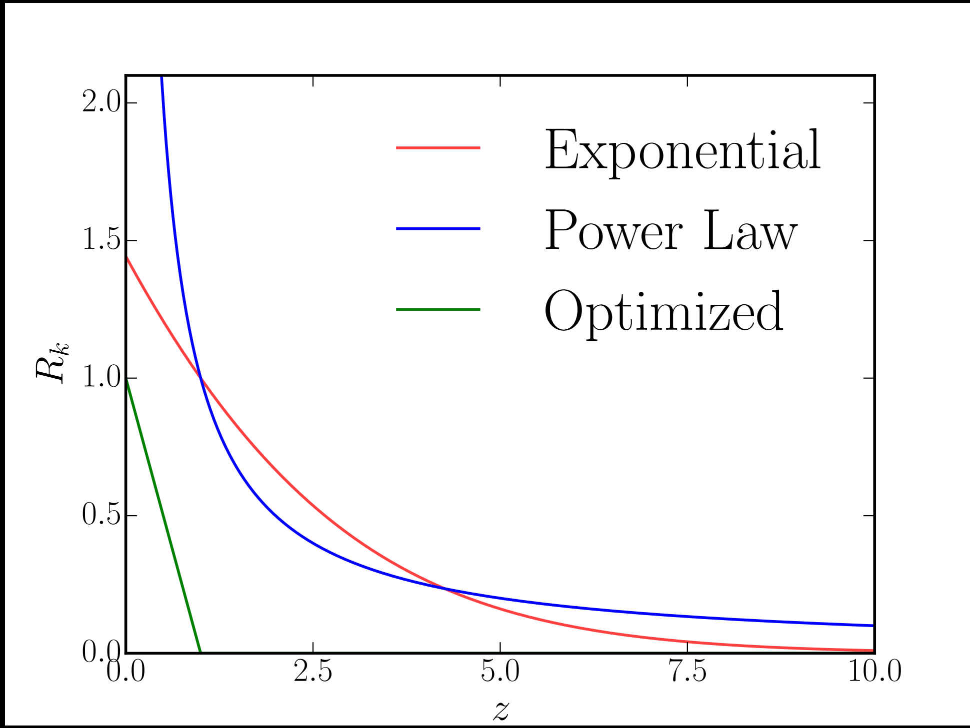
P.T. $\longleftrightarrow$ fluctuations $\longleftrightarrow$ $m = 0$

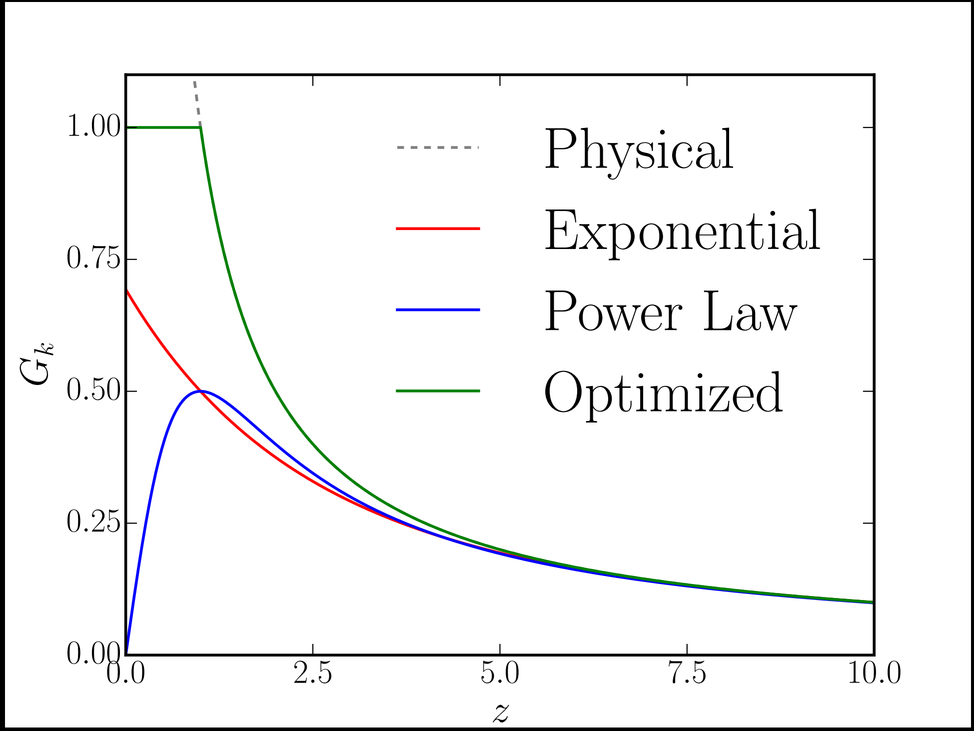
IR Regulator

$$\Delta S_k = \int \frac{d^d q}{(2\pi)^d} \varphi(q) R_k(q) \varphi(-q)$$

$$R_k(q) \gg 1 \text{ if } q > k$$

$$R_k(q) \ll 1 \text{ if } q < k$$





Local Potential Approximation

$$\Gamma_k = \int d\tau \int d^d x \{K_k \partial_\tau \varphi_i \partial_\tau \varphi_i - Z_k \varphi_i \Delta^{\frac{\sigma}{2}} \varphi - Z_{2,k} \varphi_i \Delta \varphi + U_k(\rho)\}$$

$$\partial_t U_k(\rho) = \int \partial_t R_k(\omega, q) G_k(\omega, q) \frac{d^d q}{(2\pi)^d} \frac{d\omega}{2\pi}$$

$$\partial_t K_k = \lim_{p \to 0, \nu \to 0} \frac{1}{2} \frac{d^2}{d\nu^2} \partial_t \Gamma^{(2)}(\nu, p)$$

$$\partial_t Z_k = \lim_{p \to 0, \nu \to 0} \frac{d}{dp^\sigma} \partial_t \Gamma^{(2)}(\nu, p)$$

$$\partial_t Z_{2,k} = \lim_{p \to 0, \nu \to 0} \frac{d^2}{dp^2} \partial_t \Gamma^{(2)}(\nu, p)$$

Flow Equations

$$\partial_t \bar{U}_k = (d+z)\bar{U}_k(\bar{\rho}) - (d+z-\sigma)\bar{\rho}\,\bar{U}_k'(\bar{\rho}) - \frac{\sigma}{2}(N-1)\frac{1-\frac{\eta_\tau z}{3\sigma+2d}}{1+\bar{U}_k'(\bar{\rho})} - \frac{\sigma}{2}\frac{1-\frac{\eta_\tau z}{3\sigma+2d}}{1+\bar{U}_k'(\bar{\rho})+2\bar{\rho}\,\bar{U}_k''(\bar{\rho})}$$

$$\partial_t Z_k = (2-\sigma-\eta)\, Z_k. \qquad \partial_t K_k = \eta_\tau K_k.$$

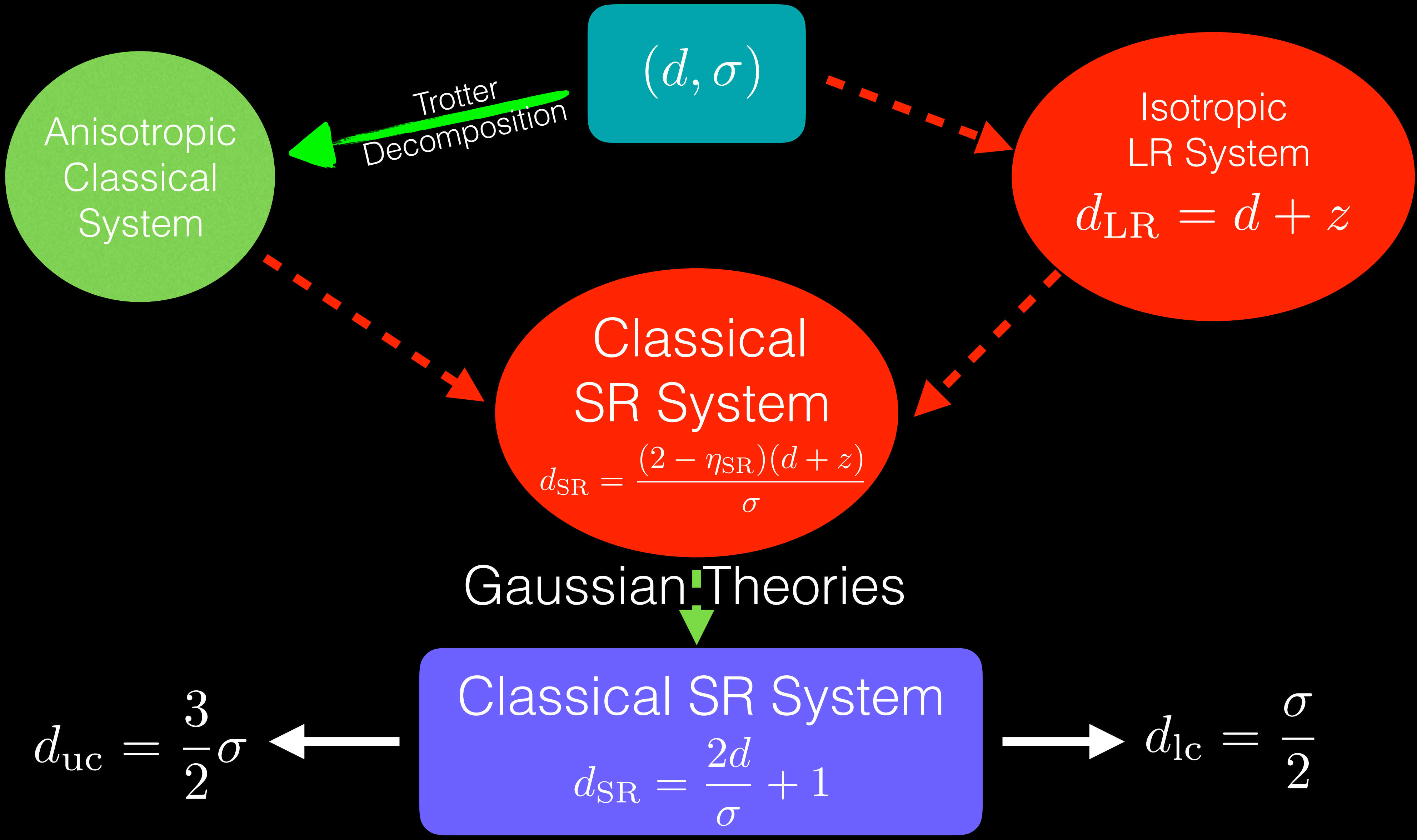
$$\partial_t Z_{2,k} = \eta Z_{2,k}.$$

$$\eta_\tau = \frac{f(\tilde{\rho}_0, \bar{U}''(\bar{\rho}_0))(3\sigma+2d)}{d+(3\sigma+d)(1+f(\bar{\rho}_0, \bar{U}''(\tilde{\rho}_0)))}$$

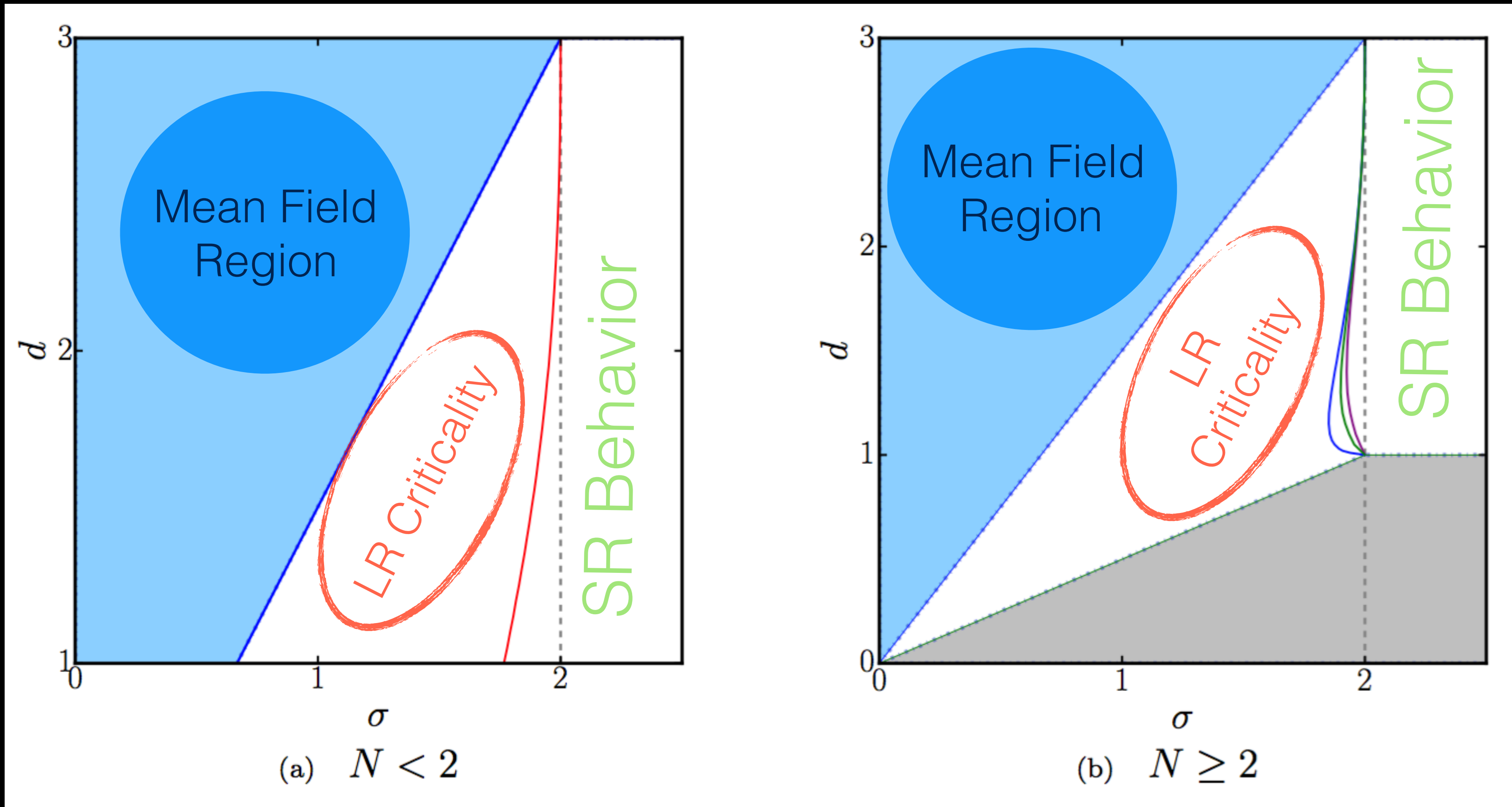
$$\eta = F(\tilde{\rho}_0, \bar{U}''(\bar{\rho}_0), Z_k) \qquad \lim_{Z_k\to 0} F(\tilde{\rho}_0, \bar{U}''(\bar{\rho}_0), Z_k) = \eta_{\mathrm{SR}}$$

Effective dimensions

LR quantum rotor models



Phase Diagram



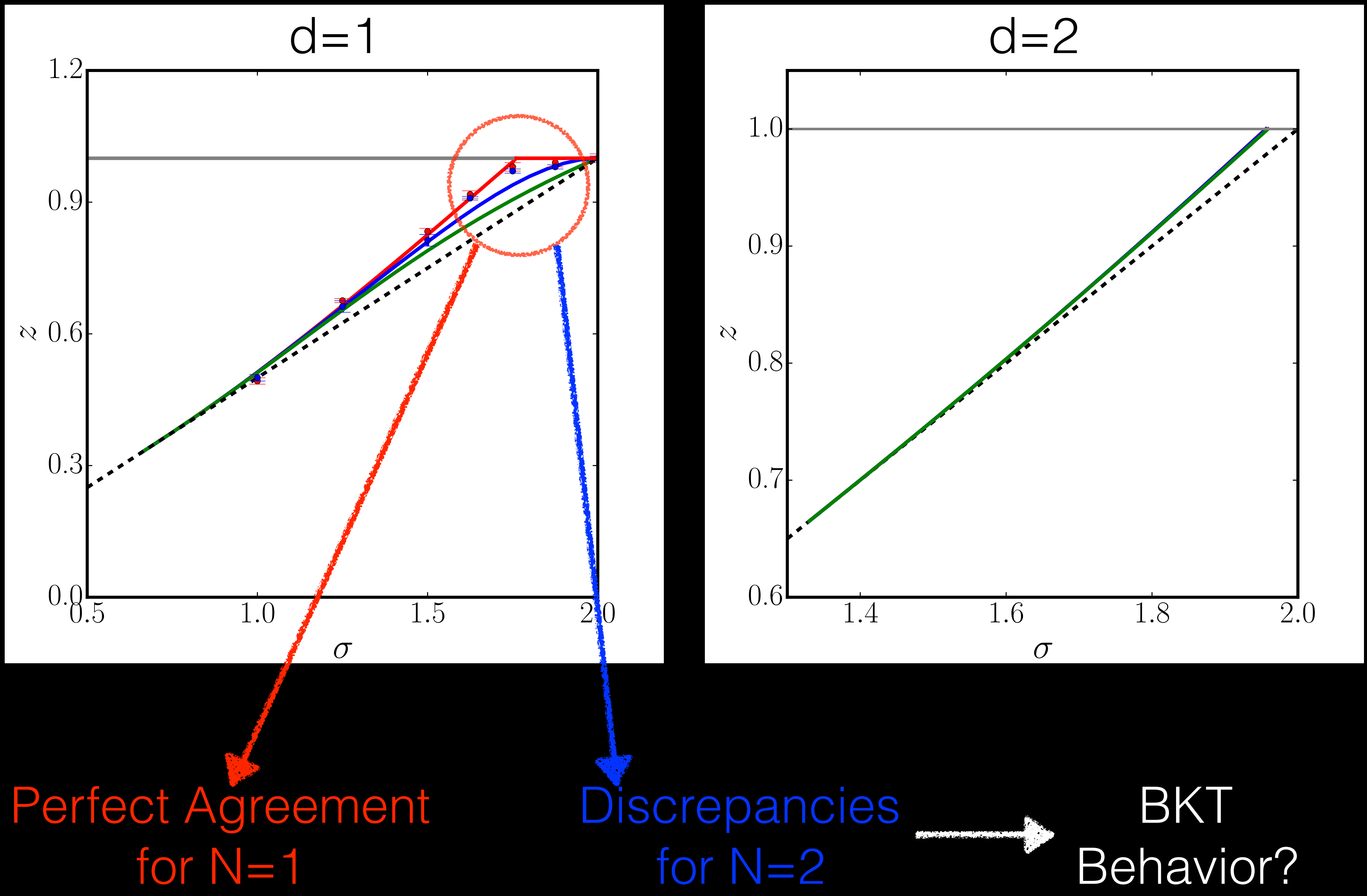
Mean field exponents

$$\eta = 2 - \sigma,$$
$$z = \frac{\sigma}{2},$$
$$\nu = \sigma^{-1}$$

Boundary region

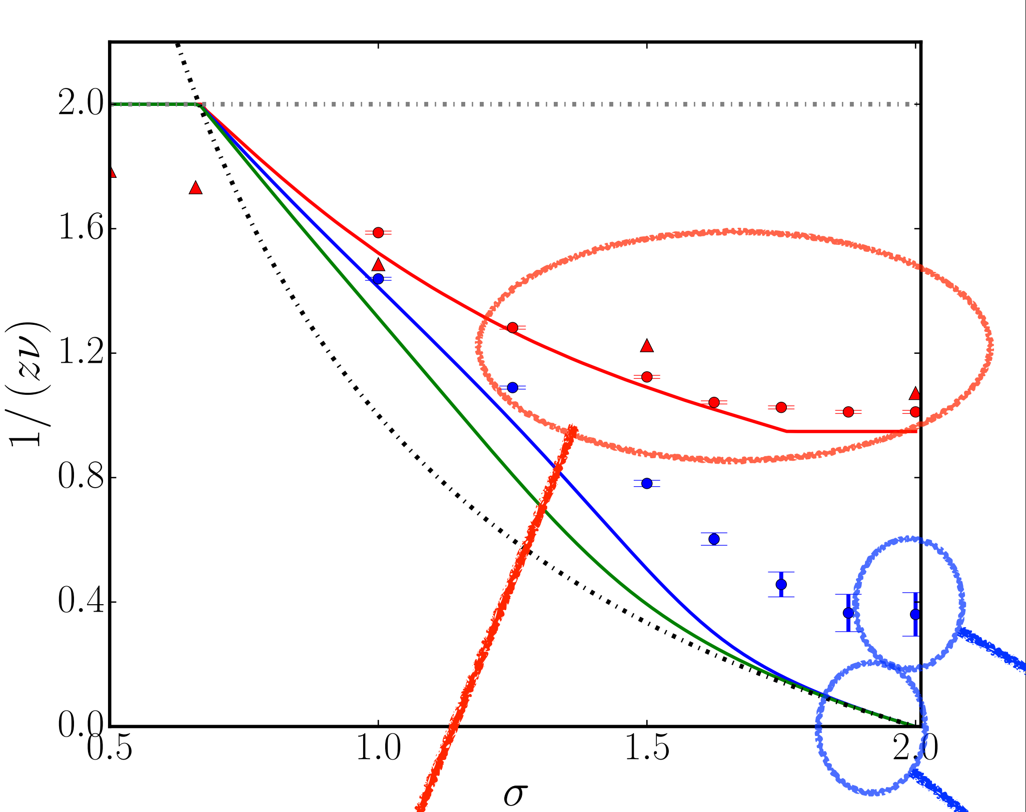
$$\sigma_* = 2 - \eta_{\mathrm{SR}}$$

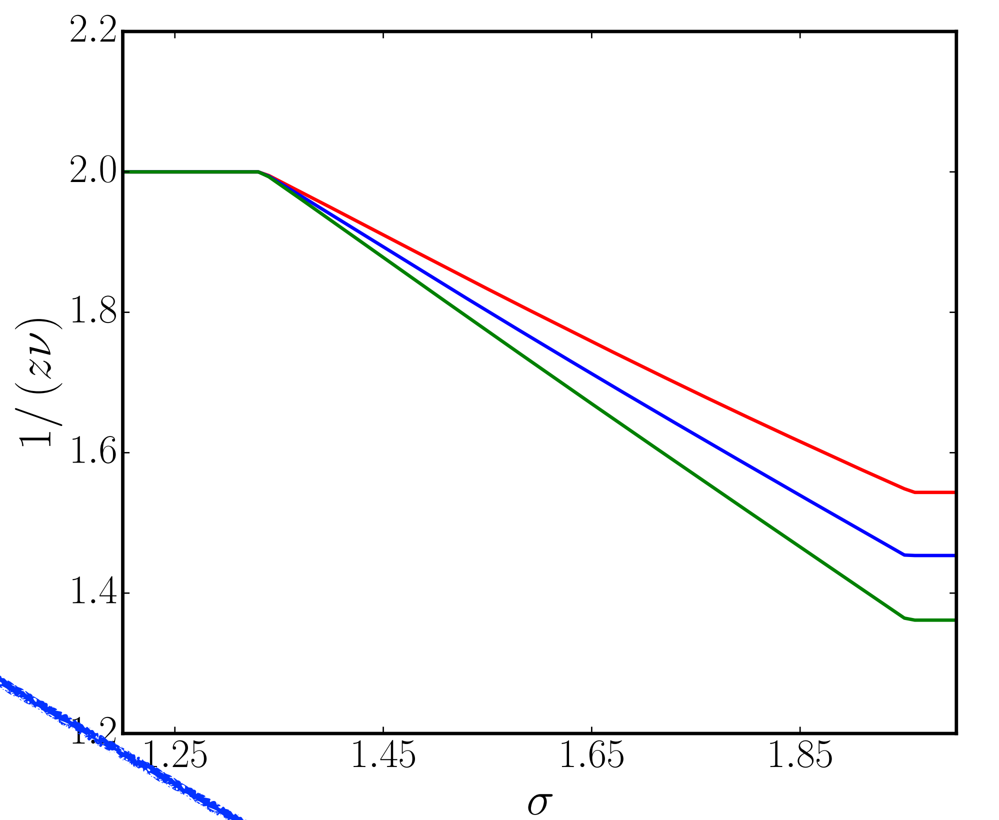
Dynamical Critical Exponent



Correlation Length Exponent

Stability analysis → $\bar{U}_k(\bar{\rho}) = \bar{U}^*(\bar{\rho}) + k^{y_t} u(\bar{\rho})$





Good (but not perfect) agreement

Exact Behavior

Finite size effects

Investigate BKT behavior

N=2 quantum
rotor model d=1

↓

Classical O(2)
model in d=2

↓

Sine-Gordon
Model

Sine-Gordon Model

XY Model $d = 2$

$$H = -J \sum_{\langle ij \rangle} \cos(\theta_i - \theta_j)$$

Kosterlitz-Thouless

Coulomb Gas

$$H = -\sum_{i \neq j} q_i q_j \log \left| \frac{r_j - r_i}{a} \right|$$

sine-Gordon

$$S = \int d^d x \left\{ \partial_\mu \varphi \partial_\mu \varphi + u(1 - \cos(\beta\varphi)) \right\}$$

U(1) field theory $d = 2$

$$S = \int d^d x \left\{ \partial_\mu \varphi^* \partial_\mu \varphi + U(\varphi^* \varphi) \right\}$$

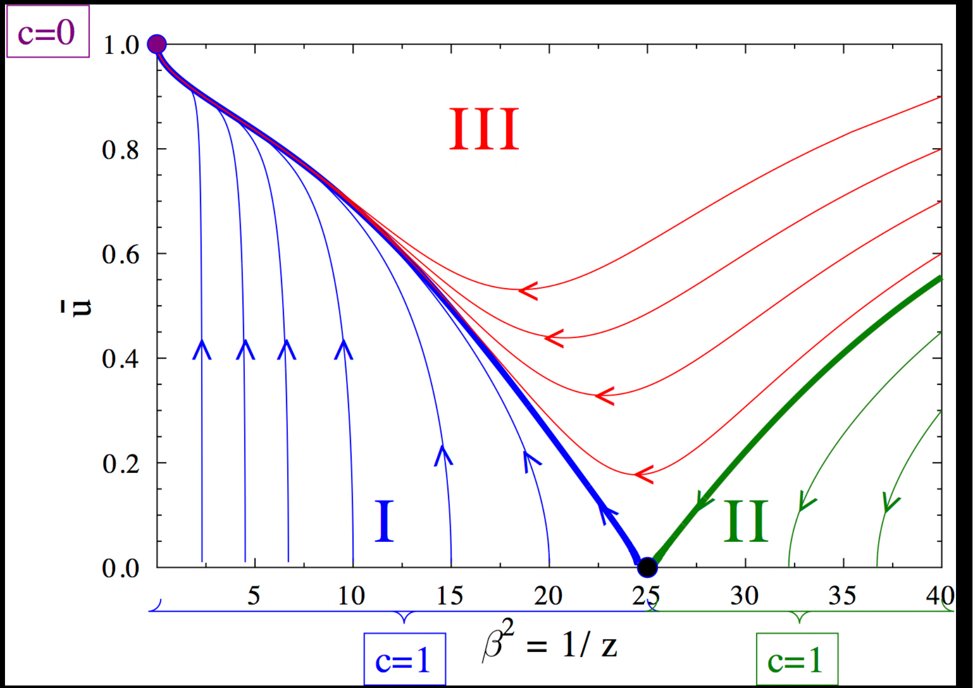
Phase Diagram

Effective Action Ansatz

$$\Gamma_k = \int d^d x \left\{ z_k \partial_\mu \tilde{\varphi} \partial_\mu \tilde{\varphi} + u_k \cos(\tilde{\varphi}) \right\}$$

$$(2 + \partial_t)\tilde{u}_k = \frac{1}{2\pi\, z_k \tilde{u}_k}\left(1 - \sqrt{1 - \tilde{u}_k^2}\right)$$

$$\partial_t z_k = -\frac{1}{24\pi}\frac{\tilde{u}_k^2}{\sqrt{1-\tilde{u}_k^2}^{\,3}}$$

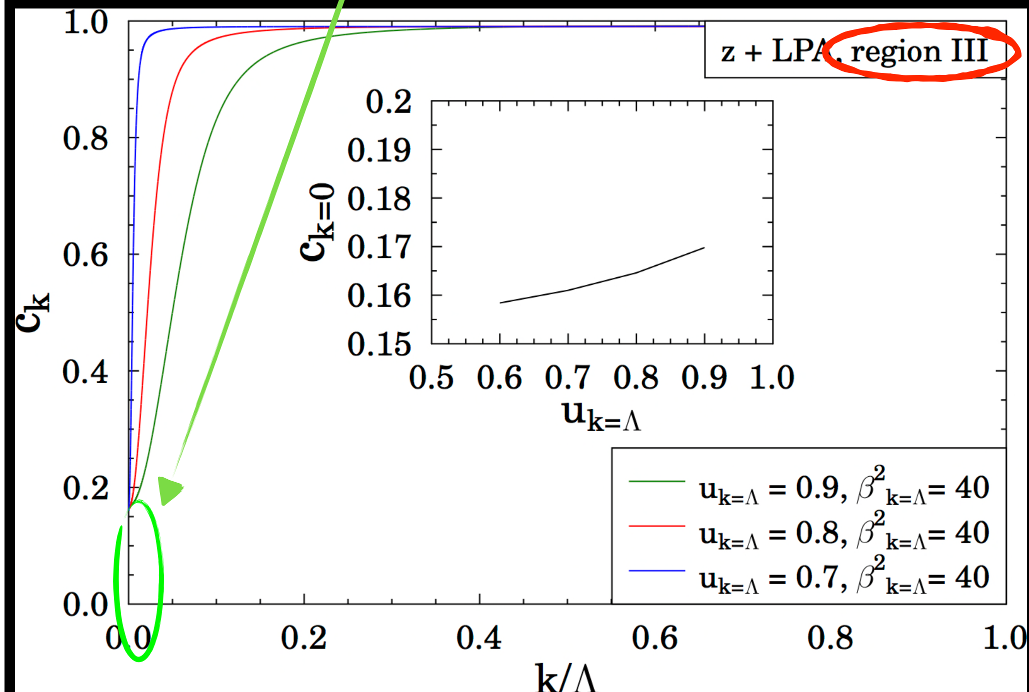


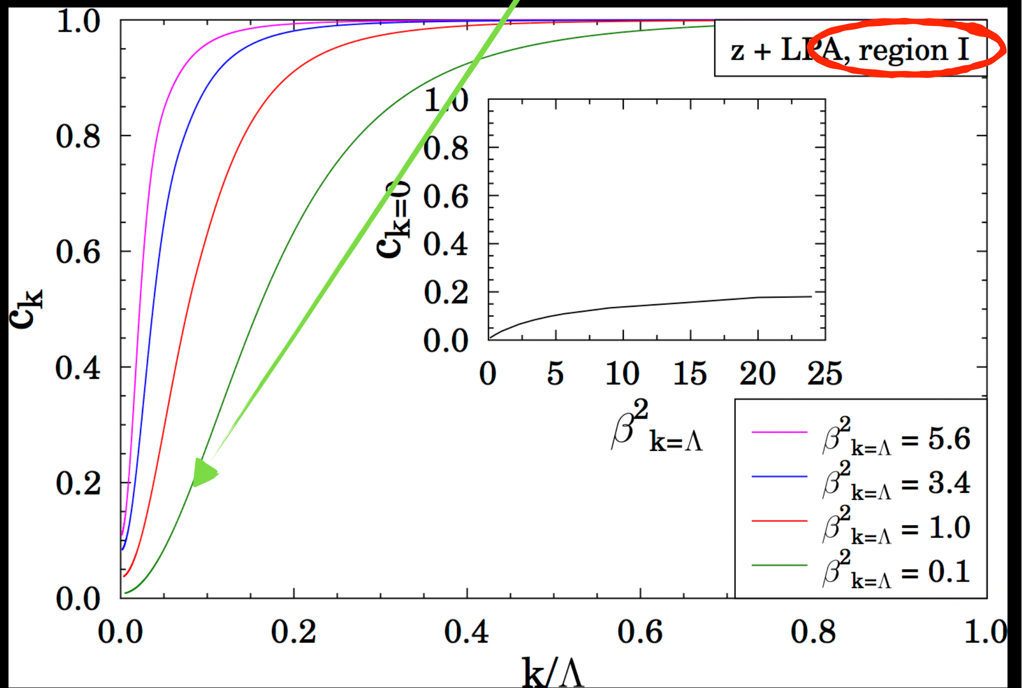
C-function

High frequency contribution?

$$\partial_t c_k = \frac{\partial_t \tilde{u}_k}{(1+\tilde{u}_k)^3}$$

Perturbative Limit





Future Perspective

- Clarify boundary behavior for N=2.
- Study out of equilibrium dynamics.
- Compute non universal quantities.
- Investigate Strong LR region $\sigma < 0$

Acknowledgements

Andrea
Trombettoni

Stefano
Ruffo

Thank You