Surface tension in a model with diffuse interface

Abstract The purpose is to compute the surface tension in a variant of a SOS model where the interface is not sharp but diffuse. The original system is the Ising model with Kac potentials that I study at the mesoscopic level. The mesoscopic states are functions $m(i,y)$, $i \in \mathbb{Z}$, $y \in \mathbb{R}$, $m(i,y) \in [-1,1]$. The excess free energy of a state is

$$\begin{aligned} F(m) &= \sum_i \{ \int dy\, f_{\beta,\lambda}(m(i,y)) + \frac{1}{4} \int \int dy dy' J(y,y') [m(i,y) - m(i,y')]^2 \\ &+ \frac{\lambda}{2} \int dy [m(i,y) - m(i+1,y)]^2 \} \end{aligned}$$

where $f_{\beta,\lambda}(m)$ is a symmetric double well with minimum 0 at $\pm m_{\beta,\lambda}$. The first two terms describe the free energy of a system with Kac potentials on "the vertical line i", the last term the interaction among lines, the vertical interaction is not local while the horizontal one is local. Both penalize departures from the minimizer $m(i,y) \equiv m_{\beta,\lambda}$ or $m(i,y) \equiv -m_{\beta,\lambda}$. We consider states $m(i,y)$ such that for all i $\lim_{y \pm \infty} m(i,y) = \pm m_{\beta,\lambda}$, so that on each vertical line there is coexistence of the plus and minus phases. We want the interface to have given slope, following Funaki and Spohn this is achieved by imposing periodicity: namely the state $\{m(i,y)\}$ is (a,L)-periodic, $a > 0$, $L \in \mathbb{N}$, if $m(i,y) = m(i+L, y+aL)$ for all i and y. Then the interface has slope a. The problem is to minimize the free energy on this class of functions. This is done in two steps:

- Find the minimizer on the class of $(a,1)$-periodic functions.
- Prove that the minimizer on the class of (a,L)-periodic functions is $(a,1)$-periodic.

The first part is complete, I have proved uniqueness of the minimizer (modulo vertical translations) and strong stability properties. I am still working on the second part.