

P. Diaconis - ZOOM - Random walk on the Rado graph and Hardy's inequality for trees

jeudi 27 octobre 2022 17:00 (1 heure)

The Rado graph R is a natural limit of the set of all finite graphs. One way to think of it is: on \mathbb{N} (natural numbers) flip a fair coin for each pair of vertices and put an edge in if it comes up heads. Each vertex has infinite degree and the diameter is 2. In joint work with Sourav Chatterjee and Laurent Miclo we study a natural laplacian: pick a positive probability $Q(j)$ on \mathbb{N} . From i , the walk chooses a nearest neighbor of i (in R) with probability proportional to $Q(j)$. This walk has a stationary distribution and one may ask about rates of convergence to stationarity. The main result studies $Q(i) = 1/2^{(i+1)}$ and shows that, starting from i , $\log_2^* i$ steps suffice for convergence and, are needed for infinitely many i . The analysis uses a novel form of weighted Hardy inequalities for trees; Hardy's inequalities with weights are familiar on \mathbb{R} but even on \mathbb{R}^d are a poorly developed tool. We develop the version on infinite trees and use it to get a spectral gap for the walk and to give a new picture of the geometry of the graph R . I will try to explain R and Hardy's inequalities (and the application) in 'mathematical English'. Understanding the problem for other Q (eg $Q(j) = 1/j^2$) is open.