ROUND MEANFIELD / crowd-opinion-cells

YPATIA WORKSHOP

Titres et Résumés — Titoli e Riassunti

2022

Nathalie Ayi & **Nastassia Pouradier Duteil**, Sorbonne Université

Continuum limits of collective dynamics with time-varying weights

Résumé/Riassunto: We introduce an augmented model for first-order opinion dynamics, in which a weight of influence is attributed to each agent. Each agent's influence on another agent's opinion is then proportional not only to the classical interaction function, but also to the agent's weight. The weights evolve in time and their equations are coupled with the opinions' evolution. We exhibit different kinds of long-term behavior, such as emergence of a single leader and emergence of two co-leaders. We then focus on two different macroscopic limits of this microscopic system, beginning with the so-called "graph limit", which translates the concept of indices to an infinite-dimensional system, and which can be derived even when the microscopic system does not preserve indistinguishability. When indistinguishability is preserved, we derive the mean-field limit, and obtain a transport equation with source, where the transport term corresponds to the opinion dynamics, and the source term comes from the weight redistribution among the agents. We show the convergence of the microscopic model to both the graph limit equation and the mean-field limit equation, and show the subordination of the mean-field equation to the graph limit one. All results are illustrated with numerical simulations.

Benoît Bonnet-Weill, CNRS LAAS Toulouse

Some results related to consensus formation in graphon dynamics

Résumé/Riassunto: The analysis of consensus formation is a fairly hold topic, which started to interest people in physics- and engineering-related communities at the end of the nineties. Shortly after, in the beginning of the years 2000, some general and rather sharp conditions ensuring the convergence to consensus and bearing on the dynamics of the system had been put forth in a series of independent papers.

Lately, a trend of mathematical research at the interface between multiagent systems analysis and kinetic theory focused on the generalisation of these stability results to the setting of macroscopic approximations ; e.g. via the meanfield procedure. While being both meaningful and flexible, the latter can only account for symmetric interaction topologies, thus excluding a large score of models which are relevant in applications.

So as to paliate this intrinsic limitation, a couple of recent papers have started studying this problem in the framework of graph limits, which is better-suited to handling asymmetric interaction.

In this talk, I will discuss two families of sufficient conditions ensuring consensus formation for macroscopic approximations of first-order multiagent dynamics described by graphons, derived in collaboration with N. Pouradier Duteil and M. Sigalotti. These latter will involve suitable infinite-dimensional generalisations of the so-called scrambling coefficient and algebraic connectivity of the interaction topology, which are known to be intimately connected to the asymptotic behaviour of cooperative systems in the finite-dimensional setting.

Giacomo Borghi & **Lorenzo Pareschi**, WTH Aachen University & Università degli Studi di Ferrara

On stochastic particle systems in global optimization

Riassunto/Résumé : In this talk we will review some recent results on the global minimization of a possibly non-smooth and non- convex high-dimensional objective function by gradient-free stochastic particle methods. In the first part we will focus on the mean field limit of such methods, considering in detail the case of the well-known particle swarm optimization (PSO) and the recently introduced consensus-based optimization (CBO). In the second part we consider some recent extensions to multi-objective problems with uniform approximation of the Pareto front. Rigorous results regarding the mean-field limit and the convergence of the methods to the global minimum are presented together with some applications to machine learning problems.

Emiliano Cristiani, IAC, CNR

A-round mean-field games in pedestrian dynamics

Riassunto/Résumé : In this talk we investigate a generalized MFG model for pedestrian flow, characterized by the fact that pedestrians do have predictive abilities, but limited in time, extending only up to Dt time units into the future, where Dt is a model parameter. 1) For Dt=0 (no predictive abilities), we recover the modeling assumptions of the Hughes's model, where people take decisions on the basis of the current position of the crowd only. 2) For a large Dt, instead, we recover the standard MFG setting, where people are able to forecast the behavior of the others at any future time and take decisions on the basis of the current and future position of the whole crowd. 3) For intermediate values of Dt we obtain something different: as in the Hughes's model, the numerical procedure to solve the problem requires to run an off-line procedure at any fixed time t, which returns the current optimal velocity field at time t by solving an associated backward-in- time Hamilton–Jacobi–Bellman equation; but, differently from the Hughes's model, here the procedure involves a prediction of the crowd behavior in the sliding time window [t,t+Dt), therefore the optimal velocity field is given by the solution to a forward-backward system which joins a Fokker–Planck equation with a Hamilton–Jacobi–Bellman equation as in the MFG approach. The fact that a different forward- backward system must be solved at any time t arises new interesting theoretical questions. Numerical tests will give some clues about the well-posedness of the problem.

Pierre Degond, CNRS Université Paul Sabatier, Toulouse

Mean-field limit of systems of topologically interacting particles

Résumé/Riassunto: We consider a system of particles subject to pairwise interactions which only depend on the proximity rank of the target particle with respect to the test particle. An example is when the particles interact only with their closest neighbor with a fixed interaction intensity. In the literature, this type of interaction is called 'topological', by contrast to the classical case where the interaction strength depends on the distance between the two interacting particles, in which case the interaction is called 'metric'. The mean-field limit of this interacting particle system is either a non- local equation in space, or a diffusion equation in space. This contrasts with the metric case, where the mean-field equation is either nonlocal or a diffusion equation in velocity. The case where the limit equation is nonlocal is treated in full mathematical rigor, while the case of the diffusion equation is formal only. This is a series of works involving collaborations with Adrien Blanchet (Toulouse Capitole) and Mario Pulvirenti (Rome 1).

Marco Di Francesco, Università degli Studi dell'Aquila

The "follow-the-leader" approximation of one-dimensional nonlinear transport equations

Riassunto/Résumé : I will review recent results contained in a series of papers in collaboration with M. Rosini (Ferrara), S. Fagioli (L'Aquila), G. Stivaletta (L'Aquila), E. Radici (EPFL), G. Russo (Catania) on the approximation of entropy solutions to 1d scalar conservation laws via "follow-the-leader" particle systems. I shall also briefly describe extensions to nonlocal interaction equations and pedestrian movements.

Marta Menci & **Roberto Natalini**, IAC, CNR

Mean-Field limit for hybrid models with chemotaxis: theory and numerics

Riassunto/Résumé : In this two-persons presentation we focus on a quite general class of hybrid mathematical models of collective motions of cells under the influence of chemical stimuli. The models are hybrid in the sense that cells are discrete entities given by ODE, while the chemoattractant is considered as a continuous signal which solves a diffusive equation. For these models it is possible to prove the mean-field limit in the Wasserstein distance to a system given by the coupling of a Vlasov-type equation with the chemoattractant equation. This approach and results are not based on empirical measures, but rather on marginals of an increasing number of individuals densities, and we show the limit with explicit bounds, by proving also existence and uniqueness for the limit system. In the monokinetic case we derive a new pressureless nonlocal Euler-type model with chemotaxis. Numerical simulation comparing the different scales are presented. These results have been obtained by Roberto Natalini and Thierry Paul in collaboration, for the numerical part, with Marta Menci.

Thierry Paul & **Emmanuel Trélat**, Sorbonne Université & CNRS, IRL LYSM (TP)

Considerations on micro/macroscopic scale equations

Résumé/Riassunto: Various ways to pass to the limit a given family of finite-dimensional particle systems are considered, either by mean field limit, deriving the Vlasov equation, or by hydrodynamic or graph limit, obtaining the Euler equation, together with convergence estimates. We also show how to pass from Liouville to Vlasov or to Euler by taking adequate moments. As a surprising consequence of our analysis solutions of any quasilinear PDE, under appropriate regularity assumptions, can be approximated by the solutions of finite-dimensional particle systems

David Poyato, Université Claude Bernard Lyon 1

Mean field limit of non-exchangeable multi-agent systems

Résumé/Riassunto: In this talk I will discuss on a recent derivation of the mean-field limit for multi-agent systems on a large class of sparse graphs. More specifically, the case of non-exchangeable multi-agent systems consisting of non-identical agents is addressed, where the heterogeneous distribution of connectivities in the network is known to have critical effects on the collective dynamics. Our method of proof does not only involve PDEs and stochastic analysis, but also graph theory through a novel concept of limits of sparse graphs (extended graphons) for the structure of the network, which can be regarded as a new non-trivial extension of the seminal works by L. Lovasz and B. Szegedy for dense graph limits. Our proof allows removing some of the main restrictive hypotheses in the previous literature on the connectivities between agents (dense graphs) and the cooperation between them (symmetric interactions). This is a joint work with Pierre-Emmanuel Jabin (Penn State University) and Juan Soler (University of Granada).

Mario Pulvirenti, Sapienza Università di Roma

Propagation of chaos for a stochastic system modelling epidemics via a coupling approach

Riassunto/Résumé : The well known SIR model for epidemics (1927) establishes an evolution equation for a population constituted by Susceptible, Infected and Recovered individuals. It is natural to investigate a more detailed (kinetic) model taking into account spatial patterns and the motion of the agents. In a series of papers in collaboration with A. Ciallella and S. Simonella we propose and study a simple stochastic model for the probability distribution of such agents and, in a suitable mean-field limit, we deduce a system of kinetic equations in agreement with the original SIR model when taking the averages. In deducing these kinetic equations we use a coupling method.

Stefano Rossi, Sapienza Università di Roma

Mean-field limit for particle systems with topological interactions

Riassunto/Résumé : In this talk, I will consider a system of self-propelled agents interacting via a "topological" interaction. This interaction does not depend on the metric distance between agents. but rather on the proximity ranking. I will focus on the existence of the N-particle dynamics for this model and on the proof of the mean-field limit. The latter, due to the singular character of the interaction, cannot be obtained by means of the usual Dobrushin approach.