Computer Algebra for Functional Equations in Combinatorics and Physics

Report of Contributions

https://indico.math.cnrs.fr/e/8115
Creative Telescoping by Shaoshi Chen, Manuel Kauers & Christoph Koutschan. 9:30-12:00. IHP, Amphitheater Hermite

Monday, 27 November 2023 09:30 (2h 30m)
Creative Telescoping by S. Chen, M. Kauers & C. Koutschan. 9:30-12:00. IHP, Amphitheater Hermite

Tuesday, 28 November 2023 09:30 (2h 30m)
Creative Telescoping by S. Chen, M. Kauers & C. Koutschan. 9:30-12:00. IHP, Amphitheater Hermite

Wednesday, 29 November 2023 09:30 (2h 30m)
Creative Telescoping by S. Chen, M. Kauers & C. Koutschan. 9:30-12:00. IHP, Amphitheater Hermite

Thursday, 30 November 2023 09:30 (2h 30m)
Creative Telescoping by S. Chen, M. Kauers & C. Koutschan. 9:30-12:00. IHP, Amphitheater Hermite

Friday, 1 December 2023 09:30 (2h 30m)
A generating function method for the determination of differentially algebraic integer sequences modulo prime powers by Christian Krattenthaler. 15:00-17:00.
IHP, Amphitheater Hermite

Monday, 27 November 2023 15:00 (2 hours)
A generating function method for the determination of differentially algebraic integer sequences modulo prime powers by C. Krattenthaler. 15:00-17:00. IHP, Amphitheater Hermite

Tuesday, 28 November 2023 15:00 (2 hours)
General audience presentation. On Fermat’s penultimate theorem by Xavier Caruso, IMB, Bordeaux. 16:00-17:00. IHP

Wednesday, 29 November 2023 16:00 (1 hour)

Abstract. While reading books, Fermat often added notes in the margin for himself. The most famous one is certainly what is called nowadays Fermat’s Last Theorem which stipulates that the equation \( x^n + y^n = z^n \) has no nontrivial solution as soon as \( n > 2 \). More than three centuries of effort were needed to eventually write down a proof of this result. In this presentation, I will discuss another margin-theorem by Fermat. It is less known but still quite impressive as it took almost two centuries to the community to obtain a full proof, opening as a byproduct new topics of research.
Abstract. A Liouville number is an irrational real number with an infinite irrationality exponent. Almost all real numbers are not Liouville numbers but it can be difficult in practice to prove that a given real number is not a Liouville one. In this talk, I will explain how a combination of results due to André, Beukers and Shidlovskii enables to prove that the real values of E-functions at algebraic points are not Liouville numbers. This is a joint work with Stéphane Fischler (Université Paris Saclay).
Abstract. Creative telescoping is an algorithmic method initiated by Zeilberger to compute definite sums or integrals by synthesizing summands or integrands that telescope, called certificates. We describe a creative telescoping algorithm that computes telescopers for definite sums or integrals of D-finite functions as well as the associated certificates in a compact form. In the integral case, the algorithm relies on a generalization of the Hermite reduction in symbolic integration. In the sum case, the algorithm relies on a discrete analogue of the generalized Hermite reduction, or equivalently, a generalization of the Abramov-Petkovsek reduction. We present a Maple implementation with good timings on a variety of examples.
Welcome coffee

Monday, 4 December 2023 09:30 (30 minutes)
Welcome coffee

Tuesday, 5 December 2023 09:30 (30 minutes)
Welcome coffee

*Wednesday, 6 December 2023 09:30 (30 minutes)*
Special Day, joint with the Flajolet Seminar.
Welcome coffee

Thursday, 7 December 2023 09:30 (30 minutes)
Welcome coffee

Friday, 8 December 2023 09:30 (30 minutes)
Persistence for a class of order-one autoregressive processes and Mallows-Riordan polynomials by Kilian Raschel

Monday, 4 December 2023 10:00 (1 hour)
Abstract. The appearance of a stretched exponential term
\[ \mu n^\sigma \]
with \( \mu > 0 \) and \( \sigma \in (0,1) \) in a counting sequence \((c_n)_{n \geq 0}\) is not common, although more and more examples are appearing lately. Proving that a sequence has a stretched exponential is often quite difficult. This is in part because such a sequence cannot be "very nice": its generating function cannot be algebraic, and it can only be D-finite if it has an irregular singularity. Previously, the saddle-point method was the only generic method for proving such a phenomenon, but it requires detailed information about the generating function. Recently, together with Andrew Elvey Price and Wenjie Fang, we have developed a new method at the level of recurrences to prove stretched exponentials. I will introduce the basics of this method and show how it can be extended to other problems. Then I will summarize recent progress (new bijections, limit laws, etc.) in the study of compacted trees, a subclass of directed acyclic graphs. Finally, I will give an outlook on how these results now allow an in-depth study of limit shapes and open many new avenues for further research.
Summation Tools for Combinatorics and Elementary Particle Physics by Carsten Schneider

Abstract. The summation theory of difference rings provides general and efficient tools to derive linear recurrences for definite sums based on parameterized telescoping and to solve recurrences within the class of indefinite nested sums defined over hypergeometric products, q-hypergeometric products and more generally, mixed hypergeometric products and their nested versions. In particular, one can use these techniques to simplify definite multi-sums to representations in terms of the class indefinite nested sums defined over indefinite nested products. A special feature is that the representation of the arising sums and products are optimal in the sense that the objects interpreted as sequences (except root of unity products) do not satisfy any algebraic relations among each other. This leads not only to compact expressions in the final output, but also speeds up significantly the underlying summation algorithms. In particular, one gains a general (Galois) machinery to prove algorithmically algebraic independence of big classes of sums and products. We will illustrate this algorithmic difference ring toolbox by non-trivial applications coming from enumerative combinatorics and elementary particle physics.
Coffee break

*Monday, 4 December 2023 16:00 (30 minutes)*
Contribution ID: 20

TBA by Igor Pak

Monday, 4 December 2023 16:30 (1 hour)
Galois group for large steps walks by Charlotte Hardouin

Tuesday, 5 December 2023 10:00 (1 hour)

Abstract. In this talk, we will present some Galois theoretic tools to study large steps walks confined in the quadrant. We generalize in particular the notion of group of the walk introduced by Bousquet-Mélou and Mishna for small steps walk to the large steps framework. This allows to develop algorithms and criteria to test the existence of invariants and decoupling functions. This is a collaboration with Pierre Bonnet (Labri).
Abstract. Quadratization problem is, given a system of ODEs with polynomial right-hand side, transform the system to a system with quadratic right-hand side by introducing new variables. Such transformations have been used, for example, as a preprocessing step by model order reduction methods and for transforming chemical reaction networks. We will present a recent algorithm for computing such transformations and its extensions including systems with control and of varying dimension. The talk is based on joint works with Andrey Bychkov, Opal Issan, and Boris Kramer.
New Software for Analytic Combinatorics by Stephen Melczer

Tuesday, 5 December 2023 15:00 (1 hour)

Abstract. This talk surveys some recently developed software for analytic combinatorics, including an extension to the Sage ore_algebra package for the asymptotics of P-recursive sequences with explicit error terms (used for certifying sequence positivity), and the new sage_acsv package for rigorous multivariate asymptotics using the tools of Analytic Combinatorics in Several Variables (ACSV).
Coffee break

*Tuesday, 5 December 2023 16:00 (30 minutes)*
Coffee break

Monday, 11 December 2023 10:45 (30 minutes)
Submodule approach to creative telescoping by
Mark van Hoeij

Tuesday, 5 December 2023 16:30 (1 hour)

Abstract. In their 1999 paper, Hendriks and Singer introduced Liouvillian solutions of linear difference equations with rational function coefficients. They proved that finding such solutions if they exist reduces to finding first order factors of certain auxiliary equations. Recently, an algorithm and implementation to find higher order factors has become available. This raises the question how to generalize the definition of Liouvillian solutions in order to benefit from this new capability, how to design a corresponding solver, and how to prove its completeness for this more general class of solutions.
Coffee break

*Thursday, 7 December 2023 16:00 (30 minutes)*
Some problems I'd like solved, from a user of computer algebra by Alan Sokal

Wednesday, 6 December 2023 10:00 (1 hour)
Efficient algorithms for differential equations satisfied by Feynman integrals by Pierre Vanhove

Wednesday, 6 December 2023 11:00 (1 hour)

Abstract. Feynman integrals are a type of mathematical function that are important for precision measurements in physics. They are notoriously difficult to evaluate, and a lot of effort has been devoted to developing efficient analytic and numerical methods for doing so. In this talk, we will present a new algorithm for determining the minimal order Picard-Fuchs operator associated with a given Feynman integral. This operator is a differential operator that governs the analytic behavior of the Feynman integral. We will first discuss an implementation of the Griffiths-Dwork algorithm for the case of Feynman integrals in integer spacetime dimension. In that case, the integrand of the Feynman integral is a rational differential form to which the Griffiths-Dwork reduction is applied for determining the Picard-Fuchs operator. We will then extend this algorithm to the case of generic spacetime dimension. In this case, one needs to consider twisted cohomology. We will show how the knowledge of the Picard-Fuchs operator can be deduced by Hodge theoretic considerations on the variation of the mixed Hodge structure associated to the Feynman integral. This talk is based on work with Pierre Lairez, Eric Pichon-Pharabod, Charles Doran, and Andrew Harder.
Cocktail. IHP, Perrin building, 2nd floor

Wednesday, 6 December 2023 18:30 (2h 15m)

Emmy Noether salon located at the second floor of the Perrin building (IHP). We’ll need to have a precise list of participants at the entrance to the building. If you were not registered before December or if you have any doubts, please register or contact the organizers.
Computer algebra for the study of two-dimensional exclusion processes by Arvind Ayyer

Thursday, 7 December 2023 15:00 (1 hour)

Abstract. The multispecies totally asymmetric long-range exclusion process (mTALREP) is an interacting particle system with multiple species of particles on a finite ring where the hopping rates are site-dependent. (The homogeneous variant on Z is also known as the Hammersley–Aldous–Diaconis process.) In its simplest variant with a single species, a particle at a given site will hop to the first available site clockwise. We show that the partition function of this process is intimately related to the classical Macdonald polynomial and to the multispecies totally asymmetric simple exclusion process (TASEP). We also show that well-known families of symmetric polynomials appear as expectations in the stationary distribution of important observables. This is joint work with James Martin and Omer Angel.
Self-avoiding walks in a square and the gerrymander sequence by Tony Guttmann

Thursday, 7 December 2023 11:00 (1 hour)

Abstract. We give an improved algorithm for the enumeration of self-avoiding walks and polygons within an N×N square as well as SAWs crossing a square. We present some proofs of the expected asymptotic behaviour as the size N of the square grows, and then show how one can numerically estimate the parameters in the asymptotic expression. We then show how the improved algorithm can be adapted to count gerrymander sequences (OEIS A348456), and prove that the asymptotics of the gerrymander sequence is similar to that of SAWs crossing a square. This work has been done in collaboration with Iwan Jensen, and in part with Aleks Owczarek.
Proving positivity for P-recursive sequences by Veronika Pillwein

Abstract. In this talk we consider the problem of automatically proving inequalities involving sequences that are only given in terms of their defining recurrence relations. We will consider sequences satisfying linear recurrences with constant coefficients (C-recursive), linear recurrences with polynomial coefficients (P-recursive or holonomic), or certain systems of polynomial non-linear recurrences (admissible). Even when restricting to the simplest class, C-recursive, and the positivity problem, decidability is only known for orders up to five. And yet, there are computer algebra methods that try to tackle this problem. Obviously, they do not succeed on all types of input and so even though correctness can be proven, termination is typically an issue. In this talk, we will give an overview on this topic, share some available methods and showcases where algorithmic proofs actually succeeded.
A new proof of Viazovska’s modular form inequalities for sphere packing in dimension 8 by Dan Romik

Thursday, 7 December 2023 16:30 (1 hour)

Maryna Viazovska in 2016 found a remarkable application of the theory of modular forms to a fundamental problem in geometry, obtaining a solution to the sphere packing problem in dimension 8 through an explicit construction of a so-called “magic function” that she defined in terms of classical functions, the Eisenstein series and Jacobi thetanull functions. The same method also led shortly afterwards to the solution of the sphere packing in dimension 24 by her and several collaborators. One component of Viazovska’s proof consisted of proving a pair of inequalities satisfied by the modular forms she constructed. Viazovska gave a proof of these inequalities that relied in an essential way on computer calculations. In this talk I will present a new proof of Viazovska’s inequalities that uses only elementary arguments that can be easily checked by a human.
A partition of a positive integer \( n \) is a non-increasing sequence of positive integers whose sum is \( n \). A partition identity is a theorem stating that for all \( n \), the number of partitions of \( n \) satisfying some conditions equals the number of partitions of \( n \) satisfying some other conditions. In this talk, we will show how functional equations and computer algebra can be used to prove such identities. In particular we will discuss a semi-automatic method using recurrences and q-difference equations, and what would be needed to make it fully automatic.
Beyond Painlevé: The need for computational tools to reveal hidden structure by Nicholas Witte

Friday, 8 December 2023 11:00 (1 hour)

Abstract. The simplest examples of integrability in mathematical physics - spectral distributions of fundamental random matrix ensembles or the diagonal spin-spin correlations of the square lattice Ising model - to give just two examples, reveal the importance and practical utility of the six Painlevé equations in our understanding of these models. Few aspects of this understanding escape the presence of these equations, whether it is the application of Riemann-Hilbert methods to the asymptotic limits as matrix ranks or spin separations tend to infinity or the relationships inherited from the affine Weyl group symmetries of the Painlevé equations. This understanding operates in both directions: fundamental understanding of the Painlevé equations provides the most powerful and rigorous tools to characterise a variety of statistics applied to the models, and these applications are driving and motivating a lot of studies into the pure mathematics at the core of these equations. However today we coming up against numerous examples where one can pose questions just outside the simplest models: e.g. the singular value distribution of a product of two Ginibre random matrices, or the diagonal correlations of the triangular Ising model or even the off-diagonal correlations of the square lattice model, and one is beyond the six Painlevé set or even the discrete Sakai scheme. Integrability is still present and some of this territory - the Garnier, Fuji-Suzuki, Sasano and matrix-Painlevé systems - has been sketched out but there are gaps in our understanding and our tool-box is missing critical tools that one needs. In some senses the complexity has grown and it is often impractical to do hand-calculations and when computer-assisted systems are employed our best results so far cannot be analysed by hand. There are hints that simplicity is achievable yet the standard computer assisted algorithms do not find them. So the challenge is to develop some flexible, model-adapted algorithms for doing what are essentially algebraic geometry calculations. A number of examples, including those mentioned previously, will illustrate this state of affairs.
Opening remarks for Topical day: Elimination for Functional Equations. IHP, Amphitheater Darboux

Monday, 11 December 2023 09:45 (15 minutes)
Differential Equation Invariance Axiomatization by André Platzer

Monday, 11 December 2023 10:00 (45 minutes)
Differential elimination ideals and spectral curves by
Sonia Rueda

Monday, 11 December 2023 11:15 (45 minutes)
Geometry-driven algorithms for combinatorial functional equations by Hadrien Notarantonio

Monday, 11 December 2023 14:00 (30 minutes)
Examples of use of functional equations obtained from the elimination theory in nonlinear models by Nathalie Verdière

Monday, 11 December 2023 14:30 (45 minutes)
Coffee break

*Monday, 11 December 2023 15:15 (30 minutes)*

Contribution ID: 42

Type: *not specified*
On some functional equations for maps by
Alexandros Singh

Monday, 11 December 2023 15:45 (30 minutes)
Solving differential elimination problems with Thomas decomposition by Daniel Robertz

Monday, 11 December 2023 16:15 (45 minutes)
Closing remarks

*Monday, 11 December 2023 17:00 (15 minutes)*
Computer algebra in my combinatorics life by Mireille Bousquet-Mélou

Thursday, 7 December 2023 10:00 (1 hour)

Abstract. Many of my papers would just not exist without computer algebra. I will describe how CA has become an essential tool in my research in enumerative combinatorics. The point of view will be that of a (sometimes naive) user, not of an expert. Many examples and questions will be taken from a joint paper with Michael Wallner dealing with the enumeration of king walks avoiding a quadrant (arXiv 2021, to appear). My hope is that some of the questions that I will raise will have an immediate answer ("yes, this is done") and/or that some people in the audience will find a question interesting enough to take it back home.
General audience presentation The Zeta Team by Wadim Zudilin, Radboud University. 15:00-16:00. IHP, Amphitheater Hermite

Wednesday, 29 November 2023 15:00 (1 hour)

Abstract. An innocent story of finding recursions for the sums of powers of binomial coefficients transformed into an intrigue involving creative telescoping and irrationality. The goalscorers performed as a dream team.
Special session of the Differential Seminar.
Rounding error analysis of linear recurrences using generating series by Marc Mezzarobba, LIX, Palaiseau. 16:00-17:00. IHP, Amphitheater Hermite

Thursday, 30 November 2023 16:00 (1 hour)

Abstract. In the computation of linearly recurrent sequences, round-off errors arising from each step tend to "cancel out" rather than just accumulate. This phenomenon is crucial to consider when aiming to establish realistic error bounds. That requires a close examination of how a given round-off error propagates through the remaining iterations of the recurrence. Doing so using traditional "sequence" notations results in intricate calculations involving nested sums. However, in other contexts, such as enumerative combinatorics, the use of generating series to encode sequences simplifies these calculations while simultaneously granting access to powerful new analytic tools. In this talk, I will show how the idea of using generating series can be adapted to the error analysis of numerical algorithms based on linear recurrence relations.
Special session of the Differential Seminar.
Combinatorics and Transcendence: Applications of Inhomogeneous order 1 iterative functional equations by Marni Mishna, Simon Fraser University.
16:00-17:00. IHP, Amphitheater Hermite

Friday, 1 December 2023 16:00 (1 hour)

The problem of understanding the structure of transcendental objects has fascinated mathematicians for well over a century. Combinatorics provides an intuitive framework to study power series. A combinatorial family is associated to a power series in $\mathbb{Q}[[t]]$ via its enumerative generating function wherein the number of objects of size $n$ is the coefficient of $t^n$. Twentieth century combinatorics and theoretical computer science provided characterizations of classes with rational and algebraic generating functions. Finding natural extensions of these correspondences has been a motivating goal of enumerative combinatorics for several decades. This talk will focus on differentially transcendental functions.

In particular, I will present recent work completed with Lucia Di Vizio and Gwladys Fernandes which characterizes solutions $f(t)$ of order 1 iterative equations of the form $f(R(t)) = a(t)f(t) + b(t)$ where $R$, $a$, and $b$ are rational functions. These appear in the study of complete trees, walks on self-similar graphs (e.g. the Sierpinski graph), and pattern avoiding permutations. The proof strategy is inspired by the Galois theory of functional equations, and relies on the property of the dynamics of $R(t)$, Liouville-Rosenlicht’s theorem and Ax’ theorem. This program and has led to progress in identifying the differential transcendence of combinatorial generating functions arising in the literature, and indeed generally.