## Computer Algebra for Functional Equations in Combinatorics and Physics



# Rapport sur les contributions 

# Creative Telescoping by Shaoshi Chen, Manuel Kauers \& Christoph Koutschan. 9:30-12:00. IHP, Amphitheater Hermite 

lundi 27 novembre 2023 09:30 (2h 30m)

# Creative Telescoping by S. Chen, M. Kauers \& C. Koutschan. 9:30-12:00. IHP, Amphitheater Hermite 

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vendredi 1 décembre 2023 09:30 (2h 30m)

# A generating function method for the determination of differentially algebraic integer sequences modulo prime powers by Christian Krattenthaler. 15:00-17:00. IHP, Amphitheater Hermite 

lundi 27 novembre 2023 15:00 (2 heures)

# A generating function method for the determination of differentially algebraic integer sequences modulo prime powers by C. Krattenthaler. 15:00-17:00. IHP, Amphitheater Hermite 

mardi 28 novembre 2023 15:00 (2 heures)

# General audience presentation. On Fermat's penultimate theorem by Xavier Caruso, IMB, Bordeaux. 16:00-17:00. IHP 

mercredi 29 novembre 2023 16:00 (1 heure)


#### Abstract

While reading books, Fermat often added notes in the margin for himself. The most famous one is certainly what is called nowadays Fermat's Last Theorem which stipulates that the equation $\mathrm{xn}+\mathrm{yn}=\mathrm{zn}$ has no nontrivial solution as soon as $\mathrm{n}>2$. More than three centuries of effort were needed to eventually write down a proof of this result. In this presentation, I will discuss another margin-theorem by Fermat. It is less known but still quite impressive as it took almost two centuries to the community to obtain a full proof, opening as a byproduct new topics of research.


# Special session of the Differential Seminar. Values of E-functions are not Liouville numbers by Tanguy Rivoal, Institut Fourier. 15:00-16:00. IHP, Amphitheater Hermite 

jeudi 30 novembre 2023 15:00 (1 heure)


#### Abstract

A Liouville number is an irrational real number with an infinite irrationality exponent. Almost all real numbers are not Liouville numbers but it can be difficult in practice to prove that a given real number is not a Liouville one. In this talk, I will explain how a combination of results due to André, Beukers and Shidlovskii enables to prove that the real values of E-functions at algebraic points are not Liouville numbers. This is a joint work with Stéphane Fischler (Université Paris Saclay).


# Special session of the Differential Seminar. <br> Computation of sums and integrals by reduction-based creative telescoping by Bruno Salvy, Inria, ENS Lyon. 15:00-16:00. IHP, Amphitheater Hermite 

vendredi 1 décembre 2023 15:00 (1 heure)


#### Abstract

Creative telescoping is an algorithmic method initiated by Zeilberger to compute definite sums or integrals by synthesizing summands or integrands that telescope, called certificates. We describe a creative telescoping algorithm that computes telescopers for definite sums or integrals of D-finite functions as well as the associated certificates in a compact form. In the integral case, the algorithm relies on a generalization of the Hermite reduction in symbolic integration. In the sum case, the algorithm relies on a discrete analogue of the generalized Hermite reduction, or equivalently, a generalization of the Abramov-Petkovsek reduction. We present a Maple implementation with good timings on a variety of examples.


## Welcome coffee

lundi 4 décembre 2023 09:30 (30 minutes)

## Welcome coffee

mardi 5 décembre 2023 09:30 (30 minutes)

## Welcome coffee

## Welcome coffee

jeudi 7 décembre 2023 09:30 (30 minutes)

Special Day, joint with the Flajolet Seminar.

## Welcome coffee

# Persistence for a class of order-one autoregressive processes and Mallows-Riordan polynomials by Kilian Raschel 

lundi 4 décembre 2023 10:00 (1 heure)


#### Abstract

We establish exact formulae for the (positivity) persistence probabilities of an autoregressive sequence with symmetric uniform innovations in terms of certain families of polynomials, most notably a family introduced by Mallows and Riordan as enumerators of finite labeled trees when ordered by inversions. The connection of these polynomials with the volumes of certain polytopes is also discussed. Two further results provide factorizations of general autoregressive models, one for negative drifts with continuous innovations, and one for positive drifts with continuous and symmetric innovations. The second factorization extends a classical universal formula of Sparre Andersen for symmetric random walks. Our results also lead to explicit asymptotic estimates for the persistence probabilities. This is a joint work with Gerold Alsmeyer, Alin Bostan and Thomas Simon (Adv. Appl. Math., 2023).


# Stretched exponentials and beyond by Michael Wallner 

lundi 4 décembre 2023 11:00 (1 heure)


#### Abstract

The appearance of a stretched exponential term $\mu^{n^{\sigma}}$ with $\mu>0$ and $\sigma \in(0,1)$ in a counting sequence $\left(c_{n}\right)_{n \geq 0}$ is not common, although more and more examples are appearing lately. Proving that a sequence has a stretched exponential is often quite difficult. This is in part because such a sequence cannot be "very nice": its generating function cannot be algebraic, and it can only be D-finite if it has an irregular singularity. Previously, the saddle-point method was the only generic method for proving such a phenomenon, but it requires detailed information about the generating function. Recently, together with Andrew Elvey Price and Wenjie Fang, we have developed a new method at the level of recurrences to prove stretched exponentials. I will introduce the basics of this method and show how it can be extended to other problems. Then I will summarize recent progress (new bijections, limit laws, etc.) in the study of compacted trees, a subclass of directed acyclic graphs. Finally, I will give an outlook on how these results now allow an in-depth study of limit shapes and open many new avenues for further research.


# Summation Tools for Combinatorics and Elementary Particle Physics by Carsten Schneider 

lundi 4 décembre 2023 15:00 (1 heure)


#### Abstract

The summation theory of difference rings provides general and efficient tools to derive linear recurrences for definite sums based on parameterized telescoping and to solve recurrences within the class of indefinite nested sums defined over hypergeometric products, $q$-hypergeometric products and more generally, mixed hypergeometric products and their nested versions. In particular, one can use these techniques to simplify definite multi-sums to representations in terms of the class indefinite nested sums defined over indefinite nested products. A special feature is that the representation of the arising sums and products are optimal in the sense that the objects interpreted as sequences (except root of unity products) do not satisfy any algebraic relations among each other. This leads not only to compact expressions in the final output, but also speeds up significantly the underlying summation algorithms. In particular, one gains a general (Galois) machinery to prove algorithmically algebraic independence of big classes of sums and products. We will illustrate this algorithmic difference ring toolbox by non-trivial applications coming from enumerative combinatorics and elementary particle physics.


## Coffee break

lundi 4 décembre 2023 16:00 (30 minutes)

## What is beyond D-finite? by Igor Pak

lundi 4 décembre 2023 16:30 (1 heure)

Abstract: I will review some classes of GFs that are of interest in Enumerative Combinatorics but remain understudied. This talk will be accessible to the general audience.

# Galois group for large steps walks by Charlotte Hardouin 

mardi 5 décembre 2023 10:00 (1 heure)


#### Abstract

In this talk, we will present some Galois theoretic tools to study large steps walks confined in the quadrant. We generalize in particular the notion of group of the walk introduced by Bousquet-Mélou and Mishna for small steps walk to the large steps framework. This allows to develop algorithms and criteria to test the existence of invariants and decoupling functions. This is a collaboration with Pierre Bonnet (Labri).


# Quadratizations of differential equations by Gleb Pogudin 

mardi 5 décembre 2023 11:00 (1 heure)


#### Abstract

Quadratization problem is, given a system of ODEs with polynomial right-hand side, transform the system to a system with quadratic right-hand side by introducing new variables. Such transformations have been used, for example, as a preprocessing step by model order reduction methods and for transforming chemical reaction networks. We will present a recent algorithm for computing such transformations and its extensions including systems with control and of varying dimension. The talk is based on joint works with Andrey Bychkov, Opal Issan, and Boris Kramer.


# New Software for Analytic Combinatorics by Stephen Melczer 

mardi 5 décembre 2023 15:00 (1 heure)


#### Abstract

This talk surveys some recently developed software for analytic combinatorics, including an extension to the Sage ore_algebra package for the asymptotics of P-recursive sequences with explicit error terms (used for certifying sequence positivity), and the new sage_acsv package for rigorous multivariate asymptotics using the tools of Analytic Combinatorics in Several Variables (ACSV).


## Coffee break

mardi 5 décembre 2023 16:00 (30 minutes)

## Coffee break

lundi 11 décembre 2023 10:45 (30 minutes)

# Submodule approach to creative telescoping by Mark van Hoeij 

mardi 5 décembre 2023 16:30 (1 heure)


#### Abstract

This talk proposes ideas to speed up the process of creative telescoping, particularly when the telescoper is reducible. One can interpret telescoping as computing an annihilator $L$ in $D$ for an element $H$ in a D-module $M$. The main idea is to look for submodules of $M$. For a non-trivial submodule $N$, constructing the minimal operator $R$ of the image of $H$ in $M / N$ gives a rightfactor of $L$ in $D$. Then $L=L^{\prime} R$ where $L^{\prime}$ is the telescoper of $R(H)$. To expedite computing $L^{\prime}$, compute the action of $D$ on a natural basis of $N$, then obtain the telescoper $L^{\prime}$ for $R(H)$ with a cyclic vector computation. The next main idea is that when $N$ has automorphisms, use them to construct submodules. An automorphism with distinct eigenvalues can be used to decompose $N$ as a direct sum of submodules $N_{1}, \ldots, N_{k}$. Then $L^{\prime}=\operatorname{LCLM}\left(L_{1}, \ldots, L_{k}\right)$ where $L_{i}$ is the telescoper of the projection of $R(H)$ on $N_{i}$. An LCLM can greatly increase the degrees of the coefficients, so $L^{\prime}$ and hence $L$ can be much larger than the factors $L_{1}, \ldots, L_{k}$ and $R$. Examples show that computing each factor $L_{i}$ and $R$ separately can save a lot of CPU time compared to computing the full telescoper $L$ all at once with standard creative telescoping.


## Coffee break

jeudi 7 décembre 2023 16:00 (30 minutes)

# Some problems l'd like solved, from a user of computer algebra by Alan Sokal 

mercredi 6 décembre 2023 10:00 (1 heure)


#### Abstract

A matrix $M$ of real numbers is called $\{$ lem totally positive $\} \backslash$ if every minor of $M$ is nonnegative. Gantmakher and Krein showed in 1937 that a Hankel matrix $H=\left(a_{i+j}\right)_{i, j \geq 0}$ of real numbers is totally positive if and only if the underlying sequence $\left(a_{n}\right)_{n \geq 0}$ is a Stieltjes moment sequence, i.e. $\sim$ the moments of a positive measure on $[0, \infty)$. Moreover, this holds if and only if the ordinary generating function $\sum_{n=0}^{\infty} a_{n} t^{n}$ can be expanded as a Stieltjes-type continued fraction with nonnegative coefficients. So totally positive Hankel matrices are closely connected with the Stieltjes moment problem and with continued fractions. Here I will introduce a generalization: a matrix $M$ of polynomials (in some set of indeterminates) will be called \{\em coefficientwise totally positive $\} \ /$ if every minor of $M$ is a polynomial with nonnegative coefficients. And a sequence $\left(a_{n}\right)_{n \geq 0}$ of polynomials will be called $\{\backslash \mathrm{em}$ coefficientwise Hankel-totally positive $\} \backslash /$ if the Hankel matrix $H=\left(a_{i+j}\right)_{i, j \geq 0}$ associated to $\left(a_{n}\right)$ is coefficientwise totally positive. It turns out that many sequences of polynomials arising naturally in enumerative combinatorics are (empirically) coefficientwise Hankel-totally positive. In some cases this can be proven using continued fractions, by either combinatorial or algebraic methods; in other cases this can be done using a more general algebraic method called $\{\backslash \mathrm{em}$ production matrices $\} \backslash /$. However, in a very large number of other cases it remains an open problem. Along the way I will mention some problems in computer algebra, the solution of which would be helpful to this research.


# Efficient algorithms for differential equations satisfied by Feynman integrals by Pierre Vanhove 

mercredi 6 décembre 2023 11:00 (1 heure)


#### Abstract

Feynman integrals are a type of mathematical function that are important for precision measurements in physics. They are notoriously difficult to evaluate, and a lot of effort has been devoted to developing efficient analytic and numerical methods for doing so. In this talk, we will present a new algorithm for determining the minimal order Picard-Fuchs operator associated with a given Feynman integral. This operator is a differential operator that governs the analytic behavior of the Feynman integral. We will first discuss an implementation of the Griffiths-Dwork algorithm for the case of Feynman integrals in integer spacetime dimension. In that case, the integrand of the Feynman integral is a rational differential form to which the Griffiths-Dwork reduction is applied for determining the Picard-Fuchs operator. We will then extend this algorithm to the case of generic spacetime dimension. In this case, one needs to consider twisted cohomology. We will show how the knowledge of the Picard-Fuchs operator can be deduced by Hodge theoretic considerations on the variation of the mixed Hodge structure associated to the Feynman integral. This talk is based on work with Pierre Lairez, Eric Pichon-Pharabod, Charles Doran, and Andrew Harder.


## Cocktail. IHP, Perrin building, 2nd floor

mercredi 6 décembre 2023 18:30 (2h 15m)

Emmy Noether salon located at the second floor of the Perrin building (IHP). We'll need to have a precise list of participants at the entrance to the building. If you were not registered before December or if you have any doubts, please register or contact the organizers.

# Computer algebra for the study of two-dimensional exclusion processes by Arvind Ayyer 

jeudi 7 décembre 2023 15:00 (1 heure)


#### Abstract

We define a new disordered asymmetric simple exclusion process (ASEP) with two species of particles, first-class and second-class, on a two-dimensional toroidal lattice. The dynamics is controlled by first-class particles, which only move horizontally, with forward and backward hopping rates $p_{i}$ and $q_{i}$ respectively if the particle is on row $I$. The motion of second-class particles depends on the relative position of these with respect to the first-class ones, and can be both horizontal and vertical. In the first part of my talk, I will illustrate how we used computer algebra software, specifically Mathematica and SageMath, to understand the stationary distribution of this process. We computed the partition function, as well as densities and currents of all particles in the steady state. We observed a novel mechanism we call the Scott Russell phenomenon: the current of second class particles in the vertical direction is the same as that of first-class particles in the horizontal direction. In the second part of my talk, I will show how we simulated the process and realized that the Scott Russell phenomenon also holds out of equilibrium. This is partly joint work with P. Nadeau (European Journal of Combinatorics, 2022).


# Self-avoiding walks in a square and the gerrymander sequence by Tony Guttmann 

jeudi 7 décembre 2023 11:00 (1 heure)


#### Abstract

We give an improved algorithm for the enumeration of self-avoiding walks and polygons within an $\mathrm{N} \times \mathrm{N}$ square as well as SAWs crossing a square. We present some proofs of the expected asymptotic behaviour as the size N of the square grows, and then show how one can numerically estimate the parameters in the asymptotic expression. We then show how the improved algorithm can be adapted to count gerrymander sequences (OEIS A348456), and prove that the asymptotics of the gerrymander sequence is similar to that of SAWs crossing a square. This work has been done in collaboration with Iwan Jensen, and in part with Aleks Owczarek.


## Proving positivity for P-recursive sequences by Veronika Pillwein


#### Abstract

In this talk we consider the problem of automatically proving inequalities involving sequences that are only given in terms of their defining recurrence relations. We will consider sequences satisfying linear recurrences with constant coefficients (C-recursive), linear recurrences with polynomial coefficients (P-recursive or holonomic), or certain systems of polynomial nonlinear recurrences (admissible). Even when restricting to the simplest class, C-recursive, and the positivity problem, decidability is only known for orders up to five. And yet, there are computer algebra methods that try to tackle this problem. Obviously, they do not succeed on all types of input and so even though correctness can be proven, termination is typically an issue. In this talk, we will give an overview on this topic, share some available methods and showcases where algorithmic proofs actually succeeded.


# A new proof of Viazovska's modular form inequalities for sphere packing in dimension 8 by Dan Romik 

jeudi 7 décembre 2023 16:30 (1 heure)

Maryna Viazovska in 2016 found a remarkable application of the theory of modular forms to a fundamental problem in geometry, obtaining a solution to the sphere packing problem in dimension 8 through an explicit construction of a so-called "magic function" that she defined in terms of classical functions, the Eisenstein series and Jacobi thetanull functions. The same method also led shortly afterwards to the solution of the sphere packing in dimension 24 by her and several collaborators. One component of Viazovska's proof consisted of proving a pair of inequalities satisfied by the modular forms she constructed. Viazovska gave a proof of these inequalities that relied in an essential way on computer calculations. In this talk I will present a new proof of Viazovska's inequalities that uses only elementary arguments that can be easily checked by a human.

# Partition identities, functional equations and computer algebra by Jehanne Dousse 

vendredi 8 décembre 2023 10:00 (1 heure)


#### Abstract

A partition of a positive integer $n$ is a non-increasing sequence of positive integers whose sum is $n$. A partition identity is a theorem stating that for all $n$, the number of partitions of $n$ satisfying some conditions equals the number of partitions of $n$ satisfying some other conditions. In this talk, we will show how functional equations and computer algebra can be used to prove such identities. In particular we will discuss a semi-automatic method using recurrences and q-difference equations, and what would be needed to make it fully automatic.


# Beyond Painlevé: The need for computational tools to reveal hidden structure by Nicholas Witte 

vendredi 8 décembre 2023 11:00 (1 heure)


#### Abstract

The simplest examples of integrability in mathematical physics - spectral distributions of fundamental random matrix ensembles or the diagonal spin-spin correlations of the square lattice Ising model - to give just two examples, reveal the importance and practical utility of the six Painlevé equations in our understanding of these models. Few aspects of this understanding escape the presence of these equations, whether it is the application of Riemann-Hilbert methods to the asymptotic limits as matrix ranks or spin separations tend to infinity or the relationships inherited from the affine Weyl group symmetries of the Painlevé equations. This understanding operates in both directions: fundamental understanding of the Painlevé equations provides the most powerful and rigorous tools to characterise a variety of statistics applied to the models, and these applications are driving and motivating a lot of studies into the pure mathematics at the core of these equations. However today we coming up against numerous examples where one can pose questions just outside the simplest models: e.g. the singular value distribution of a product of two Ginibre random matrices, or the diagonal correlations of the triangular Ising model or even the off-diagonal correlations of the square lattice model, and one is beyond the six Painlevé set or even the discrete Sakai scheme. Integrability is still present and some of this territory - the Garnier, Fuji-Suzuki, Sasano and matrix-Painlevé systems - has been sketched out but there are gaps in our understanding and our tool-box is missing critical tools that one needs. In some senses the complexity has grown and it is often impractical to do hand-calculations and when computerassisted systems are employed our best results so far cannot be analysed by hand. There are hints that simplicity is achievable yet the standard computer assisted algorithms do not find them. So the challenge is to develop some flexible, model-adapted algorithms for doing what are essentially algebraic geometry calculations. A number of examples, including those mentioned previously, will illustrate this state of affairs.


# Opening remarks for Topical day: Elimination for Functional Equations. IHP, Amphitheater Darboux 

lundi 11 décembre 2023 09:45 (15 minutes)

# Differential Equation Invariance Axiomatization by André Platzer 

lundi 11 décembre 2023 10:00 (45 minutes)

# Differential elimination ideals and spectral curves by Sonia Rueda 

lundi 11 décembre 2023 11:15 (45 minutes)


#### Abstract

Commuting pairs of ordinary differential operators have been related to plane algebraic curves at least since the work of Burchnall and Chaundy a century ago. This talk is devoted to the revision of some classical results, using now a differential algebra framework and, oriented to the development of algorithms based in the computation of differential resultants. The new concept of Burchnall-Chaundy ideal of a commuting pair will be presented, as the ideal of all constant coefficient bivariate polynomials satisfied by the pair. This prime ideal will be proved equal to the radical of a differential elimination ideal and the defining ideal of a plane algebraic curve, the spectral curve of a commuting pair. We are motivated by the development of a Picard-Vessiot theory for spectral problems, in the case of algebro-geometric ordinary differential operator, which are intrinsically linked to the study of integrable hierarchies.


# Geometry-driven algorithms for combinatorial functional equations by Hadrien Notarantonio 

lundi 11 décembre 2023 14:00 (30 minutes)


#### Abstract

Enumerative combinatorics contains a vast landscape of problems that could hardly be solved without the consideration of special functional equations called "Discrete Differential Equations". Among these problems, the enumeration of walks, planar maps carrying hard particles, etc. These functional equations relate formal power series in $n$ variables with specializations of them to some of the variables (the specializations being generating functions related to the enumeration of interest). When the involved variables are "nested", a celebrated result by Popescu (1986) implies algebraicity of the solutions. In 2006, Bousquet-Mélou and Jehanne provided an elementary proof of algebraicity of the solutions in the case $n=2$. Their proof yields an algorithm, and it has been the state-of-the-art in enumerative combinatorics for solving these equations since then. In this talk, I will present a recent approach, based on the intensive use of effective algebraic geometry, in order to solve more efficiently such equations in the case $n=2$. Also, I will introduce and discuss recent advances in the case of systems of such equations. The talk is based on joint works with Alin Bostan, Mohab Safey El Din and Sergey Yurkevich.


# Examples of use of functional equations obtained from the elimination theory in nonlinear models by Nathalie Verdière 

lundi 11 décembre 2023 14:30 (45 minutes)


#### Abstract

This talk addresses the use of functional equations obtained from computer algebra in the structural properties study of nonlinear dynamical models. For several years, they were exploited in the context of identifiability and diagnosticability studies. A final application was recently developed and concerns an a priori study for the reconstruction of some variables of interest in nonlinear complex networks with an application on a C. elegans chemotaxis network.


## Coffee break

lundi 11 décembre 2023 15:15 (30 minutes)

# On some functional equations for maps by Alexandros Singh 

lundi 11 décembre 2023 15:45 (30 minutes)


#### Abstract

The goal of this exploratory talk is to present various functional equations related to the enumeration of maps and the study of various combinatorial parameters thereof. In particular, we'll concern ourselves with questions such as: how are these equations related and can they be inter-derived by combinatorial or differentially-algebraic methods? This presentation draws from material from ongoing joint work with Olivier Bodini and Konstantinos Tsagkaris.


# Solving differential elimination problems with Thomas decomposition by Daniel Robertz 

lundi 11 décembre 2023 16:15 (45 minutes)


#### Abstract

This talk gives an introduction to the Thomas decomposition method for systems of nonlinear partial differential equations. A Thomas decomposition is a finite family of so-called simple differential systems, each of which is formally integrable, and such that the solution set of the given PDE system is the disjoint union of the solution sets of the simple systems. This versatile technique allows solving differential elimination problems, for example, for the analysis of certain structural properties of nonlinear control systems. The Thomas decomposition method has been implemented in Maple.


## Closing remarks

lundi 11 décembre 2023 17:00 (15 minutes)

# Computer algebra in my combinatorics life by Mireille Bousquet-Mélou 

jeudi 7 décembre 2023 10:00 (1 heure)


#### Abstract

Many of my papers would just not exist without computer algebra. I will describe how CA has become an essential tool in my research in enumerative combinatorics. The point of view will be that of a (sometimes naive) user, not of an expert. Many examples and questions will be taken from a joint paper with Michael Wallner dealing with the enumeration of king walks avoiding a quadrant (arXiv 2021, to appear). My hope is that some of the questions that I will raise will have an immediate answer ("yes, this is done") and/or that some people in the audience will find a question interesting enough to take it back home.


# General audience presentation The Zeta Team by Wadim Zudilin, Radboud University. 15:00-16:00. IHP, Amphitheater Hermite 

mercredi 29 novembre 2023 15:00 (1 heure)

Abstract. An innocent story of finding recursions for the sums of powers of binomial coefficients transformed into an intrigue involving creative telescoping and irrationality. The goalscorers performed as a dream team.

# Special session of the Differential Seminar. Rounding error analysis of linear recurrences using generating series by Marc Mezzarobba, LIX, Palaiseau. 16:00-17:00. IHP, Amphitheater Hermite 

jeudi 30 novembre 2023 16:00 (1 heure)


#### Abstract

In the computation of linearly recurrent sequences, round-off errors arising from each step tend to "cancel out" rather than just accumulate. This phenomenon is crucial to consider when aiming to establish realistic error bounds. That requires a close examination of how a given roundoff error propagates through the remaining iterations of the recurrence. Doing so using traditional "sequence"notations results in intricate calculations involving nested sums. However, in other contexts, such as enumerative combinatorics, the use of generating series to encode sequences simplifies these calculations while simultaneously granting access to powerful new analytic tools. In this talk, I will show how the idea of using generating series can be adapted to the error analysis of numerical algorithms based on linear recurrence relations.


# Special session of the Differential Seminar. Combinatorics and Transcendence: Applications of Inhomogeneous order 1 iterative functional equations by Marni Mishna, Simon Fraser University. 16:00-17:00. IHP, Amphitheater Hermite 

vendredi 1 décembre 2023 16:00 (1 heure)

The problem of understanding the structure of transcendental objects has fascinated mathematicians for well over a century. Combinatorics provides an intuitive framework to study power series. A combinatorial family is associated to a power series in $\mathbb{Q}[[t]]$ via its enumerative generating function wherein the number of objects of size $n$ is the coefficient of $t^{n}$. Twentieth century combinatorics and theoretical computer science provided characterizations of classes with rational and algebraic generating functions. Finding natural extensions of these correspondences has been a motivating goal of enumerative combinatorics for several decades. This talk will focus on differentially transcendental functions.
In particular, I will present recent work completed with Lucia Di Vizio and Gwladys Fernandes which characterizes solutions $f(t)$ of order 1 iterative equations of the form $f(R(t))=a(t) f(t)+$ $b(t)$ where $R, a$, and $b$ are rational functions. These appear in the study of complete trees, walks on self-similar graphs (eg. the Sierpinski graph), and pattern avoiding permutations. The proof strategy is inspired by the Galois theory of functional equations, and relies on the property of the dynamics of $R(t)$, Liouville-Rosenlicht's theorem and Ax' theorem. This program and has led to progress in identifying the differential transcendence of combinatorial generating functions arising in the literature, and indeed generally.

