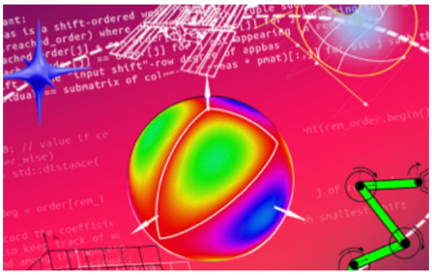

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Some problems I'd like solved, from a user of computer algebra by Alan Sokal

mercredi 6 décembre 2023 10:00 (1 heure)

Abstract. A matrix M of real numbers is called {\em totally positive}\/ if every minor of M is nonnegative. Gantmakher and Krein showed in 1937 that a Hankel matrix $H = (a_{i+j})_{i,j \geq 0}$ of real numbers is totally positive if and only if the underlying sequence $(a_n)_{n \geq 0}$ is a Stieltjes moment sequence, i.e.~the moments of a positive measure on $[0, \infty)$. Moreover, this holds if and only if the ordinary generating function $\sum_{n=0}^{\infty} a_n t^n$ can be expanded as a Stieltjes-type continued fraction with nonnegative coefficients. So totally positive Hankel matrices are closely connected with the Stieltjes moment problem and with continued fractions. Here I will introduce a generalization: a matrix M of polynomials (in some set of indeterminates) will be called {\em coefficientwise totally positive}\/ if every minor of M is a polynomial with nonnegative coefficients. And a sequence $(a_n)_{n \geq 0}$ of polynomials will be called {\em coefficientwise Hankel-totally positive}\/ if the Hankel matrix $H = (a_{i+j})_{i,j \geq 0}$ associated to (a_n) is coefficientwise totally positive. It turns out that many sequences of polynomials arising naturally in enumerative combinatorics are (empirically) coefficientwise Hankel-totally positive. In some cases this can be proven using continued fractions, by either combinatorial or algebraic methods; in other cases this can be done using a more general algebraic method called {\em production matrices}\/. However, in a very large number of other cases it remains an open problem. Along the way I will mention some problems in computer algebra, the solution of which would be helpful to this research.