Special session of the Differential Seminar.
Combinatorics and Transcendence: Applications of Inhomogeneous order 1 iterative functional equations by Marni Mishna, Simon Fraser University.
16:00-17:00. IHP, Amphitheater Hermite

Friday, 1 December 2023 16:00 (1 hour)

The problem of understanding the structure of transcendental objects has fascinated mathematicians for well over a century. Combinatorics provides an intuitive framework to study power series. A combinatorial family is associated to a power series in $\mathbb{Q}[[t]]$ via its enumerative generating function wherein the number of objects of size $n$ is the coefficient of $t^n$. Twentieth century combinatorics and theoretical computer science provided characterizations of classes with rational and algebraic generating functions. Finding natural extensions of these correspondences has been a motivating goal of enumerative combinatorics for several decades. This talk will focus on differentially transcendental functions.

In particular, I will present recent work completed with Lucia Di Vizio and Gwladys Fernandes which characterizes solutions $f(t)$ of order 1 iterative equations of the form $f(R(t)) = a(t)f(t) + b(t)$ where $R$, $a$, and $b$ are rational functions. These appear in the study of complete trees, walks on self-similar graphs (e.g. the Sierpinski graph), and pattern avoiding permutations. The proof strategy is inspired by the Galois theory of functional equations, and relies on the property of the dynamics of $R(t)$, Liouville-Rosenlicht’s theorem and Ax’ theorem. This program and has led to progress in identifying the differential transcendence of combinatorial generating functions arising in the literature, and indeed generally.