Geometry of Polynomial System Solving, Optimization and Topology
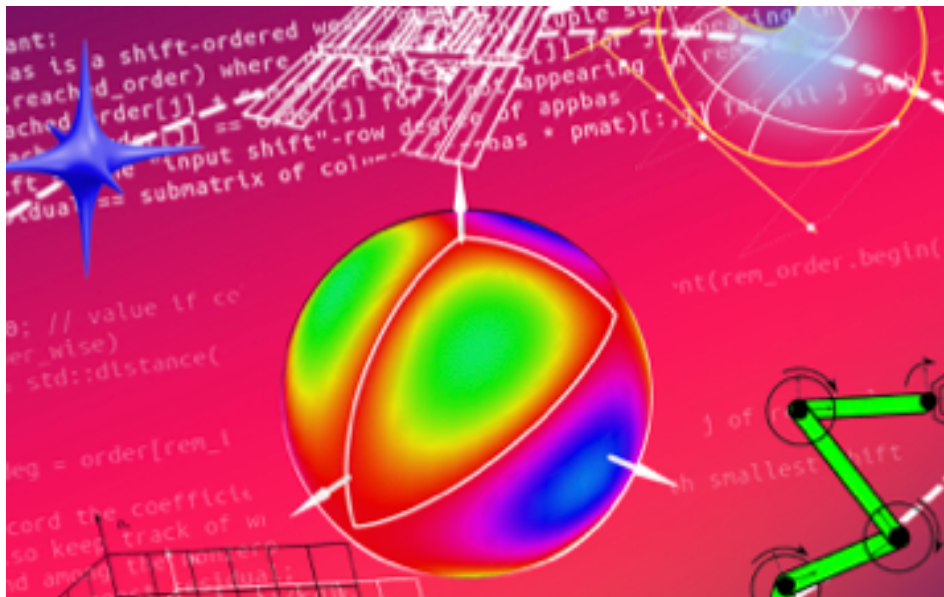

Rapport sur les contributions

ID de Contribution: **1**   Type: **Non spécifié**

Moment Method in Optimization by D. Henrion. 10:00-12:00. Amphitheater Darboux, IHP

lundi 9 octobre 2023 10:00 (2 heures)

Moment Method in Optimization by D. Henrion. 10:00-12:00. Amphitheatre Darboux, IHP

ID de Contribution: **3**　　　　　　　　　　　　　　　Type: **Non spécifié**

Moment Method in Optimization by D. Henrion. 10:00-12:00. Amphitheater Darboux, IHP

mardi 10 octobre 2023 10:00 (2 heures)

ID de Contribution: **4**   Type: **Non spécifié**

Polynomials Systems and Real Geometry by I. Emiris and D. Plaumann. 14:00-17:30. Amphitheater Darboux, IHP

mardi 10 octobre 2023 14:00 (3h 30m)

Geometry of Poly ... / Rapport sur les contributions   Polynomials Systems and Real Ge ...

May 17, 2024   Page 4

ID de Contribution: **5**   Type: **Non spécifié**

Polynomials Systems and Real Geometry by I. Emiris and D. Plaumann. 10:00-12:00. Amphitheater Darboux, IHP

mercredi 11 octobre 2023 10:00 (2 heures)

Geometry of Poly ... / Rapport sur les contributions   Polynomials Systems and Real Ge ...

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ID de Contribution: **6**   Type: **Non spécifié**

Polynomials Systems and Real Geometry by I. Emiris and D. Plaumann. 10:00-12:00. Amphitheater Darboux, IHP

jeudi 12 octobre 2023 10:00 (2 heures)

Geometry of Poly ... / Rapport sur les contributions   Polynomials Systems and Real Ge ...

May 17, 2024   Page 6

ID de Contribution: **7**   Type: **Non spécifié**

Polynomials Systems and Real Geometry by I. Emiris and D. Plaumann. 14:00-17:00. Amphitheater Darboux, IHP

jeudi 12 octobre 2023 14:00 (3 heures)

Geometry of Poly ... / Rapport sur les contributions   Polynomials Systems and Real Ge ...

May 17, 2024   Page 7

ID de Contribution: **8** Type: **Non spécifié**

General audience presentation "The quadratic equation revisited" by B. Sturmfels. 16:00-18:00. Amphitheater Darboux, IHP

mercredi 11 octobre 2023 16:00 (2 heures)

Abstract High school students learn how to express the solution of a quadratic equation in one unknown in terms of its three coefficients. What does this this formula matter? We offer an answer in terms of discriminants and data. This lecture invites the audience to a journey towards non-linear algebra.

ID de Contribution: **9**   Type: **Non spécifié**

Welcome coffee

lundi 16 octobre 2023 08:45 (30 minutes)

ID de Contribution: **10**   Type: **Non spécifié**

Opening

lundi 16 octobre 2023 09:15 (15 minutes)

ID de Contribution: **11**   Type: **Non spécifié**

Dimension results for sparse systems homogenized via rational polytopes by Pierre-Jean Spaenlehauer

lundi 16 octobre 2023 09:30 (1 heure)

Abstract. A classical method to compute with sparse polynomials is to homogenize them with respect to Newton polytopes, regarding them as homogeneous elements of Cartier degrees in the Cox ring of a projective toric variety. In this talk, we investigate subvarieties defined by generic polynomial systems in the Cox ring when the degrees are non-necessarily Cartier, with a view towards identifying alternative toric homogenizations that are suitable for practical computations. Joint work with Matías Bender.

ID de Contribution: **12**   Type: **Non spécifié**

Coffee break

lundi 16 octobre 2023 10:30 (30 minutes)

ID de Contribution: **13**

Type: **Non spécifié**

Chordal Graphs in Triangular Decomposition in Top-Down Style by Chenqi Mou

lundi 16 octobre 2023 11:00 (1 heure)

Abstract. In this talk, I will present the connections between chordal graphs from graph theory and triangular decomposition in top-down style from symbolic computation, including the underlying theories, algorithms, and applications in biology. Viewing triangular decomposition in top-down style as polynomial generalization of Gaussian elimination, we show that all the polynomial sets, including all the computed triangular sets, appearing in several typical top-down algorithms for triangular decomposition have associated graphs which are subgraphs of the chordal graph of the input polynomial set. These theoretical results can be interpreted as "triangular decomposition in top-down style preserves chordality" and are further used to design sparse triangular decomposition for polynomial sets which are sparse with respect to their variables. Sparse triangular decomposition is shown to be more efficient than ordinary one experimentally, and its application on computing equilibria of biological dynamic systems will also be reported. This talk is based on the joint work with Yang Bai, Jiahua Lai, and Wenwen Ju.

ID de Contribution: **14**   Type: **Non spécifié**

Modular algorithms for computing triangular decompositions of polynomial systems by Marc Moreno Maza

lundi 16 octobre 2023 14:30 (1 heure)

Abstract. https://rtca2023.github.io/pages_Paris/files_m5/abstract_moreno-maza.pdf

ID de Contribution: **15**   Type: **Non spécifié**

Coffee break

lundi 16 octobre 2023 15:30 (30 minutes)

ID de Contribution: **16**   Type: **Non spécifié**

Conormal Spaces and Whitney Stratifications by Martin Helmer

jeudi 19 octobre 2023 16:00 (1 heure)

Abstract. We describe a new algorithm for computing Whitney stratifications of complex projective varieties. The main ingredients are (a) an algebraic criterion, due to Lê and Teissier, which reformulates Whitney regularity in terms of conormal spaces and maps, and (b) a new interpretation of this conormal criterion via ideal saturations, which can be practically implemented on a computer. We show that this algorithm improves upon the existing state of the art by several orders of magnitude, even for relatively small input varieties. This is joint work with Vidit Nanda (Oxford).

ID de Contribution: **17**   Type: **Non spécifié**

Some advances in numerical algebraic geometry for computing real solutions by Jon Hauenstein

mercredi 18 octobre 2023 09:30 (1 heure)

Abstract. Numerical algebraic geometry provides a collection of algorithms for computing and analyzing solution sets of polynomial systems. This talk will discuss new techniques that have been developed in numerical algebraic geometry for focusing on real solution sets of polynomial systems. Several applications of these techniques will be presented such as computing smooth points on algebraic sets, approximate synthesis of mechanisms, and path planning for output mode switching.

ID de Contribution: **18**   Type: **Non spécifié**

Coffee break

mardi 17 octobre 2023 10:30 (30 minutes)

ID de Contribution: **19**
Type: **Non spécifié**

u-generation: solving systems of polynomials equation-by-equation by Anton Leykin

mercredi 18 octobre 2023 11:00 (1 heure)

Abstract. We develop a new method that improves the efficiency of equation-by-equation homotopy continuation methods for solving polynomial systems. Our method is based on a novel geometric construction and reduces the total number of homotopy paths that must be numerically continued. These improvements may be applied to the basic algorithms of numerical algebraic geometry in the settings of both projective and multiprojective varieties. (This is joint work with T. Duff and J. I. Rodriguez.)

ID de Contribution: **20**   Type: **Non spécifié**

Cocktail

mercredi 18 octobre 2023 18:30 (2h 30m)

ID de Contribution: **21**   Type: **Non spécifié**

Polynomial systems arising in the formal verification of programs by Fatemeh Mohammadi

mardi 17 octobre 2023 09:30 (1 heure)

Abstract. Multiple classical problems in the formal verification of programs such as reachability, termination, and template-based synthesis can be reduced to solving polynomial systems of equations. In this talk, I will describe the primary objects and these connections. In particular, I will show how the algebraic and geometric techniques can be applied, enhancing the scalability and completeness for such problems.

ID de Contribution: **22**   Type: **Non spécifié**

Positive solutions to polynomial systems and applications to reaction networks by Elisenda Feliu

mardi 17 octobre 2023 11:00 (1 heure)

Abstract. The main object of study in the (algebraic) theory of reaction networks is the solution se of a system of parametric polynomial equations in the positive orthant. This system consists of polynomials with fixed support, the coefficients are linear in the parameters, but there might be some (proportionality) dependencies among the coefficients. The questions of interest concern properties of this system, and of its intersection with a family of parallel linear subspaces of complementary dimension. In this case, of relevance is to determine the possible number of zeros the system has when the parameters vary. In this talk I will introduce the framework and the families of polynomial systems under study in full generality, and having the reaction networks as the main application. I will proceed to discuss recent results addressing the expected dimension of the solution sets and on how to decide whether the solution set admits a toric parametrization for all parameter values. The latter is relevant for the problem of counting solutions, and this connection will also be explained in the talk.

ID de Contribution: **23**   Type: **Non spécifié**

Polynomial Systems Arising in Paradoxical 6R Linkages by Zijia Li

mardi 17 octobre 2023 14:30 (1 heure)

Abstract. In this talk, we first provide a comprehensive definition of closed n-linkages and explain their mobility, typically denoted as n-6. We then focus on the intriguing subset of closed n-linkages with a mobility higher than n-6, known as paradoxical linkages. Based on the powerful tools of Bond Theory and the freezing technique, we present a thorough classification of n-linkages with a mobility of n-4 or higher, incorporating revolute, prismatic, or helical joints. Additionally, we explicitly derive strong necessary conditions for nR-linkages with a mobility of n-5. Utilizing these necessary conditions, we explore and discuss possible polynomial systems that arise in paradoxical 6R linkages.

ID de Contribution: **24**   Type: **Non spécifié**

Coffee break

mardi 17 octobre 2023 15:30 (30 minutes)

ID de Contribution: **25**   Type: **Non spécifié**

Coffee break

mercredi 18 octobre 2023 10:30 (30 minutes)

ID de Contribution: **26**

Type: **Non spécifié**

Sums of squares approximations in polynomial optimization: performance analysis and degree bounds by Monique Laurent

jeudi 19 octobre 2023 09:30 (1 heure)

Abstract. Polynomial optimization deals with optimizing a polynomial function over a feasible region defined by polynomial inequalities, thus modeling a broad range of hard nonlinear nonconvex optimization problems. Hierarchies of tractable semidefinite relaxations have been introduced that are based on using sums of squares of polynomials as a "proxy" for global nonnegativity. These hierarchies give bounds on the global minimum of the original problem with asymptotic convergence (under a minor compactness assumption). In this lecture we discuss recent results on the performance analysis of these hierarchies and related effective degree bounds for dedicated sums of squares representations of positive polynomials on some classes of compact semi-algebraic sets (including the hypercube, the sphere or the ball).

ID de Contribution: **27**   Type: **Non spécifié**

Coffee break

jeudi 19 octobre 2023 10:30 (30 minutes)

ID de Contribution: **28**   Type: **Non spécifié**

Solving by duality by Bernard Mourrain

jeudi 19 octobre 2023 11:00 (1 heure)

Abstract. Finding the common roots of a set of polynomial equations is a problem that appears in many contexts and applications. Standard approaches for solving this difficult question, such as Grobner bases, border basis, triangular sets, etc. are based on polynomial reductions but their instability against numerical approximations can be critical. In this talk, we will describe a dual approach which focuses on linear functionals vanishing at the roots. We will review the properties of Truncated Normal Forms, the connexion with classical computer algebra approaches and resultants. We will also detail the dual approach in the context of optimisation problems and for analysing isolated singularities. Examples from geometric modeling, robotics and tensor decomposition will illustrate the numerical behavior of these dual methods.

ID de Contribution: **29**   Type: **Non spécifié**

Non-negativity and rational sums of squares over zero-dimensional varieties by Teresa Krick

jeudi 19 octobre 2023 14:30 (1 heure)

Abstract. In this work in progress with Lorenzo Baldi and Bernard Mourrain, we extend previous results on univariate rational sums of squares, obtained with Bernard and Agnes Szanto, to the case of a non-negative rational polynomial on a basic zero-dimensional semi-algebraic set defined by rational polynomials.

ID de Contribution: **30**   Type: **Non spécifié**

Coffee break

jeudi 19 octobre 2023 15:30 (30 minutes)

ID de Contribution: **31** Type: **Non spécifié**

The geometry of the Vandermonde map at infinity by Cordian Riener

vendredi 20 octobre 2023 09:30 (1 heure)

Abstract. The Vandermonde map is the polynomial map given by the power-sum polynomials. We study the geometry of the image of the nonnegative orthant under under this map and focus on the limit as the number of variables approaches infinity. We will show, the geometry of this limit is the key to new undecidability results in nonnegativity of symmetric polynomials and deciding validity of trace inequalities in linear algebra.

ID de Contribution: **32**   Type: **Non spécifié**

Coffee break

vendredi 20 octobre 2023 10:30 (30 minutes)

ID de Contribution: **33**
Type: **Non spécifié**

Recent advances in Gröbner basis algorithms and geometric applications by Christian Eder

vendredi 20 octobre 2023 11:00 (1 heure)

Abstract. The tasks of designing innovative mathematical software and of solving complex research problems using computational methods are strongly mutually dependent. Developing a new generation of algorithms to considerably push the computational boundaries of nonlinear algebra, notably addressing polynomial system solving, is thus envitable. One important task of this process is to no longer use Gröbner bases only as a black box in higher level algorthms, but to optimize their computation with the geometric context in mind. In this talk, we will illustrate this idea by presenting a new algorithm for computing Gröbner bases of saturated polynomial ideals. Moreover, we introduce msolve, an open source software package build to provide this new generation of efficient and optimized algorithms for the community. The contents of this talk are based on joint work with Jérémy Berthomieu and Mohab Safey El Din.

ID de Contribution: **34**   Type: **Non spécifié**

Topical day: Mechanism Design and Computer Algebra. Galois Theory for Planar Mechanisms by Josef Schicho. 9:30-10h30. Amphitheater Darboux

mardi 24 octobre 2023 09:30 (1 heure)

Let G be a graph with n vertices and e edges. The computation of the position of n points in the plane such that for any two vertices in the graph connected by an edge, the distance between the two corresponding points is given, is equivalent to the inverse kinematic problem for a (highly parallel) planar mechanism with revolute joints. If the graph is a Laman graph, then the solution set is generically a finite set of orbits under the group of Euclidean displacements, and can be assigned a Galois group (which is associated to the field extension needed to express the solutions exactly). We explain some geometric ideas for analyzing the Galois group. Using these ideas, we determine the number of components of the solution set for graphs that have the property that the above position problem is generically solvable.

ID de Contribution: **35**   Type: **Non spécifié**

Mechanisms from Motions –Rational and Algebraic by H.-P. Schröcker. 11:00-12:00

mardi 24 octobre 2023 11:00 (1 heure)

The factorization of rational motions has been introduced about a decade ago. On a kinematics level it corresponds to the decomposition of a rational motion into elementary motions (rotations, translations, ...) The mathematics behind is the factorization of special polynomials over non-commutative rings into linear factors. This talk gives an overview about the past decade of motion factorization and explains many of the underlying constructions of mechanisms at hand of animations. It will also feature a new geometric factorization algorithm that highlights the importance of "kinematics at infinity"and gives rise to an extension of the factorization theory from rational to algebraic motions.

ID de Contribution: **36**   Type: **Non spécifié**

Inverse kinematics with computer algebra and the Lasserre hierarchy by Didier Henrion. 14:00-15:00

mardi 24 octobre 2023 14:00 (1 heure)

The Inverse Kinematics (IK) problem consists of finding robot control parameters to bring it into a desired position under kinematics and collision constraints. We describe a global solution to the optimal IK problem for a general serial manipulator with 7 degrees of freedom (7DOF) with revolute joints. Classical modeling yields a polynomial optimization problem with constraints of degree four. A direct application of the moment-SOS (sums of squares) aka Lasserre hierarchy generates semidefinite optimization problems which are too large for state-of-the-art numerical solvers. Using computer algebra (Groebner basis computations), we show that the kinematic constraints due to rotations can all be generated by degree two polynomials. On this reduced problem, we demonstrate that the second relaxation of the Lasserre hierarchy is sufficient to solve the 7DOF IK problem on a KUKA LBR IIWA manipulator and we show that we are able to compute the optimal IK or certify infeasibility in 99% of the tested poses. This is joint work with Pavel Trutman (Prague), Tomas Pajdla (Prague) and Mohab Safey El Din (Paris).

ID de Contribution: **37**   Type: **Non spécifié**

Cloth State Representation Using a Derivative of the Gauss Linking Integral by Maria Alberich-Carraminana. 15:00-16:00

mardi 24 octobre 2023 15:00 (1 heure)

Robotic manipulation of cloth presents a complex challenge due to the infinite-dimensional shape-state space of textiles. This complexity makes accurate state estimation a daunting task. To address this issue, we introduce the concept of dGLI Cloth Coordinates—a finite, low-dimensional representation of cloth states. This novel approach enables us to effectively distinguish among a wide range of folded states, paving the way for efficient learning methods in cloth manipulation planning and control.

ID de Contribution: **38**   Type: **Non spécifié**

Polynomials Systems and Real Geometry by I. Emiris and D. Plaumann. 10:00-12:00. Amphitheater Darboux, IHP