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Homological stability and non-stability for configuration spaces on closed manifolds

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Unordered configuration spaces of points (or particles) on connected manifolds are basic objects that appear in many different areas within topology. When the manifold M is non-compact, a theorem of McDuff and Segal states that these spaces are homologically stable, with integral coefficients, as the number of points goes to infinity. When M is closed, however, these spaces are in general homologically *unstable* - one can see this already in the degree-1 homology of configuration spaces on the 2-sphere. Moreover, there are natural "stabilisation" maps between configuration spaces in the non-compact case, which do not exist when M is closed.

I will describe some joint work with Federico Cantero, in which we prove several results that show that configuration spaces on closed manifolds nevertheless exhibit some more subtle kinds of stable behaviour. For example, we prove homological stability for odd-dimensional M after inverting 2 in the coefficients, and for even-dimensional M (with *non-vanishing* Euler characteristic) we prove that the mod-p homology of the configuration spaces is eventually periodic, with an explicit upper bound for the period. We also construct so-called "replication maps" between configuration spaces (when M has *vanishing* Euler characteristic), which induce homology isomorphisms in a stable range after inverting certain primes.

This builds on and improves previous work of several others, including O. Randal-Williams and [M. Bendersky - J. Miller]. The periodicity result is very similar to a theorem of R. Nagpal, although we have different estimates for the period. Very recently, the upper bound for the period has been improved in work of [A. Kupers - J. Miller], who also recover our result for odd-dimensional M with coefficients in Z[1/2] and give a more explicit description of the corresponding isomorphisms. There is also very recent work of [S. Galatius - O. Randal-Williams], who prove analogous "stability and non-stability" results for classifying spaces of diffeomorphism groups of high-dimensional closed manifolds. If time permits, I will also briefly describe some of this subsequent work, as well as new directions to explore.

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