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Shocks in dispersive media

The Euler equations of compressible fluids, hyperelasticity, MHD, etc. are typical examples of hyperbolic systems of conservation laws admitting shock solutions, i.e. discontinuous solutions satisfying the governing equations in a weak sense.

The corresponding equations of motion are the Euler-Lagrange equations for a functional which is the Hamilton action.

The dispersive regularizations of these models based on the modification of the corresponding Lagrangian aim at avoiding discontinuities by replacing them by “dispersive shocks”, i.e. by strongly oscillating non stationary fronts. We show that in some cases dispersive regularization produces solutions that are “almost” classical shocks. Such solutions must necessarily satisfy special jump relations (generalized Rankine-Hugoniot relations) that follow naturally from the variational structure of the governing equations. I will consider the Benjamin-Bona-Mahony (BBM) equation as a toy model to study these unusual shock solutions. The BBM equation is a simplest unidirectional model of shallow water waves. As we will see, it has stable discontinuous solutions.

I will also explain how such generalized jump relations can be obtained for the second gradient fluids and bubbly fluids.

This is a joint work with H. Gouin (AMU, Marseille) and K. M. Shyue (NTU, Taiwan).