

Jim Bryan : The geometry and arithmetic of banana nano-manifolds.

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The Hodge numbers of a Calabi-Yau threefold X are determined by the two numbers $h^{1,1}(X)$ and $h^{1,2}(X)$ which can be interpreted respectively as the dimensions of the space of Kähler forms and complex deformations respectively. We construct four new examples X_N , where $N \in \{5,6,8,9\}$, of rigid Calabi-Yau threefolds ($h^{2,1}=0$) with Picard number 4 ($h^{1,1}=4$). These manifolds are of “banana type” and have among the smallest known values for Calabi-Yau Hodge numbers. We (partially) compute the Donaldson-Thomas partition functions of these manifolds and in particular show that the associated genus g Gromov-Witten potential is given by a weight $2g-2$ Siegel paramodular form of index N . These manifolds are also modular in the arithmetic sense: there is a weight 4 modular form whose Fourier coefficients are obtained by counting points over finite fields on X_N . We compute this form and observe that it is the unique cusp form of weight 4 and index N . This is joint work with Stephen Pietromonaco.