Scattered thoughts on
Quantum Geometry and **Strings**

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Closing Colloquium of the program on

“ The Quantum World ”

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DISCLAIMER

I (think I) was asked by the organizers to talk about String Theory, and what it tells us about "The Quantum World", in a way that can be of (some) use to non-specialists.

With this in mind, I will not speak about my recent results; rather, I will try to put in perspective few well-established facts, and comment on some of the recent literature that might be of relevance to issues raised in this Quantum Trimester.

" a poor-man's version of a Bourbaki seminar "

from the blog of a young american mathematician

http://blogs.ams.org/phdplus/2013/06/24/an-afternoon-at-the-seminaire-bourbaki/#sthash.LqCbSWiv.dpbs

…. who dropped by, but did not understand french :

“ Something interesting happens when you’re really lost. You notice things that otherwise might not register.
For example, I noticed I was one of two women in a crowd of about 40+ people.
From the picture on the left you can also see that the median age is probably about 50.

Also, it’s apparently OK to fall asleep at the Seminaire Bourbaki. ”

same here !

A	STRINGS & QUANTUM GRAVITY	(general remarks)
B	SINGULARITIES	(resolutions)
Γ	ENTROPY & HORIZONS	(counting, fuzzballs)
Δ	EMERGENT GEOMETRY	(BH interior)

1

STRINGS & QUANTUM GRAVITY

$$M_{\rm Pl}^2 \int d^4x \sqrt{g}(R - \Lambda) + \int d^4x \, \mathcal{L}_{\rm SM}(\phi_i, g)$$

Einstein - Hilbert
classical action

Relativistic ***QFT***
in backrgound g

Treating as QFT in classical geometry does not run into any direct clash with present-day observations *(but such may hide in the sky)*

But: - (*Optional*) problem of **dark energy**;

- **conceptually incomplete** [*geometries as coherent or mixed quantum states ? information paradox*]

- **math. incomplete**: GR singularities

So something must be done at or before $\sim \ell_{\rm Planck}$

The most timid ideas run quickly into difficulties:

Deform Einstein's theory:

$$\mathcal{L} = \mathcal{L}_{\rm EH} + aR^2$$

ok for Euclidean, but ghosts for Lorentzian signature

Induced-emergent gravity:

Sakharov 1967

Gravity from some regular QFT, in same sense that hydrodynamics emerges from atomic/molecular physics.

In its simplest version, ruled out by " **Weinberg-Witten theorem** "

Weinberg-Witten: (Coleman) '80

No massless spin-2 state in a theory with a conserved energy-momentum tensor

<u>Lorentz covariance</u> and conserved $E = \int d^3x\, T^{00}$ implies

$$\lim_{p \to p'} \langle p' | T^{\mu\nu}(t,0) | p \rangle = \frac{p^\mu p^\nu}{E(2\pi)^3}$$

Inconsistent, for $j > 1$, with Lorentz transformation

$$e^{\pm 2ij\phi} \langle p', \pm j | T^{\mu\nu} | p, \pm j \rangle = \Lambda^\mu{}_\rho(\phi) \Lambda^\nu{}_\sigma(\phi) \langle p', \pm j | T^{\rho\sigma} | p, \pm j \rangle$$

Possibility to evade the theorem if Lorentz symmetry is spontaneously broken, very contrived

Bjorken ’63; Kraus+Tomboulis ’02 , ….

Plausible: Other things, such as spacetime dimensions, must also emerge together with gravity

But, a more modest proposal (miraculously) circumvents both obstructions: **Perturbative String Theory**

An Extensible model of the electron
Paul A.M. Dirac (Cambridge U.). Feb 1962.
in **Proc.Roy.Soc.Lond. A268 (1962) 57-67**

“I might have thought that the new ideas were correct if they had not been so ugly”

Dyson quoting Dirac on renormalization.

Sigma-model or α' deformation

$$S_{\rm ws} = \frac{1}{4\pi\alpha'}\int d^2\sigma\sqrt{-h}\,[h^{ab}G_{\mu\nu}(X)\partial_a X^\mu \partial_b X^\nu + \alpha'\Phi(X)R(h) + \cdots]$$

(β-function) equations give gradient flow derived from the action (c-function):

$$I_{10,\rm het} = \frac{1}{2\kappa^2}\int d^{10}x\,\sqrt{-G}\,e^{-2\Phi}[R + 4\partial_\mu\Phi\partial^\mu\Phi + \frac{\alpha'}{8}R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} + \cdots]$$

Friedan '80
Callan + Friedan + Martinec + Perry '85
Fradkin + Tseytlin '85

Conformal-invariant sigma models ⟷ solutions of deformed Einstein equations

- This deformation avoids ghosts

- The spin-2 graviton is necessarily in the spectrum

$$\alpha' m^2 = 2 \sum \mathcal{N}_n n - 1$$

Scherk, Schwarz ; Yoneya '74

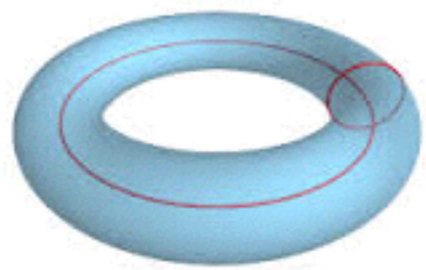
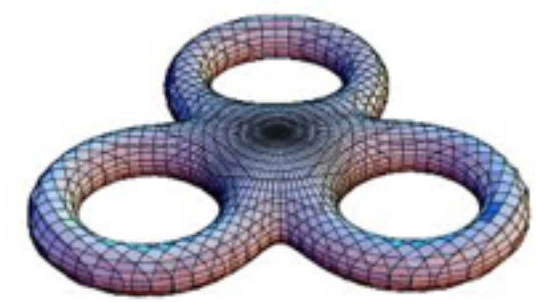
quantum (Casimir) mass

Fascinating <u>deformation</u> of classical geometry:

- <u>Mirror</u> symmetry — Candelas, de la Ossa, P. Green, Parks '90; Kontsevich '95; Strominger, Yau, Zaslow '96; ...

- " <u>Non-geometric</u> " backgrounds — Narain, Sarmadi, Vafa ; Antoniadis, CB, Kounnas ; Kawai, Lewellen, Tye ; '87 ...

Deformation in $g_s = e^{\Phi_0}$ appears perturbatively finite in stable (supersymmetric) vacua

 $+ \; g_s^2$  $+ \; g_s^4$  $+ \; g_s^6$ 

D'Hoker + Phong;
Green, Vanhove, …..

???

One more remark:

No external sources, because off-shell extensions are divergent [$\sim$ " no mouse " of quantum mechanics]

So WHY AREN'T WE DONE ?

<u>Pragmatic</u> : Connection to the observed (low-E) world incomplete

Vacuum **selection** and **stability** (de Sitter ? supersymmetry ?)

Infrared properties not understood

<u>Foundational</u> : The problem is **TIME**

Πάντα ῥεῖ

Time flows

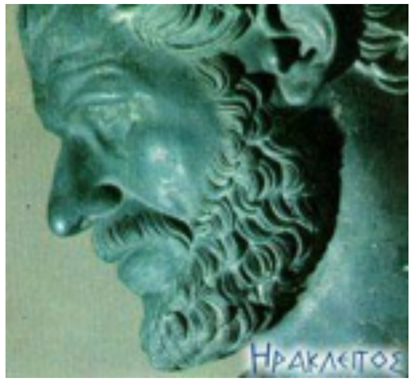
ποταμοῖσι τοῖσιν αὐτοῖσιν ἐμβαίνουσιν,
ἕτερα καὶ ἕτερα ὕδατα ἐπιρρεῖ

No closed timelike geodescis

Αἰὼν παῖς ἐστι παίζων πεσσεύων·
παιδὸς ἡ βασιληίη

Time is a child playing at draughts,
a child's kingdom

“ Phenomenology ” of Heraclitus



To summarize:

String theory avoids some ‘traps’ on the road to a theory of quantum gravity; but will now see issues with TIME

2

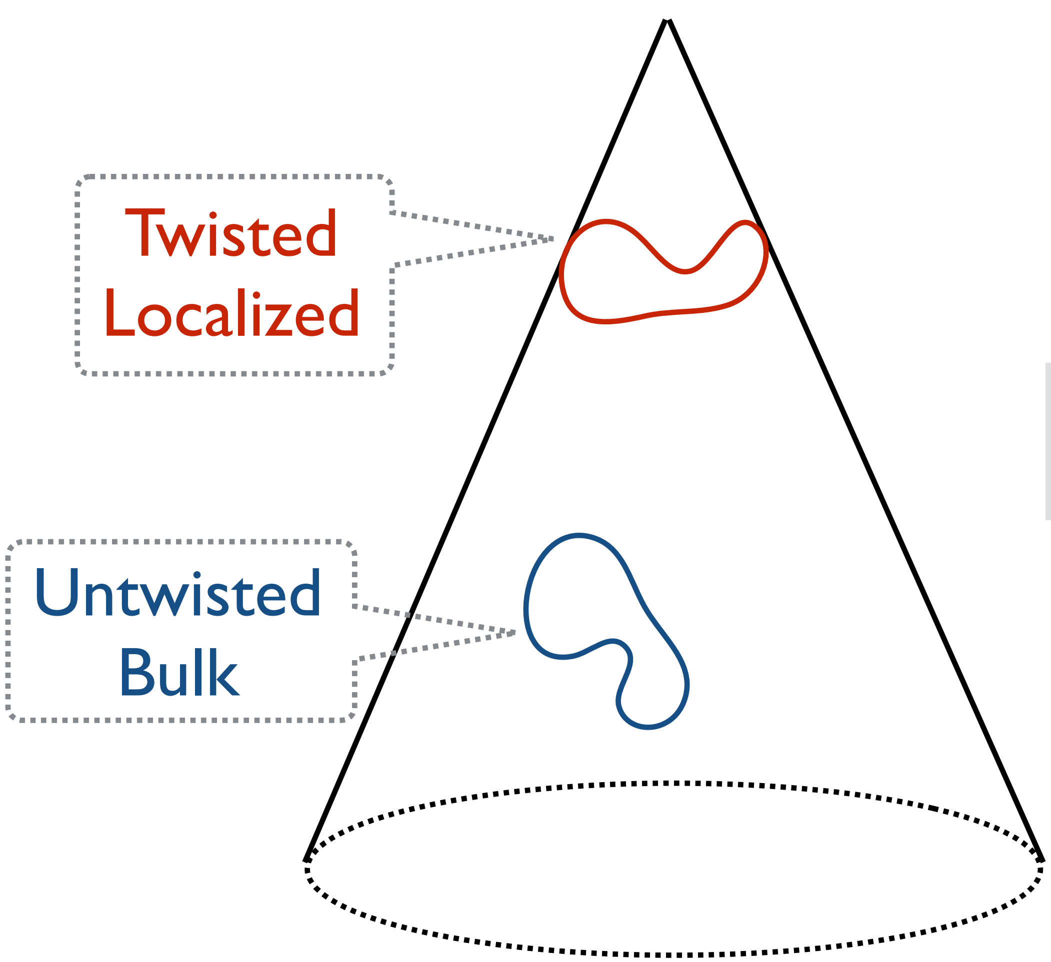
SINGULARITIES

String Theory was very successful in ***resolving*** singularities;
The simplest kind are **orbifolds**, shown below.

GR and QFT must be supplemented by adhoc bnry conditions at the tip of the cone; In string theory there is no such ambiguity.

Singular behavior seems to arise only because one fails to recognize that twisted modes may have **zero mass**.

All issues are INFRARED



$$(\mathbb{R}^n - \{0\})/\Gamma$$

Three other important examples:

D-branes: The localized modes are **open strings**

Correspond to (in general) singular
10d supergravity solutions

Polchinski '95

CY conifolds: best-known example the quintic

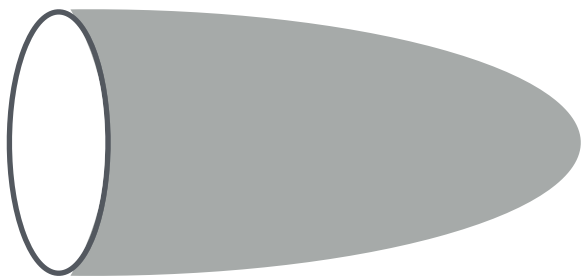
$(z_1, \cdots, z_5) \in \mathbb{C}P^4$ with $z_1^5 + z_2^5 + z_3^5 + z_4^5 + z_5^5 - 5 z_1 z_2 z_3 z_4 z_5 = 0$

which near $z_1 = z_2 = \cdots = z_5$ is a cone over $S^2 \times S^3$

The localized modes are **wrapped D2-branes** (or D3-branes)

Greene, Morrison, Strominger '95

__Dimensional__ __reduction:__ for instance the Taub-NUT metric ….



$$ds^2 = V d\vec{x} \cdot d\vec{x} + V^{-1}(d\tau + \vec{A} \cdot d\vec{x})^2$$

$$V = 1 + \frac{2M}{|\vec{x}|} \qquad \vec{\nabla} \times \vec{A} = \pm \vec{\nabla} V$$

… is singular in 3d. The localized modes are **KK modes**

All these "nice cases" look singular because we forgot some potentially light modes, that can **look different in various parameter regions**

ex. Taub-NUT of 11d sugra ⟷ D6-brane of IIA string theory

Duality

BUT, all these singularities are **time-like,** time is idle spectator

With **space-like** *(light-like)* singularities string theory has had remarkably little success:

$\mathbb{R}^{1,2}/\Gamma$

hyperbolic (parabolic) element of O(1,2)

Wick-rotations of Taub-NUT ?

s-branes (spacelike D-branes) ?

dS/CFT ?

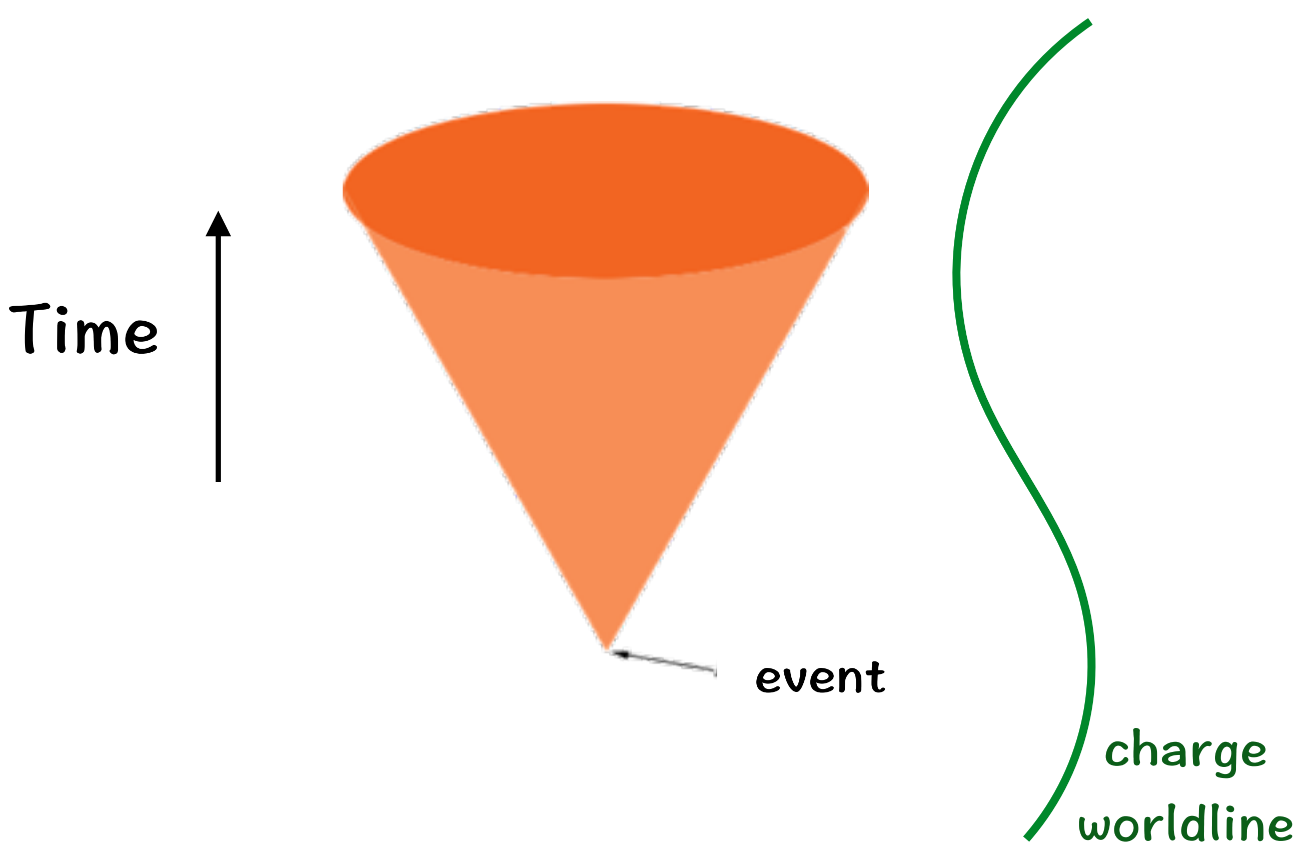
CB, Bunster, Henneaux '09

An amusing analogy may help clarify this point

Dirac events in (2+1)d electrodynamics:

$$\partial_\mu F^{\mu\nu} = j_e{}^\nu \ , \qquad \epsilon^{\mu\nu\rho}\partial_\mu F_{\nu\rho} = g\delta(x)$$

Equation for a Dirac monopole, but in $\mathbb{R}^{1,2}$ rather than $\mathbb{R}^3$.



Event: Creation of magnetic flux g which spreads out at speed c

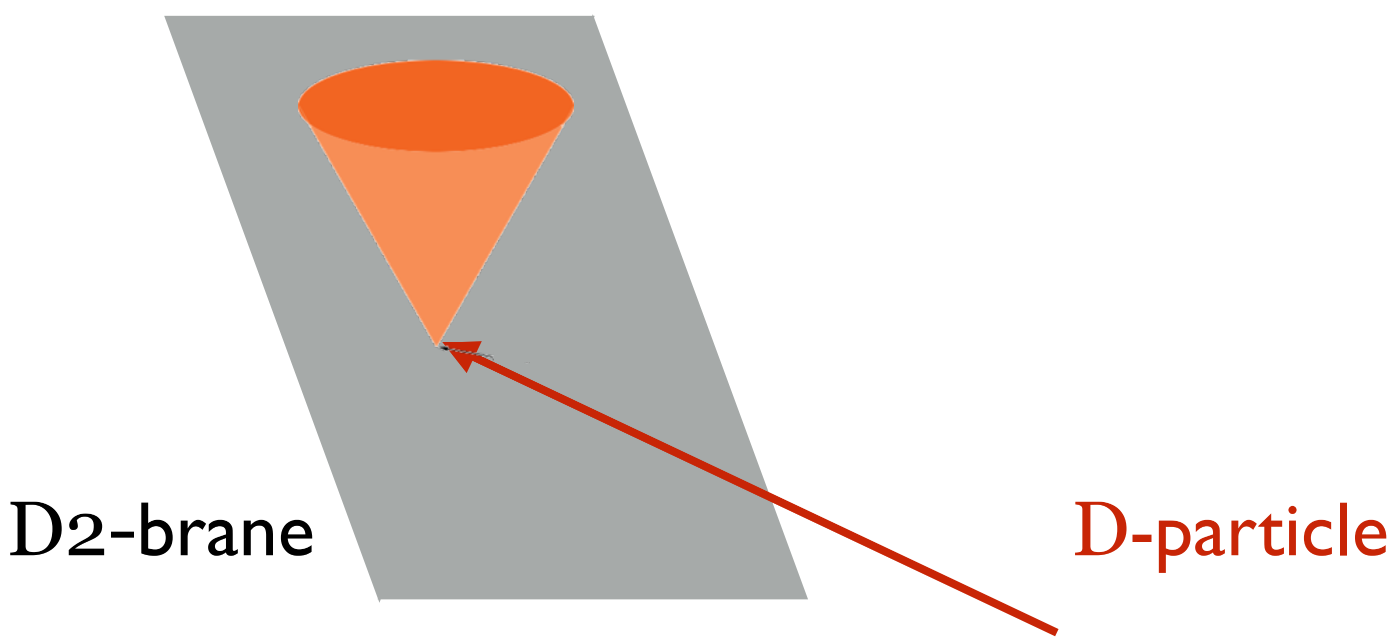
Anti-event: Focussing and disappearance of g units of magnetic flux

Dirac quantization still holds, but for a different reason:

A charged particle acquires angular momentum from the event

$$|\Delta L| = eg/2\pi = n\hbar$$

String theory resolves this singularity with the help of an **extra dimension**:   flux is confined outside a D2-brane and carried by D-particles, which can depose it as shown:



Could be observed in S-I-S Josephson junction

In this example, singular **events** are generic while **anti-events** require fine-tuned initial data. Can easily generalize to extended **events,** e.g. D-string hitting and being dissolved in a D3-brane.

But the main point: resolution unlike that of timelike singularities ; e.g. embedding in a non-abelian theory, which would resolve the singularity of a Dirac monopole, does not help.

't Hooft, Polyakov

The resolution depends on initial conditions in an extra dimension

How to transplant this to a gravity theory?
Would it make us wiser?

cf. s-branes, ekpyrotic universe, ...

To summarize:

String theory resolves all sorts of timelike singularities

Spacelike singularities pose different challenge

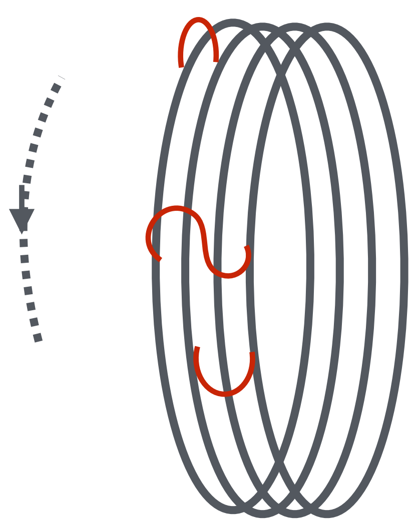
3

ENTROPY & HORIZONS

The singularities of the previous section have no degeneracy when all localized modes are in their ground state.

The simplest example of degenerate singularities are the **extremal 2-charge " black holes "**

e.g.



heterotic, or type-I D-string with

momentum = $\dfrac{n}{R}$ winding = w

$$M \geq M_{\text{ext}} = |\frac{n}{R} + 2\pi T w R|$$

$$S_{\text{ext}} = 4\pi\sqrt{nw}$$

The corresponding 9d supergravity solutions are singular because some scalars (here the string coupling e^{Φ}) run away.

In appropriate duality frames, higher-order corrections $\alpha' R^2 + \cdots$ can remove the singularity, and reproduce microscopic details.

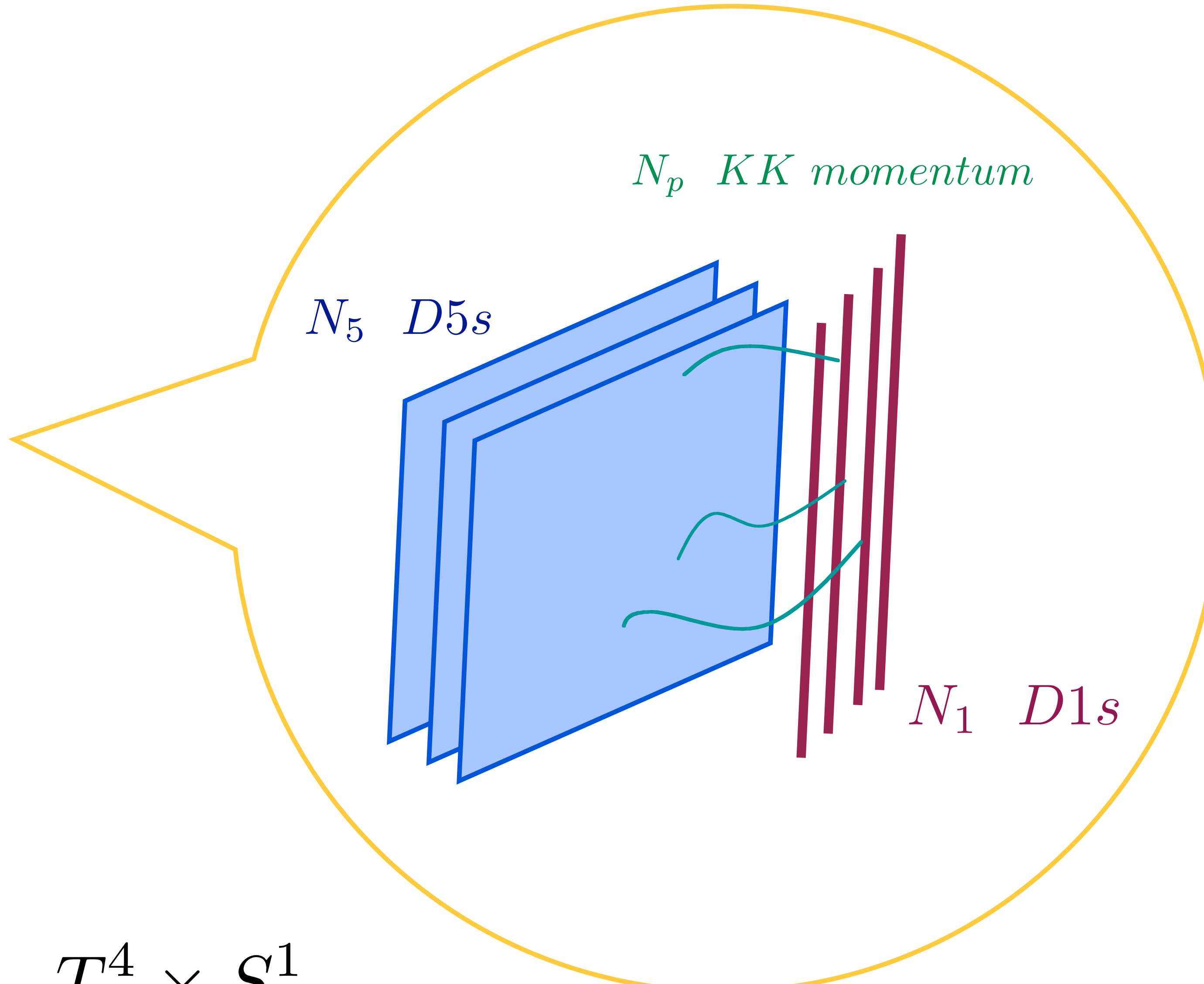
"small black holes" Sen; Mathur; Dabholkar; Wald ….

Very interesting, but micro-details are in "stretched horizon" at string or Planck scale.

What about **large black holes** that exist for $\geq$ 3 charges ?

The famous example is the Strominger-Vafa D1-D5 black hole →

Microscopic description of 3-charge BH



The branes wrap a compact $T^4 \times S^1$

The 5d sugra solution has a large smooth horizon with

$$S_{\rm BH} = \frac{\rm Area}{4{\rm G}\hbar} = 2\pi\sqrt{N_1 N_5 N} \simeq S_{\rm micro}$$

The 3-charge BH generalizes the Reissner-Nordstrom solution

$$ds^2 = -f dt^2 + f^{-1} dr^2 + r^2 d\Omega_2^2 \; , \quad \text{where}$$

$$f(r) = (1 - \frac{r_+}{r})(1 - \frac{r_-}{r}) \quad \text{with} \quad r_\pm = G_N M \pm \sqrt{G_N^2 M^2 - G_N Q^2}$$

charge in units where Coulomb's constant =1 .

$$T = \frac{r_+ - r_-}{4\pi r_+^2} \qquad S_{\rm BH} = \frac{\pi r_+^2}{G}$$

(Where) Is the information about the BH microstate stored ?

At the singularity ? At the (outer) horizon? In between ?

" featureless if Q>>1 "

Proposal: replace BH geometry with smooth, horizonless

fuzzballs. Many such examples for ≥ 3 charge BHs

Giusto, **Mathur** '04
Bena, Warner '05
Berglund, Gimon, Levi '05
....

Folklore : no gravitational solitons other than black holes

Many evasion windows and pitfalls, with illustrious prehistory !

A. Einstein and W. Pauli, On the non-existence of regular stationary solutions of relativistic field equations. Ann. Math.,44:131, 1943

Taub-NUT = Kaluza-Klein monopole is counterexample

Sorkin '83; Gross, Perry '83

Key to evasion: <u>non-trivial topology</u>. In a nice paper Gibbons + Warner arXives 1305.0957

have shown how Chern-Simons terms and non-trivial second homology of spatial sections can give globally-hyperbolic non-singular 5d solutions asymptotic to $M^{1,4}$

Consider a 5d smooth metric with a time-like Killing vector K

Conserved ADM mass:

$$\frac{32\pi}{3} G\, M = \int_{S^3} *dK \Big|_\infty \quad = \int_\Sigma d*dK + \int_{S_{\rm int}} *dK$$

0 *if no horizon*

$$= -2 \int_\Sigma *(K^\mu R_{\mu\nu} dx^\nu)$$

This usually vanishes by Einstein's equations + invariance of the matter form fields:

$$R_{\mu\nu} = 8\pi G (T_{\mu\nu} - \frac{1}{3} g_{\mu\nu} T_\rho{}^\rho) \qquad \mathcal{L}_K \omega = i_K d\omega + d(i_K\omega) = 0$$

But (minimal) N=2 5d sugra has three vector fields that obey

$$dF = 0 \ , \quad *(d * F) = F \wedge F$$

One finds after some algebra:

$$K^\rho F_{\rho\mu} = \partial_\mu \lambda, \quad K^\rho (*F)_{\rho\mu\nu} = -\frac{1}{2}\lambda F_{\mu\nu} + H_{\mu\nu} \ , \quad *(K^\rho R_{\rho\mu} dx^\mu) = \frac{1}{3} F \wedge H + \text{exact}$$

closed, not exact

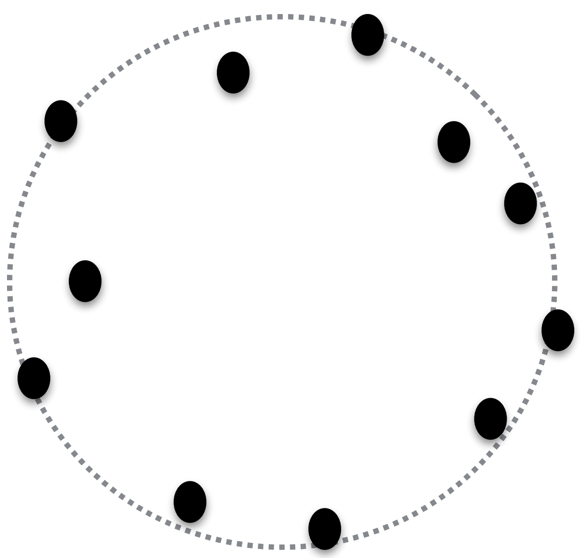
So if Σ has non-trivial 2-cycles, may have $M \neq 0$

A lot of hard work has gone into trying to generate enough fuzzball solutions to account for the entropy of the 3-charge BH

It would be a great mathematical achievement if the entropy of extremal 3-charge BHs can be accounted for by smooth 11d sugra geometries

"topological stars"

But reason for skepticism: multi-center Taub-NUT are non-singular in 5d KK theory, and mimic extremal 4d charged BH geometry far from horizon.



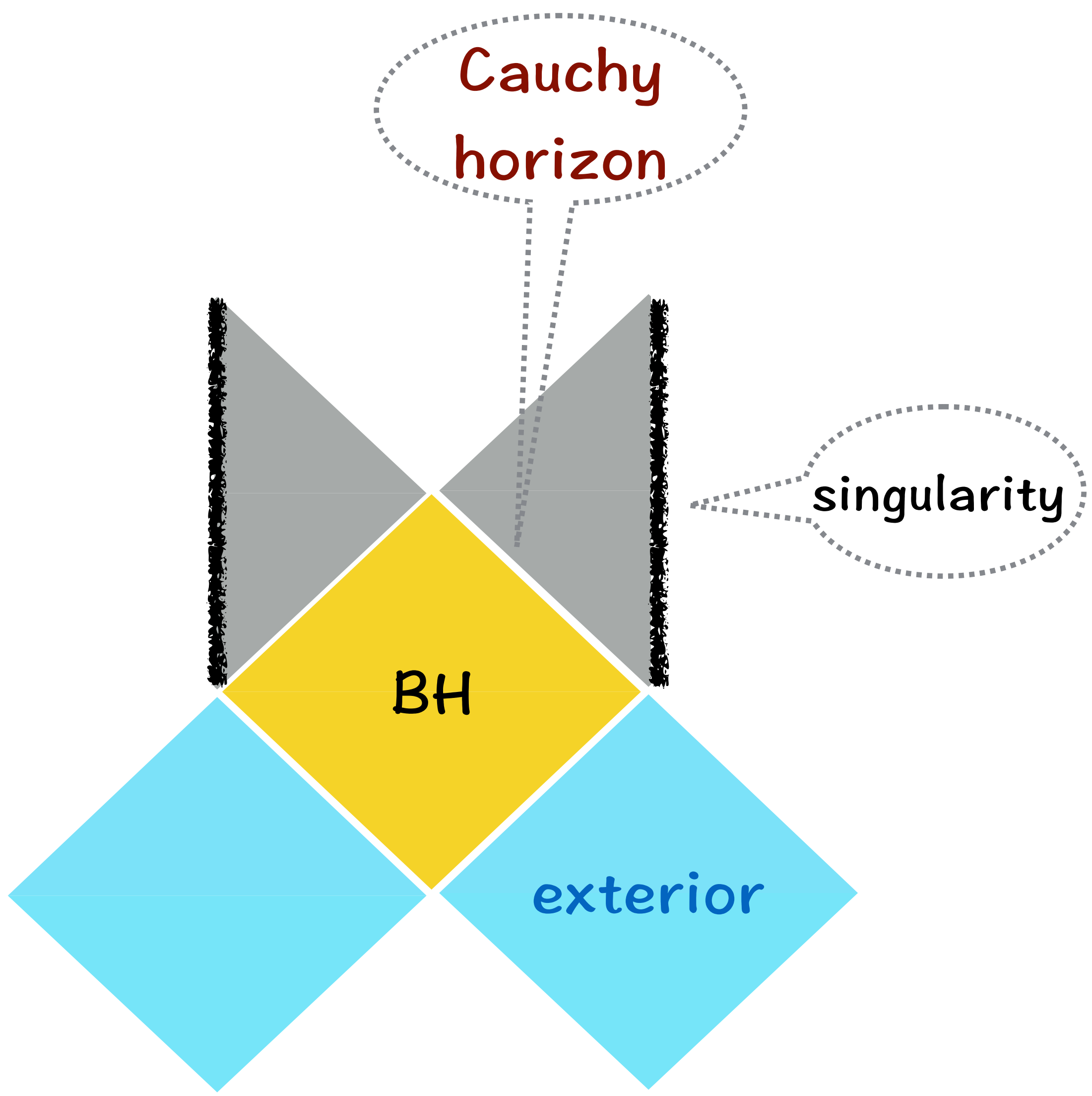
$$ds^2 = V d\vec{x} \cdot d\vec{x} + V^{-1}(d\tau + \vec{A} \cdot d\vec{x})^2$$

$$\vec{\nabla} \times \vec{A} = \pm \vec{\nabla} V \qquad V = 1 + \sum_{i=1}^{N} \frac{1}{|\vec{x} - \vec{x}_i|}$$

Could this imply breakdown of effective field theory for infalling observer ?

Genericity is crucial in most GR "theorems", as for thermodynamics.
What happens for generic non-extremal BHs ?

Penrose diagram of Reissner-Nordstrom BH



The singularity is <u>timelike</u>;
Cauchy horizon at $r = r_-$.

<u>Artificial:</u> Cauchy horizon collapse to space-like singularity

cf. Dafermos, Annals of Mathematics, 158 (2003), 875

To summarize:

Fuzzballs get rid of BH horizons, replace them by normal ‘topo-stars’

Supporting evidence still slim (extremality? enough states?)

If true would be a conventional resolution of info paradoxe

Do away with BH horizon and singularity

Why (how) does effective field theory hold/fail ?

4

EMERGENT GEOMETRY

Such issues come into sharper focus in the context of **AdS/CFT**

This conjectures that on-shell quantum gravity with asymptotic AdS_{d+1} boundary conditions at spatial infinity is equivalent to an ordinary relativistic CFT_d

e.g. $\mathrm{AdS}_5 \times \mathrm{S}^5 \longleftrightarrow \mathcal{N} = 4\ \ \mathrm{SYM}$

Duality

SYM theory is unitary and, if the correspondence is right, it should have states that resemble black holes. So here we have a well-posed problem: how does geometry emerge from CFT ?

Consider empty AdS: how does **locality in the bulk** emerge ?

$$ds^2 = \frac{L^2}{z^2}(dz^2 + dx_\mu dx^\mu)$$

$$\mathcal{O}_p = \int d^4x \, e^{ipx} \mathcal{O}(x)$$

single-trace operator of dimension Δ.

$$(\Box - m^2)\xi_p = 0$$

normalizable modes for $\Delta(\Delta - 4) = m^2 L^2$

Then one can define the "generalized free field" in AdS_5

$$\phi_{\mathrm{CFT}}(x,z) = \int_{p^0>0} \frac{d^4p}{(2\pi)^d} \left[\mathcal{O}_p \, \xi_p(x,z) + \mathcal{O}_p^\dagger \, \xi_p^*(x,z)\right]$$

This has only 2-point function at $N \to \infty$, and right causal structure

But is 4-point scattering local in z at scales $\ll L$?

The belief is that this is true only in the limit $\lambda_{\prime \mathrm{tH}} = N g_{\mathrm{YM}}^2 \gg 1$

Impressive progress in computing 4-point functions from integrability, but only for $p_\mu p^\mu = 0$

Basso, Sever, Vieira '14

So even this simple fact has not been yet fully tested.

Indirect arguments, from the existence of a mass gap for spins > 2 , and from the structure of conformal blocks are quite convincing.

.....

e.g. Hemskerk, Penedones, Polchinski, Sully '09

I used in the previous slide Poincaré coordinates, but it is simpler to change now to global coordinates, where AdS is "a box"

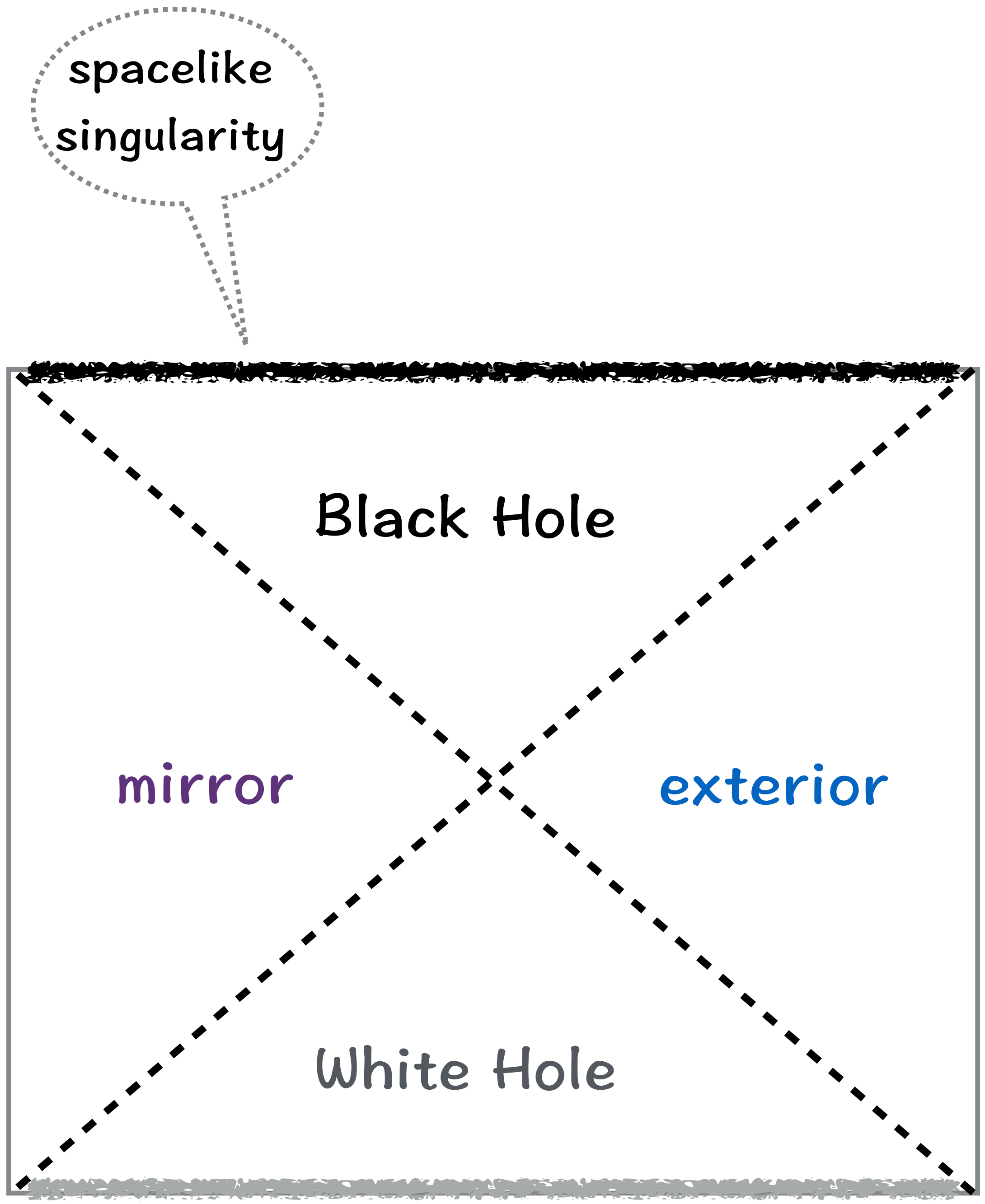
Radiation is reflected at the boundary, so there are two types of BH:

- **small** AdS black holes that evaporate
- **large** AdS black holes in thermal equilibrium

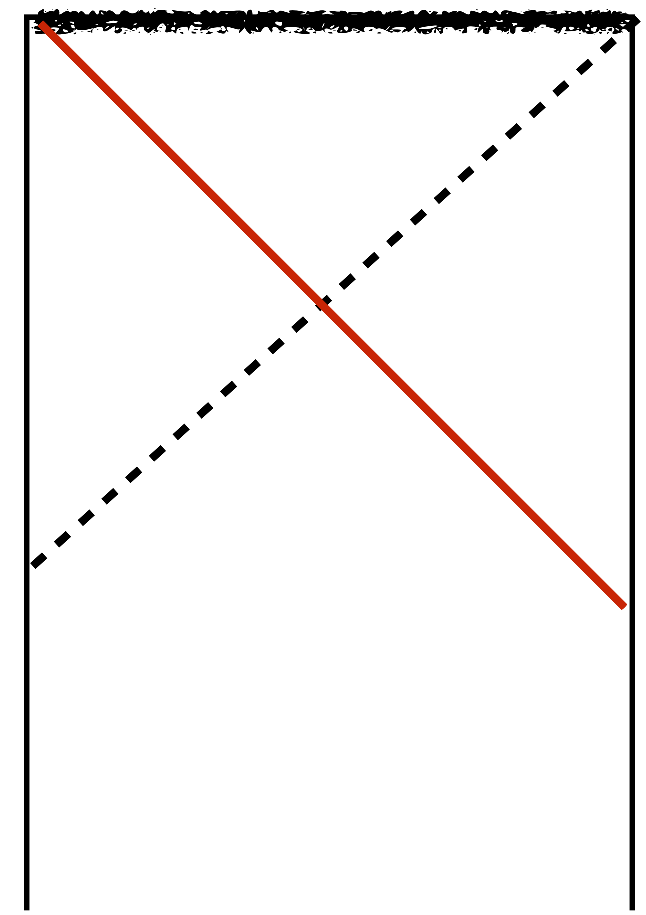
At a sufficient energy, a typical state in the CFT made out of $O(N^2)$ single-trace operators should resemble a large AdS black hole

<u>Question</u>: can one reconstruct its geometry, and in particular the smooth ride of an infalling observer as she crosses the horizon ? *or is this ride incredibly bumpy ?*

Firewall



Penrose diagram of
Eternal AdS BH



AdS BH formed
by collapsing shell

Since the BH does not evaporate, the final singularity is unavoidable

If this *is* String Theory = SYM , must be possible to understand
why and how the effective field theory breaks down !

A Hartle-Hawking assumption will not help, need to understand
the 'end of time'

Many exotic ideas have been evoked, e.g. post-selection:
final-state b.cn. at singularity

$$P(a_1, a_2 \cdots , a_n) = \mathrm{tr}(\rho_{\mathrm{F}}\, \Pi_{a_n} \cdots \Pi_{a_2} \Pi_{a_1}\, \rho\, \Pi_{a_1} \Pi_{a_2} \cdots \Pi_{a_n})$$

Horowitz, Maldacena '03

$$\Longrightarrow \quad P(a_1, a_2 \cdots , a_{n-1}) \neq \sum_{a_n} P(a_1, a_2 \cdots , a_n)$$

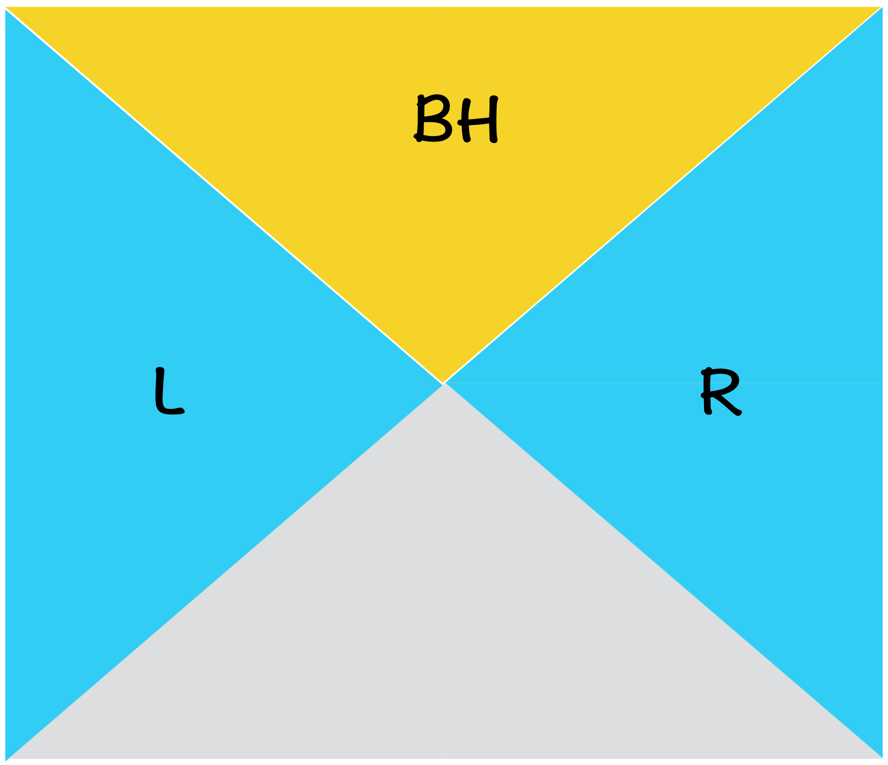
Probabilities depend on what we try to measure at later times !

Let's take a closer look: long after the BH has formed, near its horizon the geometry believed to settle to an empty, locally Minkowski region.

In terms of Rindler coordinates, vacuum looks thermal to R observer:

$$|0\rangle = \sum_n e^{-\beta E_n/2} |n\rangle_R \otimes |\, n\rangle_L$$

$$x \pm t = -e^{\tilde{\xi} \mp \tilde{\tau}}$$



$$x \pm t = e^{\xi \pm \tau}$$

Only R modes can propagate to the boundary where the CFT is defined.

Can the L modes be made of operators in same Hilbert space ?

NO: then why should the independent L modes be exactly entangled, so as to avoid a firewall for the infalling observer ?

Almheiri, Marolf, Polchinski, Sully
‘12

YES: then the infalling observer can act on the same Hilbert space as the one left outside: why cann’t she send signals to him ?

(perfect ‘complimentarity') ’t Hooft, Susskind, …

Yes, in a restricted sense, and the construction is **state-dependent**

Papadodimas, Raju
’12, ’13, ‘15

Idea looks promising, but it must overcome many hurdles:
state dependence, causality, final singularity

For a typical highly-excited state $|\Psi\rangle$, consider only a small 'algebra' of observables that do not alter drastically the state

$$\mathcal{A} = \{\mathcal{O}\} \quad \text{with} \quad \mathcal{O}|\Psi\rangle \neq 0 \quad \text{for all} \quad \mathcal{O} \in \mathcal{A}$$

O(N^2) products not allowed

Then show that one may construct an isomorphic algebra $\tilde{\mathcal{A}} \equiv \mathcal{A}$ such that $[\tilde{\mathcal{A}}, \mathcal{A}] = 0$, and the $\tilde{\mathcal{O}}$ are entangled thermally with the $\mathcal{O}$.

This mimics the Tomita-Takesaki construction, but should be valid an approximate sense. Can one avoid possible contradictions with causality, and ultimate fate of infalling observer ?

CLOSING REMARK

AdS/CFT helps to sharpen *some* of the BH puzzles, *i.e.* put them in a context where with (unlimited) prowess they could be answered.

But actual string theory has been hardly used in this debate.

From high-energy scattering we know that strings can stretch out to arbitrary transverse size, when undergoing large relative boosts

Amati, Ciafaloni, Veneziano '87
Gross, Mende '87
CB '95

Does this change our notion of locality near the BH horizon ?

Amati, Ciafaloni, Veneziano '07 ; Giddings, Gross, Maharana '07
….. Silverstein '14

Thank you very much

for your attention