Information Loss and Entanglement in Quantum Theory

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Some basic claims (to be justified, afterwards)

1. The *quantum-mechanical state* (the "wave function") does *not* have an "ontological status". It does *not* describe "what is". It is a mathematical device enabling us to make bets about what may happen in the future.
2. In the presence of information loss and entanglement with lost degrees of freedom, *pure states* typically evolve into *mixed states* (without violation of any basic principles of quantum theory), and it is this fact that makes a rational theory of *measurements/observations* possible!
3. Without *fundamental loss of information* and *entanglement* of observed degrees of freedom with *unobservable/"lost"* degrees of freedom, retrieval of information about quantum systems by measurements/observations would actually be *impossible*.

Basic claims, ctd.

4. No *information-* or *unitarity paradoxes* in quantum theory, (even if space-time is curved, e.g., in the presence of black holes)! Time evolution of states of quantum systems exhibiting information loss and entanglement with unobservable degrees of freedom (that, for example, may have disappeared in a heat bath, or escaped towards infinity, or have fallen through a horizon of a black hole) is actually never unitary – it is "tree-like"!
5. Operator algebras (including type III_1 factors!) and other sophisticated mathematical tools have been invented to be *used* in Quantum Theory, rather than to be ignored!

Credits and Contents:

Motivation from recent experiments, in particular the ones of the *Haroche-Raimond* group; papers by *Bauer & Bernard*, *Maassen & Kümmerer*, and others; joint work with my former PhD student *Baptiste Schubnel*; (some joint efforts with *Ballesteros, Faupin, Fraas, Pickl, Schilling, Schubnel*); discussions with Detlev Buchholz and others.

1. *Introduction*
2. *Information Loss*
3. *Projective (von Neumann) measurements*
4. *Conclusions*

1. Introduction – Questions to be Addressed

In our courses, we tend to describe quantum-mechanical systems as pairs of a Hilbert space, $\mathcal{H}$, and a propagator, $U(t,s)$, describing time-evolution. Unfortunately, these data encode almost *no invariant structure* (besides spectral properties of $U(t,s)$) and give the erroneous impression that quantum theory might be deterministic. Thus, among *fundamental problems of quantum theory* are:

- What do we have to add to the usual formalism of quantum theory in order to arrive at a mathematical structure that (through "interpretation") can be given physical meaning, *independently of "observers"*?

Questions, ctd.

- Where does *intrinsic randomness* in quantum theory come from, given the deterministic character of the Schrödinger equation? In which way does it differ from classical randomness?
- Do we understand the *effective dynamics* of quantum systems, e.g., ones in contact with a heat bath (*"Quantum Brownian Motion"*), or systems under repeated measurements (exhibiting *"quantum jumps"*)?
- What do we mean by an *"isolated system"* in quantum mechanics, and why is this an important notion? How does one *prepare* a system in a *prescribed state*?

Answers and insight come from understanding roles of

Information Loss & Entanglement

2. Information Loss

A simple-minded definition of quantum-mechanical systems:

An *isolated* quantum system, S, is characterized by following choices:

(i) $(\mathcal{H}, U(t,s)), \mathbb{R} \ni t, s$ ($U =$ unitary propagator)

(ii) a list, $\mathcal{O}_S = \{a_i\}_{i \in I_S}$, of bounded, selfadjoint operators on $\mathcal{H}$ representing *physical quantities/ potential properties* of S that can be measured/ observed in direct *"projective measurements"*;

(S must be an *isolated* system *chosen large enough* for quantities represented by $a_i, i \in I_S$, to be measureable).

Information Loss, ctd.

Choose fiducial time, t_0, and define (Heisenberg picture)

$$a(t) := U(t_0, t) a U(t, t_0), \qquad a \in \mathcal{O}_S,$$

to be the operator representing the pot. prop. corresp. to $a \in \mathcal{O}_S$ at time t ; ➔ list of ops. $\mathcal{O}_S(t)$

Pot. properties, $a(s)$, measureable/observable at times $s \geq t$ generate a *W*-alg.,* $\mathcal{E}_{\geq t}$:

$$\mathcal{E}_{\geq t} := \langle \sum \prod_i a_i(t_i) | a_i \in \mathcal{O}_S, t_i \geq t \rangle^{-}$$

$$\mathcal{A}_S := \mathcal{E}_{\geq -\infty}, \qquad \mathcal{S}_S \text{ (states)} \qquad (1)$$

$$B(\mathcal{H}) \supseteq \mathcal{A}_S \supseteq \mathcal{E}_{\geq t} \supseteq \mathcal{E}_{\geq s} \supseteq \mathcal{O}_S(s), \; s \geq t \qquad (2)$$

$\neq$ ⟵ *Information Loss!*

Information Loss, ctd.

Define $$\tau_s(a(t)) := a(s+t)$$

so that $$\tau_s : \mathcal{E}_{\geq t} \to \mathcal{E}_{\geq (t+s)}$$

τ_s is a *endom; τ_s *not* a *autom $\Leftrightarrow$ *information loss*

$\Rightarrow$ *entanglement with "lost" degrees of freedom!*

It is easy to construct examples of (generally *non-autonomous*) quantum systems exhibiting *information loss*, in the sense of Eq. (2):

(1) Independent "probes", E_j, $j = 1,2,3,\ldots$, with E_j being destroyed at time τ j, (τ discrete time step)

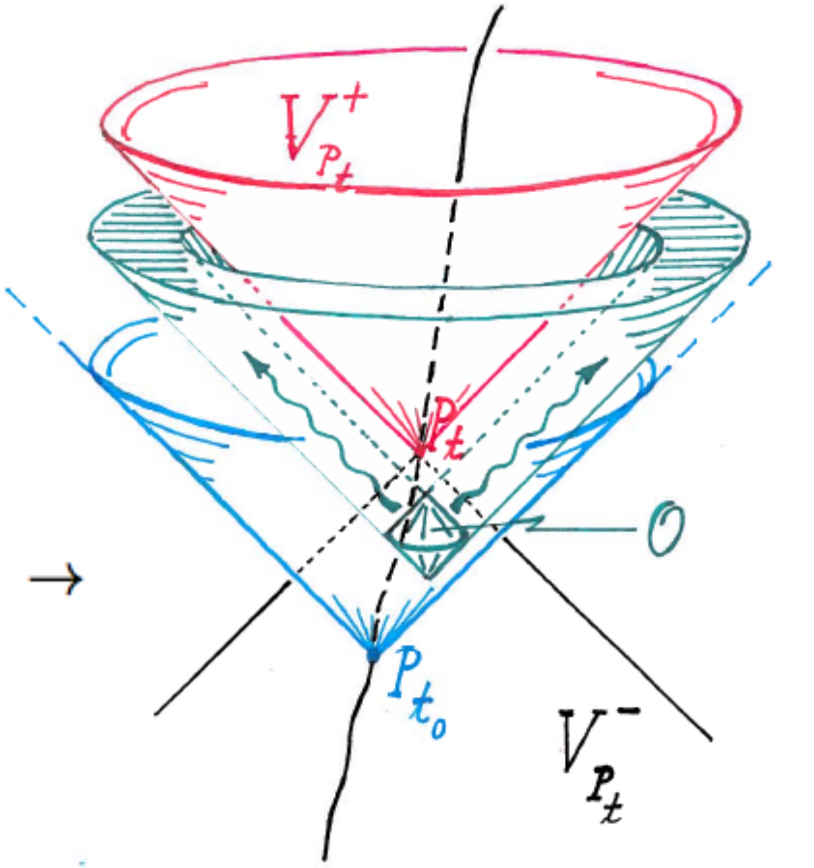
(2) "Small" systems *temporarily* interacting with quantized wave medium, (e.g., photons, phonons, etc.)

(3) *Information loss in theories like QED* – see Buchh.-Rob.

All ops. in $\mathcal{E}_{\geq t}$ are localized in $V^+_{P_t}$, $P_t = (t, \vec{x})$

$\mathcal{E}_{\geq t_0} \supset \mathcal{E}_{\geq t}$, for $t > t_0$

$(\mathcal{E}_{\geq t})' \cap \mathcal{E}_{\geq t_0} \supset \mathcal{A}^{\text{out}}_{\mathcal{O}}$ $\rightarrow$



worldline of *JF*

3. An application: Projective (von Neumann) measurement

We will now discuss how phys. quantities are measured projectively.

Some fundamental questions to be addressed:

(1) What is meant by a "measurement" of $a \in \mathcal{O}_S$? Around which time t does it take place ? A measurement of a ought to result in "a having a value", i.e., become an "empirical/objective property" of *S*
$\Leftrightarrow$ state on $\mathcal{E}_{\geq t} \simeq$ incoherent mixture of eigenstates of $a(t)$,
at some time *t*. *(Proj. measnts. vs. indirect (Kraus) measnts.)*

(2) Given a state of *S*, does *QM predict* which $a \in \mathcal{O}_S$ will be measured first; what does *QM* predict about the outcome of measnt. of a ? In which way is *QM* intrinsically *indeterministic*? Why does a measnt. of a have a *random outcome*?

Projective measurements, ctd.

Projective measurements

We have to clarify what may be meant by a *projective measurement* of a potential property $a \in \mathcal{O}_S$ and what the role of *information loss & entanglement* in measnts. is:

$$\mathcal{O}_S \ni a = a^* \quad \text{with eigenvalues} \quad \alpha_1, \alpha_2, \ldots, \alpha_k,$$

$$a(t) = \sum_{j=1}^{k} \alpha_j \Pi_j(t) \qquad (3)$$

"Measurement/observation" of a around time *t*

$\Leftrightarrow$ a is an *"empirical/objective property"* of *S* around time *t* :

Projective measurements, ctd.

State ρ is an *incoherent superposition* of eigenstates of a,

$$\rho(b) = \sum_{j=1}^{k} \rho(\Pi_j(t) b \Pi_j(t)), \qquad \text{for all } b \in \mathcal{E}_{\geq t}\,, \quad (4)$$

where ρ is the state of S right before measnt. of a, i.e.,

$$\rho = \rho_t := \rho|_{\mathcal{E}_{\geq t}} \quad (5)$$

Information Loss $\Rightarrow \rho = \rho_t$ is usually a mixed state on $\mathcal{E}_{\geq t}$ *even* if the initial state of S has been *pure*, as a state on $\mathcal{A}_S$!

Projective measurements, ctd.

Suppose, for simplicity, that $\mathcal{E}_{\geq t}$ is isomorphic to some $B(\mathcal{H}_t)$, (i.e., $\mathcal{E}_{\geq t}$ is of type $I_\infty \Rightarrow$ syst. *non-autonomous*)

$$\text{Eq. (4)} \quad \Leftrightarrow \quad [a(t), P_t] = 0, \qquad (6)$$

where P_t is the density matrix on $\mathcal{E}_{\geq t}$ corresp. to ρ_t

Definition

$a \in \mathcal{O}_S$ is measured/observed around time $t \Leftrightarrow a$ is an *"empirical/objective prop."* of *S* around time *t* iff

$$a(t)|_{Range P_t} \approx_t F(P_t) \cdot z, \text{ for some bd. fu. } F, \qquad (7)$$

and some z in the center of $\mathcal{E}_{\geq t}$. More generally, $a(t)$ belongs to the *center of the centralizer of the state* ρ_t, $\mathcal{Z}_{\geq t}$. Note that R.S. of (7) is *cond.expectation* of $a(t)$ w.r. to $\mathcal{Z}_{\geq t}$. Eq. (7) $\Rightarrow$ Eq. (4)! ➚ Tomita-Takesaki theory!)

Projective measurements, ctd.

Axiom A

If a is measured (i.e., an empirical/objective prop. of *S*) around time t then a has a *value* $\in \{\alpha_1, \ldots, \alpha_k\}$ around time *t.*

The value α_j of a is observed w. probability

$$p_j(t) = \rho(\Pi_j(t)) \tag{8}$$

If α_j is observed around time t then the state

$$\rho_j^a(\cdot) := p_j(t)^{-1} \cdot \rho(\Pi_j(t)(\cdot)\Pi_j(t)) \text{ on } \mathcal{E}_{\geq t} \tag{9}$$

should be used for improved predictions of future after time *t.* (Eq. (8) is *Born's Rule*, eq. (9) *"collapse postulate"*.)

Projective measurements – *summary*

(1) Given the *initial state* of the system *S*, *time evolution*, {*U(t,s)*}, <u>*determines*</u> which pot. prop. $a \in \mathcal{O}_S$ will first be measured (i.e., become empirical/objective), and around which time!

(2) Measnt. of a_2 is *independent* of an *earlier* measnt. of a_1 iff a_2 becomes empirical/objective *after* time of measnt. of a_1, no matter what the outcome of measnt. of a_1 was, i.e., *for all states* $\rho_j^{a_1}(\cdot)$ *, j= 1, … , k,* with $\rho_j^{a_1}(\cdot)$ as in (9).

$\Rightarrow$ *Decoherence, "consistent histories".*

(3) Time of measurement*:* Time, t_*, of observation of a det. by minimizing in t the fu. $\|a(t)|_{RangeP_t} - F(P_t) \cdot z\|$, where $F(P_t) \cdot z$ is the "*cond. exp.*" of $a(t)$ onto $\mathcal{Z}_{\geq t}$.

(4) $\mathcal{Z}_{\geq t}$ (= center of centralizer of $\mathcal{E}_{\geq t}$) contains all phys. quantities observable at time *t*.

Projective measurements – *summary*

(5) A state is called *"passive"* iff the center, $\mathcal{Z}_{\geq t}$, of the centralizer of $\mathcal{E}_{\geq t}$ is *time-independent*. There are plenty of examples of passive states:

- Equilibrium (KMS) states at positive temperature in QFT; KMS states of a QFT in the space-time of a static black hole.
- Perturbations of the vacuum state by coherent clouds of massless particles (e.g., of photons – courtesy of D. Buchholz).

Passive states have the property that they do *not admit any projective measurements/observations* of *any* physical quantities – besides measurements of *time-independent* parameters characteristic of the state in question, e.g., the temperature or a chemical potential of an equilibrium state, (which, indeed, are time-independent quantities).

4. Conclusions

We have to learn more about which states and which types of time evolutions of isolated systems allow for non-trivial measurements/observations of time-dependent physical quantities satisfying Axiom A.

Need for illuminating examples and models! – Ex.: Dynamics of a two-level atom in a heat bath monitored by photon detectors, (see, e.g., De Roeck-Derezinski, Bauer-Bernard-…):

$$dP_t = -\alpha(P_t - p) + \gamma P_t(1 - P_t)dW_t,$$

P_t : occupation prob. of upper level,

dW_t : Brownian motion; (α, γ , p: consts.)

States of quantum systems do not have any "ontological" significance. The *"ontology"* of QT resides in time-ordered sequences of *"events"* (= results of projective or indirect observations accompanied by info. loss and entanglement with unobservable degrees of freedom).

Effective Dynamics

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Paris, March 2015

"Why should I blame anyone but myself if I cannot understand what I know nothing about?" – Pablo Picasso

Credits and Contents

Credits:

Abou Salem, Albanese, Bach, Ballesteros, Bauerschmidt, Benettin, Blanchard, De Roeck, Faupin, Fraas, Gang, Giorgilli, Griesemer, Knowles, Merkli, Pizzo, Schenker, Schlein, Schnelli, Schubnel, Schwarz, Sigal, Spencer, Ueltschi, Wayne. –

Today, two recent examples from quantum theory, with: *Schenker; Ballesteros, Fraas, Schubnel*

Contents:

1. Introduction – survey of examples
2. Thermal noise kills Anderson localization
3. Dynamics of a Quantum System under repeated measurements
4. Indirect measurements & "pointer observables"
5. Analogy to classical statistical mechanics
6. Effective dynamics of pointer observables
7. Conclusions

1. Introduction – survey of examples

- Return to equilibrium – Master equations, kinetic limit, etc. (examples: equilibration of spins or oscillators, thermal ionization, easy half of *0th Law*,…; with B & Sig; M – see also Jac & Pil)
- Relaxation to a ground state – scattering techniques, cluster expansions in real-time variable - DeR (example: decay of excited atom or molecule; preparation of states in QM, etc.; with Gr,S; Schub)
- Approach to a NESS or to a NE time-periodic state – scattering techniques, resonance theory for Liouvillians, dynamics with *entropy production,* Onsager relations (application: e.g., *2nd Law of Thermodynamics*; with M & Ue, A-S)
- Anderson Localization – **multi-scale analysis**, small denominators (examples: absence of diffusion for a quantum particle in a random- or 1D qp potential, absence of heat transport in disordered arrays of anharmonic oscillators, …; with Sp; Sp & W; A; B & G)

Survey of examples, ctd.

- Quantum Brownian Motion, position-space decoherence, equipartition – derivation of diffusive motion "from scratch" (example: atom with finitely many internal states coupled to a qm heat bath and hopping on a lattice; with DeR, Piz)
- Static disorder *and* thermal noise: Weak diffusive transport (examples: dyn. of a BEC, electron transport in a sparsely populated conduction band of a disordered semi-conductor: with J. Schenker)
- Quantum- and Hamiltonian Friction – friction through emission of *Cherenkov radiation* (examples: heavy atom moving through a Bose gas exhibiting BEC; dipole moving through optically dense medium; with Gang Zhou & Sof)
- Dynamics of quantum systems subjected to repeated measurements, emergence of facts in QM – statistics of measurement protocols, stochastic evolution eqs., large deviations; "theory of knowledge acquisition".

Survey of examples, ctd.

(example: experiments of Haroche-Raimond group, electron conduction in presence of Coulomb blockade, etc.; with B, F, Sch)

- **Dynamics in limiting regimes** – mean-field limit, kinetic limit (Gross-Pitaevskii description of Bose gases, (Bogliubov-)Hatree-Fock: e.g., neutron stars; point-particle limit of NL Hartree Eq., etc.; with Schwarz, Kn. and others)

"Postmodern" examples of effective dynamics:

- Quantum chaos vs. quantum integr. behavior; e.v. statistics, …
- Quantum quenches, dynamical problems related to hard half of *0^{th} Law of Thermodynamics* (e.g., "ETH")
- Many-body localization (see also results w. A; Sp & W; B & G)
- Very fast processes, such as ionization, involving (laser) light (F-P-S)
- Dynamics of inverted populations (dynamics of negative-temperature initial states; entanglement dynamics; …)

Unfortunately, I don't have much to say about these examples, yet – *who has?*

I will limit my attention to *quantum systems!*

2. Thermal noise kills Anderson localization

(with J. Schenker)

In this section we consider a quantum particle hopping on the simple cubic lattice $\mathbb{Z}^d$, d = 2,3, under the influence of a *random potential* and coupled to a *heat bath* at some positive temperature β^{-1}. The state of the particle is described by a one-particle density matrix,

$$\rho(x,y), x,y \in \mathbb{Z}^d. \qquad (1)$$

The dynamics of ρ is described by a Liouville equation

$$\partial_t \rho_t = \mathcal{L}(\rho_t), \qquad (2)$$

where t denotes time, and $\mathcal{L}$ is the 'Liouvillian' given by a *'Lindblad generator'* of the form

Thermal noise, ctd.

$$\mathcal{L} = -i \; ad_{H_\omega} + g(G - L), \;\; g > 0, \qquad (3)$$

where the Hamiltonian

$$H_\omega = -\Delta + v_\omega \qquad (4)$$

is a standard *random Schrödinger op.* (Anderson Hamiltonian), with $v_\omega = \{\omega(x)\}_{x \in \mathbb{Z}^d}$ a random potential (the $\omega(x)$ are iid rv's with, e.g., bd. distr., μ , of cpt. support), G is a *"gain term"* and L is a *"loss term"*. Introducing the variables $X = x + y$ and $\xi = x - y$, we can write the integral kernel of ρ as a function of X and ξ, i.e., as $\rho(X, \xi)$. Then G and L are given

Thermal noise, ctd.

by the formulae

$$(G\rho)(X,\xi) := \sum_{\eta\in\mathbb{Z}^d} r(\xi,\eta)\rho(X,\eta),$$

and

$$(L\rho)(X,\xi) := \sum_{\eta\in\mathbb{Z}^d} r(\xi-\eta,0)\rho(X,\eta),$$

(5)

where the kernel $r(\xi,\eta)$ satisfies a detailed-balance condition at the temperature β^{-1} of the heat bath (w.r.t. the kinetic energy, $-\Delta$, of the particle).

Our main result is the following theorem.

Thermal noise, ctd.

Theorem. Consider Eq. (2) for ρ_t, with

$$\rho_{t=0}(x,y) = \delta_{x0}\delta_{y0}\,.$$

If the coupling constant $g > 0$ then the *diffusion constant*

$$D := \lim_{t\to\infty} t^{-1} \sum_x |x|^2 \mathbb{E}\rho_t(x,x), \qquad (6)$$

where $\mathbb{E}$ denotes an expectation w.r.t. to the product measure $\prod_{x\in\mathbb{Z}^d}\mu(\omega(x))$, exists and satisfies

$(i) \qquad 0 < D < \infty$

(ii) if the disorder is large enough for $D|_{g=0}$ to vanish (complete localization in absence of thermal noise) then

Thermal noise, ctd.

$$\lim_{g \searrow 0} \frac{D(g)}{g}$$

exists and is finite.

The *proof* of this theorem makes use of the following formalism. Let Ω denote the space of random variables $\omega(\text{-})$. We define the Hilbert space

$$\mathcal{H} := l_2(\mathbb{Z}^d \times \mathbb{Z}^d) \otimes L^2(\Omega, \Pi_x \mu(\omega(x)))$$

Fourier transformation of vectors, Ψ, in $\mathcal{H}$ is defined by

$$\widehat{\Psi}(k, x; \omega) := \sum_{a \in \mathbb{Z}^d} e^{-ik\cdot a} \Psi(x + a, a; \tau_a \omega)$$

Let $\hat{\rho}_t(k, x; \omega)$ denote the Fourier transform of ρ_t .

Thermal noise, ctd.

Then $\hat{\rho}_t(k, x; \omega)$ satisfies the equation

$$\partial_t \hat{\rho}_t(k) = -\mathcal{G}_k \hat{\rho}_t(k),$$

where $\mathcal{G}_k$ = "Fourier transform of $\mathcal{L}$ " (see blackboard).

Then the diffusion constant D/d is given by the expression

$$-\lim_{t\to\infty} \frac{1}{t} \partial_{k_1}^2 \sum_x e^{-ik\cdot x} \mathbb{E}\rho_t(x,x)|_{k=0} = -\lim_{t\to\infty} \frac{1}{t} \partial_{k_1}^2 \mathbb{E}\hat{\rho}_t(k,0;\cdot)|_{k=0}.$$

Defining the vector $\phi := (\delta_{e_1} - \delta_{-e_1}) \otimes \mathbf{1}$, we find that

$$D/d \; = \; \lim_{\epsilon\searrow 0} \langle \phi, \frac{1}{\mathcal{G}_0 + \epsilon} \phi \rangle$$

The R.S. can be estimated using the Feshbach-Schur map.

Well, this is a little sketchy! (But details are fairly easy.)