CLASSICAL PREFERABLE BASIS IN QUANTUM MECHANICS

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A way to specify the preferable basis which determines the ensemble of universes in the many-worlds conception of quantum theory is proposed. This way is based on a consideration of the classical limit in quantum mechanics. The specified basis is shown to be necessary and sufficient for comparison of theory with observations.

1. Introduction

In quantum theory, a physical system is characterized by the vector of the corresponding Hilbert space, $|\psi\rangle$, which contains all the information on the state of the system. To interpret the vector $|\psi\rangle$, it is usually necessary to find the expansion of this vector in a certain basis of the initial Hilbert space. Although all the various expansions are equivalent from the point of view of the mathematics of the theory, in comparison of the elements of the mathematics with the elements of reality this equivalence is violated. So, in the Copenhagen conception of quantum theory in a consideration of a concrete experiment, preferable is the set of vectors to which the initial vector of the "state" of the system may be reduced as a result of measurement. In the many-worlds version of quantum theory [1,2] there also exists a preferable basis which characterizes the ensemble of universes described by quantum mechanics [3]. At each time moment a concrete element of this preferable basis is assigned a concrete universe. Thus, a preferable basis determines the scheme of comparison of an abstract vector $|\psi\rangle$ of a Hilbert space with elements of reality (ensemble of universes). In what follows we are dealing only with the many-worlds conception of quantum theory.

Several papers [3–5] proposed different ways to define a preferable basis. A detailed analysis of these preferable bases will be given elsewhere. The aim of the present paper is to describe the classical preferable basis (and its possible quasiclassical extension) which in our opinion is not only the most natural one but also necessary and sufficient for comparison of theory with observations. To this end we first of all briefly dwell on a not very trivial question concerning the classical limit of quantum mechanics.

2. The classical limit of quantum mechanics

As is well known, the behaviour of macroscopic objects obeys the laws of classical mechanics. On the other hand, since any macro-object consists of micro-objects each of which is described by quantum mechanics and there is no reason to exclude application of quantum laws to systems containing a large number of particles [6]; it must be also possible for macro-objects to be described by quantum mechanics. In the corresponding limiting case, therefore, the classical laws must follow from the quantum-mechanical laws. The question about passing over from the Schrödinger equation to the equations of classical physics is usually clearly formulated and solved only for simple concrete physical systems [7]. The concept of a classical system can also be defined only in the framework of a concrete physical problem.

In any experiment, an observer perceives *directly* only a set of classical data $\mathcal{M}^{\alpha}$ ($\alpha = 1, 2, ...$) on the state of the subsystems of an investigated physical system. These data are always known with a limited degree of accuracy of $\delta\mathcal{M}^{\alpha}$. We will call classical such subsystems of a physical system whose state in the framework of the formulated problem is uniquely characterized by a set of *macroscopical* operators $\hat{M}^\alpha$ asigned to observed classical quantities $\mathscr{M}^\alpha$. An operator $\hat{M}$ is called macroscopic if there exists a (super) *complete* set of such vectors $|\psi_i\rangle$ in the Hilbert space of an investigated *subsystem* such that the quantities

$$\bar{M}_i = \langle\psi_i|\hat{M}|\psi_i\rangle \qquad (1)$$

satisfy the classical equations of motion with an accuracy exceeding $\delta\mathscr{M}^\alpha$.

It can be shown that for any vector $|\psi_i\rangle$ the following relations hold in this case,

$$\langle\psi_i|(\hat{M}^\alpha - \bar{M}^\alpha)^2|\psi_i\rangle^{1/2} < \delta\mathscr{M}^\alpha \ll \bar{M}^\alpha, \dots . \qquad (2)$$

Besides the vectors $|\psi_i\rangle$ and $|\psi_j\rangle$ can be distinguished only if they correspond to classically distinct states, i.e. if for a certain α we have

$$|\bar{M}_i^\alpha - \bar{M}_j^\alpha| > \delta\mathscr{M}^\alpha . \qquad (3)$$

If for the states i and j the conditions (3) do not hold, we will identify these states and henceforth use the indices i only for distinguishable states. For superposition of the wave functions $|\psi_i\rangle$ and $|\psi_j\rangle$ $(i \neq j)$ (a state of the "Schrödinger cat" type [8])

$$|\psi\rangle = \alpha|\psi_i\rangle + \beta|\psi_j\rangle , \qquad (4)$$

the equations for the mean $\bar{M}^\alpha = \langle\psi|\hat{M}|\psi\rangle$ derived from the Schrödinger equation are the sum of two classical equations (with weights $|\alpha|^2$ and $|\beta|^2$ for $\bar{M}_i^\alpha = \langle\psi_i|\hat{M}^\alpha|\psi_i\rangle$ and $\bar{M}_j^\alpha = \langle\psi_j|\hat{M}^\alpha|\psi_j\rangle$). It is natural to assume therefore that the wave function (4) described two types of classically different ensembles of universes. In this case the vectors $|\psi_i\rangle$ and $|\psi_j\rangle$ are the components of the preferable basis which specify different universes.

It is natural that classical equations are not exact. Quantum corrections to these equations are responsible for an interference interaction between different universes. These corrections are however small as compared with $\delta\mathscr{M}^\alpha$, and therefore the interaction between universes is smaller than the accuracy of the specification of universes. Classically different universes do not interact in this approximation and develop independently.

3. Preferable basis

On the basis of the previous consideration one can formulate the general rule for the definition of the preferable basis which determines an ensemble of universes described by a given wave function. Since an observer perceives the microworls only in the form of macroscopic phenomena, different universes have an interpretational sense for him only because they are macroscopically (classically) distinct. It is useless to speak of microscopically distinct universes because the interaction (interference) of such universes is too strong.

The definition of a preferable basis is based on the following fact: the observed world of macro-objects is a classical world.

Consider a system described by a complete wave function $|\psi(\dots)\rangle$. Fixing accuracies of determination of classical observables of a system, single out in the physical system all possible classical subsystems $M_1, M_2, \dots$ characterized by sets of macroscopic operators $\hat{M}_1^\alpha, \hat{M}_2^\alpha \dots$. Each of these subsystems is described by its own Hilbert space $\mathscr{H}^{M_n}$. In the Hilbert spaces $\mathscr{H}^{M_1}$, $\mathscr{H}^{M_2}$, ... single out the subspaces $\mathscr{H}_i^{M_n}$ ($i = 1, 2, \dots$) whose vectors correspond to certain classical states (see section 2), so that for any $|\psi_i^{M_n}\rangle \in \mathscr{H}_i^{M_n}$ the following relations hold,

$$\langle\psi_i^{M_n}|(\hat{M}_n^\delta - \bar{M}_n^\delta)^2|\psi_i^{M_n}\rangle^{1/2} < \delta\mathscr{M}_n^\delta \ll \bar{M}_n^\delta, \dots . \qquad (5)$$

The Hilbert space $\mathscr{H}$ of the whole system is constructed as the direct product of the corresponding Hilbert spaces,

$$\mathscr{H} = \mathscr{H}^{M_1} \otimes \mathscr{H}^{M_2} \otimes \dots \otimes \mathscr{H}^{M_n} \otimes \dots \otimes \mathscr{H}^{\text{mic}} , \qquad (6)$$

where

$$\mathscr{H}^{M_1} = \mathscr{H}_1^{M_1} \oplus \mathscr{H}_2^{M_1} \oplus \dots, \dots \qquad (7)$$

and $\mathscr{H}^{\text{mic}}$ is the Hilbert space of microscopic subsystems of the dynamical system.

Define the classical preferable basis as follows:

$$|\psi_{11\dots1\dots}^{\text{pref}}\rangle = |\psi_1^{M_1}\rangle\,|\psi_1^{M_2}\rangle \dots |\psi_1^{M_n}\rangle \dots |\psi^{\text{mic}}\rangle ,$$
$$|\psi_{21\dots1\dots}^{\text{pref}}\rangle = |\psi_2^{M_1}\rangle\,|\psi_1^{M_2}\rangle \dots |\psi_1^{M_n}\rangle \dots |\psi^{\text{mic}}\rangle ,$$
$$|\psi_{ij\dots k\dots}^{\text{pref}}\rangle = |\psi_i^{M_1}\rangle\,|\psi_j^{M_2}\rangle \dots |\psi_k^{M_n}\rangle \dots |\psi^{\text{mic}}\rangle , \qquad (8)$$

The accuracy of the definition of the vectors (8) is sufficient for one-to-one correspondence between theory and observations. Note that the corresponding basis $|\psi\rangle \in \mathscr{H}^{\text{mic}}$ is fixed automatically within the accuracy determined by the errors of the classical observables.

4. Theory of measurements

We shall demonstrate how the proposed way to define the basis works in the theory of measurements. Consider a quantum system S with a wave function $|\psi^S\rangle$ interacting with a device M which measures the quantity $\mathscr{A}\leftrightarrow\hat{A}$ of the system S. The device should be a classical object and its indications are characterized by a classical observable $\mathscr{M}$ (declination of the arrow) which is assigned a macroscopic operator $\hat{M}$. In the space of the wave functions of the device one can single out subspaces $\mathscr{H}^M_{A_n}$ such that for $|\psi^M_{A_n}\rangle \in \mathscr{H}^M_{A_n}$ we have

$$\langle\psi^M_{A_n}|(\hat{M}-\bar{M}[A_n])^2|\psi^M_{A_n}\rangle \ll \delta\mathscr{M}[A_n], \ldots, \tag{9}$$

where $\bar{M}[A_n] = \langle\psi^M_{A_n}|\hat{M}|\psi^M_{A_n}\rangle$, and $\delta\mathscr{M}[A_n]$ is the error in "arrow" fixation. If the initial wave vector of the system S is the eigenvector of the operator $\hat{A}$ which corresponds to the eigenvalue A_k, then the result of the interaction between the system S and the device M should have the form

$$|\psi^S_{A_k}\rangle|\psi^M_0\rangle \to |\psi^S_{A_k}\rangle|\psi^M_{A_k}\rangle, \tag{10}$$

where $|\psi^M_0\rangle$ is the vector corresponding to the initial state of the device. The condition (10) is *necessary* for realizing a good measurement of the quantity $\mathscr{A}$ by the device M [9].

If

$$|\psi^S\rangle = \sum_l a_l|\psi^S_{A_l}\rangle, \tag{11}$$

then (10) and the linearity of the Schrödinger equation imply

$$|\psi^S\rangle|\psi^M_0\rangle \to \sum_l a_l|\psi^S_{A_l}\rangle|\psi^M_{A_l}\rangle. \tag{12}$$

Different terms of the superposition (12) correspond to classically different universes ("indications of the arrow" of the device $\mathscr{M}\equiv\bar{M}[A_l]$ are distinct for them). The basis $|\psi^S_{A_l}\rangle|\psi^M_{A_l}\rangle$ is preferable because for its vectors the conditions (5) are satisfied.

For the microsystem S the components of the preferable basis are eigenvectors of the operator $\hat{A}$: $|\psi^S_{A_k}\rangle$. The choice of components of the preferable basis is unique. Indeed, suppose that there exists another preferable basis $|\tilde{\psi}^S_m\rangle$ for the system S. Then the wave function of the whole system after measurements can be represented in the form

$$|\psi\rangle = \sum_m \tilde{a}_m|\tilde{\psi}^S_m\rangle|\tilde{\psi}^M_m\rangle, \tag{13}$$

where $|\psi^M_m\rangle$ are the vectors of the Hilbert space of the device. Expand $|\psi^S_{A_l}\rangle$ in a power series of $|\psi^S_m\rangle$:

$$|\psi^S_{A_l}\rangle = \sum_m c_{ml}|\tilde{\psi}^S_m\rangle. \tag{14}$$

At least two of the coefficients c_{lm} essentially differ from zero because $|\psi^S_m\rangle \neq |\psi^S_{A_m}\rangle$ on no A_m. Substituting (14) into the right-hand side of (12) and comparing the expression obtained with (13), we find

$$|\tilde{\psi}^M_m\rangle = \sum_l a_l c_{lm}|\psi^M_{A_l}\rangle\left(\sum_k |a_k|^2|c_{km}|^2\right)^{-1/2}. \tag{15}$$

It is easy to verify that $|\tilde{\psi}^M_m\rangle \notin \mathscr{H}^M_{A_l}$ for no A_l because for (15) the conditions (5) are not satisfied. Consequently, the basis $|\psi_m\rangle$ is not preferable.

5. Concluding remarks

Above, setting the error $\delta\mathscr{M}^\alpha$ of the values of the classical observables $\mathscr{M}^\alpha$ we defined an ensemble of universes which can be considered as classically distinct non-interacting universes. The term "non-interacting" implies here that an interference influence of some universes upon others leads to corrections in $\mathscr{M}^\alpha$ smaller than $\delta\mathscr{M}^\alpha$ and cannor therefore be discovered. By decreasing $\delta\mathscr{M}^\alpha$ (i.e. increasing the accuracy of a classical device) we finally reach the values equal to the quantum mechanical dispersions $\delta\mathscr{M}^\alpha_q$ of the operators $\hat{M}^\alpha$ for the states $|\psi_i\rangle \in \mathscr{H}^M_i$. The quantities $\delta\mathscr{M}^\alpha_q$ cannot be smaller than certain minimal values determined by the parameters of the system. This is due to non-commutativity of canonically conjugate operators, that is, to the uncertainty principle. The accuracy of a classical device cannot exceed the quantum-mechanical uncertainty of $\delta\mathcal{M}_q^\alpha$. If we decide to impart a more precise meaning to the operators $\hat{M}^\alpha$, we will have to consider the "device" in a quantum-mechanical manner.

The idea of reality as a set of ensembles of "non-interacting" universes is also meaningful in the absence of an observer. In this case, when quantum fluctuations $\delta\mathcal{M}_q^\alpha$ are small as compared with the mean values of the operators which characterize the states of macroscopic subsystems ($\delta M_q^\alpha \ll \bar{M}^\alpha$, ...) the quasiclassical approximation is valid, and for $\delta\mathcal{M}^\alpha$ it is natural to choose δM_q^α, that is, $\delta\mathcal{M}^\alpha \sim \delta M_q^\alpha$. In this case different universes are weakly interacting. It is just in this case that the Hartle–Hawking wave function [10] can be interpreted as the one describing an ensemble of classically distinct universes [11].

If the dispersions of all the operators $\hat{M}^\alpha$ are comparable with the mean values (that is, $\delta M_q \sim M$) it is senseless to speak of *distinct* universes because corrections due to the interaction (the interference) for universes specified in an arbitrary way will exceed the accuracy of specification of these universes. It is more natural in this case to think of reality as of an ensemble of tightly bound universes without individual properties which would make them different from each other because the interference interaction is too strong.

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