Quantum Cosmology:
From Einstein to Everett, DeWitt, et al, and back

Thibault DAMOUR
Institut des Hautes Études Scientifiques

Einstein and Quantum Mechanics (1926)

The Einstein-Heisenberg discussion (1926)

Einstein: "From a heuristic point of view, it could be useful to remember what one really observes. But, at the level of principles, it is completely erroneous to want to found a theory uniquely on observable quantities. For, in reality, things happen in exactly the opposite way. It is only the theory which decides what can be observed."

Lesson learned from General Relativity

Confusion brought by the Neo-Newtonian interpretation of GR (Eisenstaedt)

Similarly: The Copenhagen interpretation is a kind of Neo-Classical interpretation of QM

Einstein and Quantum Mechanics (1954)

Einstein's last seminar (Princeton, 14 April 1954)

Einstein expressed his discomfort about QM:

"It is difficult to believe that [the] description [given by ψ] is complete. It seems to make the world quite nebulous unless somebody, like a mouse, is looking at it."

on QM and GR:

"There is much reason to be attracted to a theory with no space, no time. But nobody has an idea how to build it up."

Everett on Quantum Mechanics and Quantum Gravity

Everett (23) attended Einstein's last seminar and was struck by the sentence on the mouse and the universe. He also attended lectures by Bohr, in Princeton, in the fall of 1954. After having had his new idea on QM, he had problems publishing his ideas because of Wheeler/Bohr/Petersen/Rosenfeld/...

Everett 1957: "How is one to apply the conventional formulation of QM to the space-time geometry itself? The issue becomes especially acute in the case of a closed universe[3]. There is no place to stand outside the system to observe it."

"This paper proposes to regard pure wave mechanics (Process 2$[i\hbar\, \partial_t\, \psi = H\psi]$ only) as a complete theory. It postulates that a wave function that obeys a linear wave equation everywhere and at all times supplies a complete mathematical model for every isolated physical system without exception ... The wave function is taken as the basic physical entity with *no a priori interpretation*. Interpretation only comes after an investigation of the logical structure of the theory. Here as always the theory itself sets the framework for its interpretation.[5]"

Bryce DeWitt on Everett and Quantum Gravity

DeWitt April 1966 preprint: "... the formalism itself determines its own interpretation. This idea, which seems to crop up in general relativity more than in any other theory, has been stated in its most drastic form by Everett (1957) in connection with the measurement problem of ordinary nonrelativistic quantum mechanics. [...] The wave function ψ has physical reality quite independently of any "classical" observers, and it is *not* nonsense to ascribe a quantum state (and hence a wave function) to the universe as a whole. Because of its inner complexity the universe has a wave function which automatically splits into a stupendous number of branches, each one of which corresponds to a "classical" universe. The "classical" universes move independently in parallel without interfering with one another except insofar as the quantum Poincaré cycles allow anomalies to occur. The splitting processes go on continuously, each providing a mathematical model, on a local scale, of the statistical nature of quantum mechanics.

A split occurs whenever a strictly quantum process is magnified (whether by observation or otherwise) to a “classical” level. The statistical interpretation of the wave function thus emerges from the Schrödinger equation itself without having to be imposed from the outside. Within each branch various automata (e.g., human beings) can communicate the contents of their memory banks to one another in a mutually consistent “classical” fashion, and all will agree on the fundamentally statistical nature of physical phenomena at the *micro* level. Subject to the Poincaré-cycle limitation there is no communication between branches. (This is virtually a matter of definition, since the branches would no longer be distinct if communication were possible.) Nevertheless it is the total ensemble of all branches which constitutes physical reality and not merely one alone[1].

[1] Needless to say, to one who finds the size of the “actual” universe frightening, Everett’s conception must appear vastly more unsettling.

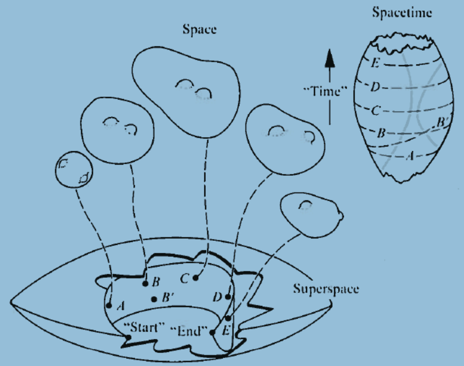


Figure 43.2.

Space, spacetime, and superspace. Upper left: Five sample configurations, A, B, C, D, E, attained by space in the course of its expansion and recontraction. Below: Superspace and these five sample configurations, each represented by a point in superspace. Upper right: Spacetime. A spacelike cut, like A, through spacetime gives a momentary configuration of space. There is no compulsion to limit attention to a one-parameter family of slices, A, B, C, D, E through spacetime. The phrase "many-fingered time" is a slogan telling one not to so limit one's slices, and B' is an example of this freedom in action. The 3-geometries B' and A, B, C, D, E, like all 3-geometries obtained by all spacelike slices whatsoever through the given classical spacetime, lie on a single bent leaf of history, indicated in the diagram, and cutting its thin slice through superspace. A different spacetime, in other words, a different solution of Einstein's field equation, means a different leaf of history (not indicated) slicing through superspace.

Figure 1. (from MTW)

Wave function in Wheeler's metric representation

Wave functional *field* representation: $\psi[\varphi(\mathbf{x})]$ for scalar field

e.g. wave functional for electromagnetic field in ground state $|0\rangle$ of field oscillators

$$\psi_0\left[\mathbf{B}(\mathbf{x})\right] = N \exp\left(-\int\int \frac{\mathbf{B}(\mathbf{x}_1)\cdot\mathbf{B}(\mathbf{x}_2)}{16\pi^3\hbar c\,|\mathbf{x}_1-\mathbf{x}_2|^2} d^3\mathbf{x}_1 d^3\mathbf{x}_2\right)$$

wave functional for a spatial metric:

$$\psi\left[g_{ij}(\mathbf{x})\right]$$

diffeomorphism invariance:

$$\nabla_j\left(\frac{\delta\psi}{\delta g_{ij}(\mathbf{x})}\right) = 0$$

The Wheeler-DeWitt equation

Einstein-Schrödinger equation for $\psi[g_{ij}(\mathbf{x})]$ (pure gravity)

$$\widehat{\mathcal{H}}_g \, \psi[g_{ij}] = 0$$

Hamiltonian constraint operator $(16\pi G = 1)$: $\widehat{p} = \frac{\hbar}{i} \frac{\partial}{\partial q} \to \widehat{\pi}^{ij} = \frac{\hbar}{i} \frac{\delta}{\delta g_{ij}(\mathbf{x})}$

$$\begin{aligned}
\widehat{\mathcal{H}}_g &= \frac{1}{2\sqrt{g}} \, G_{ijkl}(\mathbf{x}) \, \widehat{\pi}^{ij} \, \widehat{\pi}^{kl} - \sqrt{g} \, R^{(3)} \\
&= -\frac{\hbar^2}{2\sqrt{g}} \, (g_{ik} \, g_{jl} + g_{il} \, g_{jk} - g_{ij} \, g_{kl}) \, \frac{\delta}{\delta g_{ij}(\mathbf{x})} \, \frac{\delta}{\delta g_{kl}(\mathbf{x})} - \sqrt{g} \, R[g_{ij}(\mathbf{x})]
\end{aligned}$$

When adding matter

$$(\widehat{\mathcal{H}}_g + \widehat{\mathcal{H}}_m) \, \psi(g_{ij}, \varphi, \psi) = 0$$

Wave functional in superspace

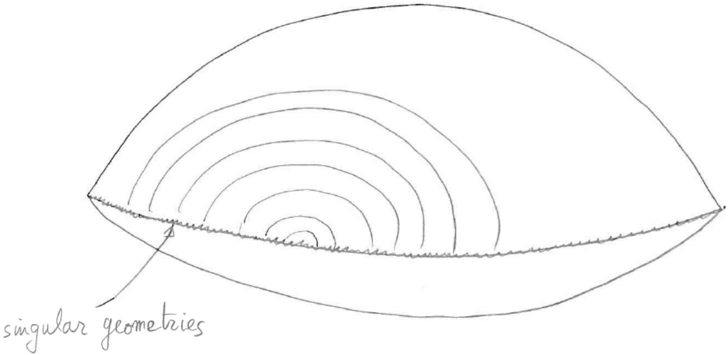


Figure 2.

Issues in Quantum Cosmology

- Is there a natural boundary condition on $\psi[g_{ij}]$ at the "boundary" of superspace made of singular geometries?
  E.g. $\psi[g_{ij}] \to 0$ on the singular boundary (DeWitt 1967)
- Is there a principle selecting a unique solution of the WDW equation?
  E.g. Hartle-Hawking no-boundary proposal
- Do quantum effects lead to singularity avoidance in gravitational collapse (either at a big bang, or at a big crunch, e.g. within a black hole)?
- "Problem of time" in $\mathcal{H}\psi = 0$?
- Link between $\psi[g_{ij}, \varphi, \psi]$ and probabilities?
- Is there a quantum to classical transition, or is the present cosmological universe the ultimate Schrödinger's cat (in need of Einstein's mouse look to exit its quantum fuzziness)?

Quantum origin of the large scale structure of our universe

Linear quantum perturbations of a classical inflationary cosmology (Mukhanov-Chibisov 1981)

$$\widehat{g}_{\mu\nu}(t,\mathbf{x}) = g^{\text{class}}_{\mu\nu}(t,\mathbf{x}) + \widehat{h}_{\mu\nu}(t,\mathbf{x})$$

$$\widehat{\varphi}(t,\mathbf{x}) = \varphi^{\text{class}}(t,\mathbf{x}) + \delta\widehat{\varphi}(t,\mathbf{x})$$

$$v := \sqrt{\varepsilon_{,x}}\, a \left(\overline{\delta\varphi} + \frac{\varphi_0'}{\mathcal{H}} \Psi \right)$$

$$\widehat{v}(\eta,\mathbf{x}) = \frac{1}{\sqrt{2}} \int \frac{d^3k}{(2\pi)^{3/2}} \left[v^*_{\mathbf{k}}(\eta) e^{i\mathbf{k}\cdot\mathbf{x}} \widehat{a}_{\mathbf{k}} + v_{\mathbf{k}}(\eta) e^{-i\mathbf{k}\cdot\mathbf{x}} \widehat{a}^\dagger_{\mathbf{k}} \right]$$

$$\left[a_{\mathbf{k}}, a^\dagger_{\mathbf{k}'} \right] = \delta(\mathbf{k} - \mathbf{k}')$$

$\Rightarrow$ quantum seeds of inhomogeneities in our universe

Quantum Mini-superspace Cosmology Models

(DeWitt 1967, Misner 1969, Vilenkin, Hawking, Linde, ...)

Gravity $+$ massive scalar field, reduced to an homogeneous, closed Friedmann cosmology: $ds^2 = -N^2 dt^2 + a^2(t)\, d\Omega_3^2$; $\varphi = \varphi(t)$

WDW eq: $\left[\frac{1}{a^p}\frac{\partial}{\partial a}\, a^p \frac{\partial}{\partial a} - a^2 + \lambda a^4 - \frac{1}{a^2}\frac{\partial^2}{\partial \varphi^2} + a^4 m^2 \varphi^2\right] \psi(a, \varphi) = 0$

with $p = 1$, $a \equiv e^\alpha$: $\left[\frac{\partial^2}{\partial \alpha^2} - \frac{\partial^2}{\partial \varphi^2} - e^{4\alpha} + \lambda e^{6\alpha} + e^{6\alpha} m^2 \varphi^2\right] \psi(\alpha, \varphi) = 0$

Klein-Gordon equation, with potential $V(\alpha, \varphi)$, in (α, φ) Minkowski space

$\exists$ solutions describing quantum versions of inflationary models

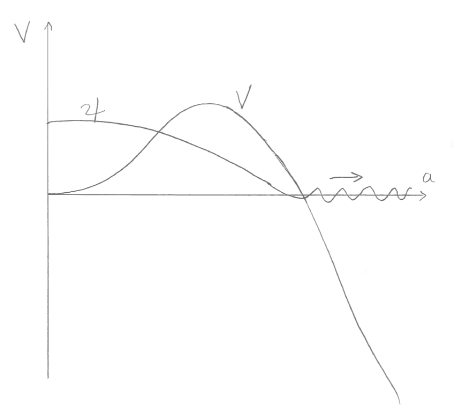
Quantum Creation of the Universe from Nothing?

(Tryon 1973, Fomin 75, Brout-Englert-Gunzig 78, Vilenkin 82)

WDW eq. with λ: zero-energy Schrödinger eq: $\left[-\frac{\partial^2}{\partial a^2} + V(a)\right]\psi(a) = 0$

$$V(a) = a^2 - \lambda a^4$$

possibility of tunelling from “nothing”, i.e. $a = 0$



Extended Superspace Models

(Halliwell-Hawking 1985) Perturbed Friedmann Models

$$g_{ij} = a^2 \left[\Omega_{ij}^{S_3} + \epsilon_{ij}(a_n, b_n, c_n, d_n, f_n) \right]$$

with an infinite number of parameters, representing all the possible perturbations of the geometry of a round sphere

$$\psi = \psi_0(\alpha, \varphi) \prod_n \psi_n(\alpha, \varphi, a_n, b_n, c_n, d_n, f_n)$$

WDW: infinite-dimensional second-order differential equation in $\alpha \equiv \ln a, \varphi, a_n, b_n, \ldots$

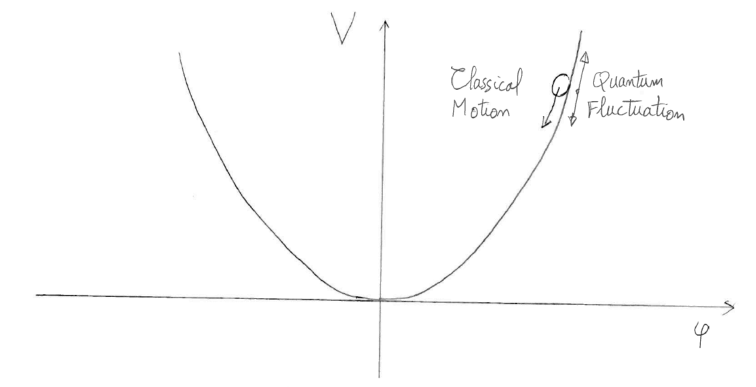
$$\Big[\frac{\partial^2}{\partial \alpha^2} - \frac{\partial^2}{\partial \varphi^2} + e^{6\alpha} m^2 \varphi^2 - e^{4\alpha} - \frac{n^2-1}{n^2-4} \frac{\partial^2}{\partial b_n^2} - \ldots$$

$$- e^{4\alpha} \left(\frac{(n^2-7)(n^2-4)}{3(n^2-1)} b_n^2 + \ldots \right) \Big] \psi = 0$$

One approximately recovers the results of quantum cosmological perturbations.

Eternal (Chaotic) Inflation (Steinhardt, Vilenkin, Guth, Linde, ...)

When $\Delta\varphi_{\text{quantum}} \simeq \frac{H}{2\pi} > 0.61\Delta\varphi_{\text{classical}} = 0.61|\dot{\varphi}_{\text{classical}}|H^{-1}$, the volume of space where $\varphi(t)$ increases (rather than decreases) grows.



Quantum Supersymmetric Anisotropic Cosmology

Quantum dynamics of a supersymmetric, triaxially squashed ($SO(3)$-homogeneous) three-sphere (Damour-Spindel 2013, 2014).

Both bosonic ($a = e^{-\beta^1}, b = e^{-\beta^2}, c = e^{-\beta^3}$) and fermionic (homogeneous gravitino field) degrees of freedom in a mini-superspace.

Supergravity $\rightarrow$ the wave function of the universe is a 64-component spinor of $\mathrm{Spin}(8,4)$ that depends on the three logarithmic squashing parameters $\beta^1, \beta^2, \beta^3$. $\psi_\sigma(\beta^a)$, $(\sigma = 1, \ldots, 64)$, instead of satisfying a simple (scalar) Klein-Gordon-like second-order WDW equation, satisfies four different (spinorial) Dirac-like first-order equations in β-space

$$\left[\Phi_A^a \frac{\hbar}{i} \frac{\partial}{\partial \beta^a} + V_A(\Phi, \beta)\right] \psi(\beta) = 0$$

where Φ_A^a are 64×64 gamma matrices of $\mathrm{Spin}(8,4)$: $\Phi_A^a \Phi_B^b + \Phi_B^b \Phi_A^a = G^{ab} \delta_{AB}$

Solutions of Quantum Susy Cosmology

• These solutions (hopefully) describe the quantum dynamics of the universe near a cosmological singularity (big bang or big crunch)

• $\exists$ a set of discrete quantum states, and notably a "ground state"-like wave function

$$\psi_0(\beta) = abc \left[(b^2 - a^2)(c^2 - b^2)(a^2 - c^2)\right]^{3/8} e^{-\frac{1}{2}(a^2+b^2+c^2)} |0\rangle_-$$

which describes a quantum universe which oscillates in shape and size, but stays of Planckian size $\sim L_P = \left(\frac{\hbar G}{c^3}\right)^{\frac{1}{2}} \simeq 1.616 \times 10^{-33}\,\mathrm{cm}$

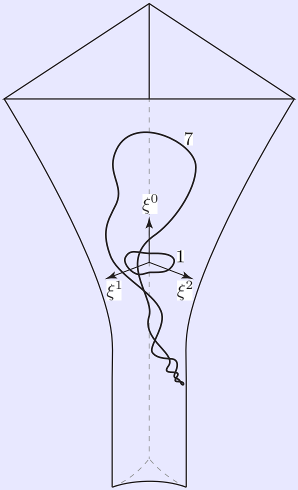
• In addition there are "continuous-spectrum"-like solutions whose dynamics has several remarkable features:

(i) it exhibits hidden-hyperbolic Kac-Moody structures

(ii) the quantum effects quartic in fermions dominate the dynamics near a small-volume singularity and can generically lead to a quantum avoidance of a singularity, i.e. a bounce of the universe.

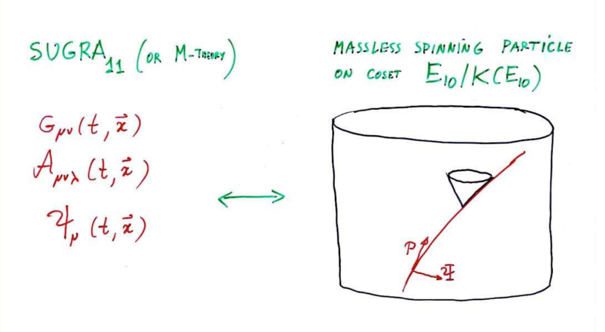
Quantum Bottle Effect?

A possible quantum storage ring of near-singularity states, ready for tunnelling, via inflation, toward large universes



Einstein's vision: "There is much reason to be attracted to a theory with no space, no time."

The appearance of hidden hyperbolic Kac-Moody structures (AE_n, E_{10}) in the "near cosmological singularity limit" of supergravity (and the low-energy limit of various string theories) suggests the existence of a gravity/Kac-Moody coset correspondence (Damour, Henneaux, Nicolai 2002). [related suggestions: E_{10}, Ganor'99; E_{11}, West'01]



Quantum Gravity $\leftrightarrow$ Quantum Coset Model

At the quantum level, it suggests that, near a singularity, "space" $(g_{ij}(t,\mathbf{x}), \mathcal{A}_{ijk}(t,\mathbf{x}), \ldots)$ is replaced by a Lie-algebraic variable $g(t) \in E_{10}(\mathbb{Z})\backslash E_{10}(\mathbb{R})/K_{10}(\mathbb{R})$ satisfying (as a consequence of Dirac-like equations) an infinite-dimensional Klein-Gordon-type equation on the (discretely quotiented, à la Hull-Townsend'95) coset space $E_{10}(\mathbb{Z})\backslash E_{10}(\mathbb{R})/K_{10}(\mathbb{R})$

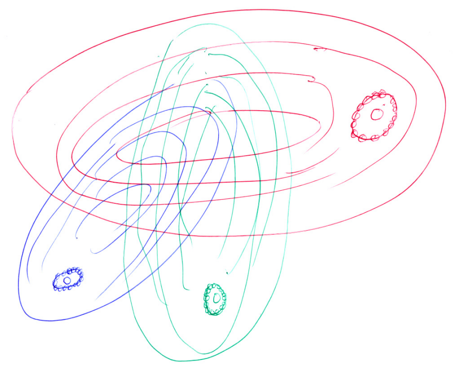
$$\Box_{E_{10}/K_{10}} \Psi(\beta^a, \nu^\alpha) = 0$$

$$\left[-G^{ab} \partial_{\beta^a} \partial_{\beta^b} - \sum_{\alpha > 0} e^{-2\alpha(\beta)} \partial^2_{\nu^\alpha} + \ldots \right] \Psi(\beta^a, \nu^\alpha) = 0$$

Remarks on Quantum Cosmology (QC)

• $\exists$ interesting technical (and conceptual) challenges that await a good resolution: "back-reaction" of quantum fluctuations" in QC [$G_{\mu\nu}(g_{\alpha\beta}^{\text{bckgrd}}) = \langle T_{\mu\nu} \rangle$ is not the answer]

• Even if we forget about the most speculative parts of QC (near singularities, quantum multiverse from eternal inflation), the theory of linear quantum cosmological perturbations shows that our local universe (Earth, Moon, Sun, Galaxy, ... is a (a priori smeared) quantum superposition



Everett's view (1)

31 May 1957 letter to DeWitt

"One of the basic criticisms leveled against the Copernican theory was that "the mobility of the earth as a real physical fact is incompatible with the common sense interpretation of nature". [...] Thus, in order to decide whether or not a theory contradicts our experience, it is necessary to see what the theory itself predicts our experience will be. Now in your letter you say, "the trajectory of the memory configuration of a real physical observer, on the other hand, does *not* branch. I can testify to this from personal introspection, as can you. I simply do *not* branch." I can't resist asking: Do you feel the motion of the earth? [...] The theory is in full accord with our experience (at least insofar as ordinary quantum mechanics is). It is in full accord just because it it *is* possible to show that no observer would ever be aware of any "branching", which is alien to our experience as you point out.

The whole issue of the "transition from the possible to the actual" is taken care of in a very simple way – there is no such transition, *nor is such a transition necessary for the theory to be in accord with our experience*. From the viewpoint of the theory, all elements of a superposition (all "branches") are "actual", none any more "real" than another. It is completely unnecessary to suppose that after an observation somehow one element of the final superposition is selected to be awarded with a mysterious quality called "reality" and the others condemned to oblivion.

Everett's view (2)

• I personally find Everett's attitude as the most rational (and economical) way of thinking about the quantum world. It is consistent both with Kant's metaphysics ("Hitherto it has been assumed that all our knowledge must conform to objects. [...] Let us try to see whether we may not have more success in the tasks of metaphysics, by assuming that objects must conform to our knowledge." Le Kantique du Quantique (TD)), and with Einstein's GR-inspired epistemology ("It is only the theory which decides what can be observed").

• Assuming psycho-physical parallelism, Everett's view is no more paradoxical than other well-known philosophical paradoxes such as the "many times" and the "many minds" problems.

Conclusion: Bryce DeWitt's last words on Everett (2002)

"Everett's interpretation has been adopted by the author out of practical necessity: He knows of no other. At least he knows of no other that imposes no artificial limitations or fuzzy metaphysics while remaining able to serve the varied needs of quantum cosmology, mesoscopic quantum physics, and the looming discipline of quantum computation."