Inhomogeneity of rotating gluon plasma and Tolman-Ehrenfest law in imaginary time

Maxim Chernodub

Institut Denis Poisson, CNRS, Tours, France

1. **Motivation: vorticity in quark-gluon plasma**

   Finite-temperature phase diagram of QCD; Non-central collisions and vorticity

2. **Overview: interacting quarks in rotation and chiral phase transition**

   Nambu—Jona-Lasinio model

3. **Rotation and confinement of color (a puzzle)**

   Lattice; holography; hadron resonance model; compact electrodynamics;
   (Minkowski and Euclidean) Tolman-Ehrenfest law and inhomogeneity of plasmas;
   ~~no-go theorem for analytical transformation Euclidean → Minkowski; fractals.~~

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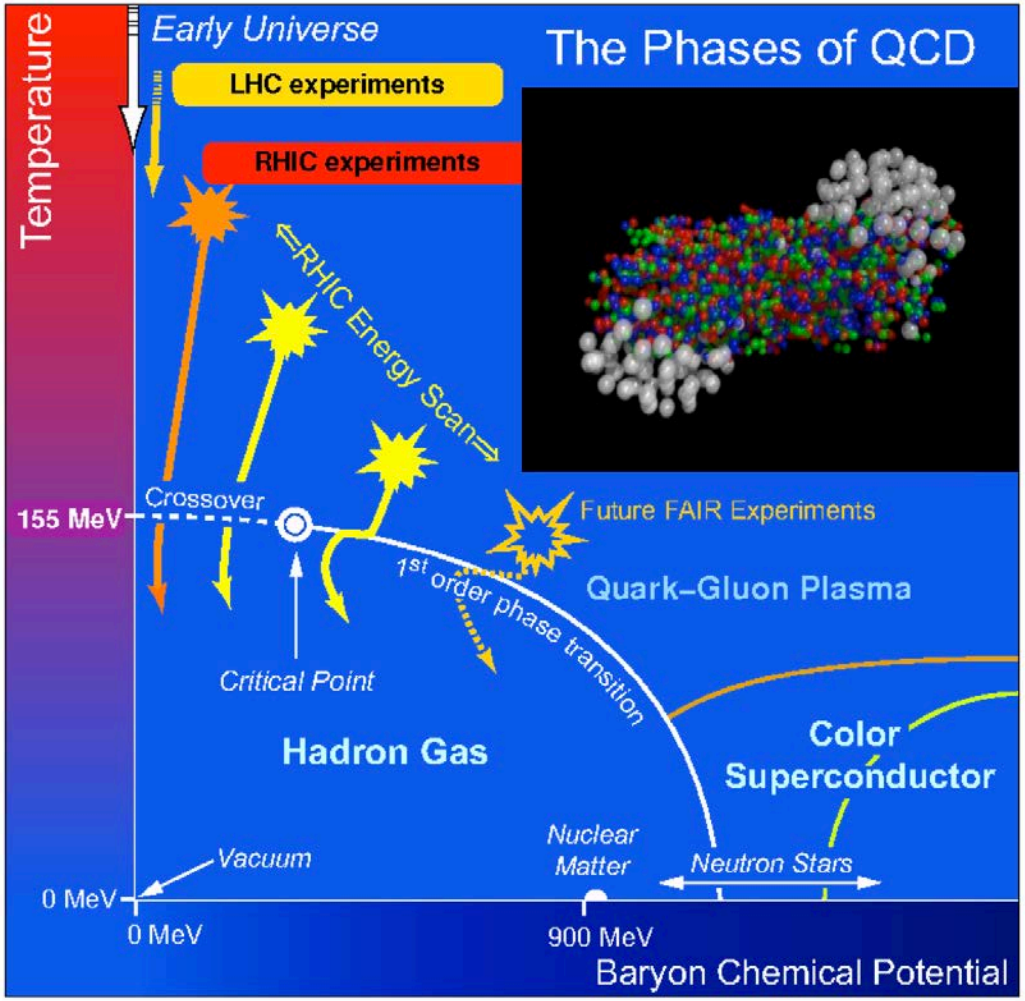
Phase diagram of QCD

1) Hot quark-gluon plasma phase and cold hadron phase constitute, basically, one single phase because they are separated by a nonsingular transition ("crossover").

2) The color superconducting phases at high baryonic chemical potential μ were extensively studied theoretically [they are out of reach of both lattice simulations and Earth-based experiments]

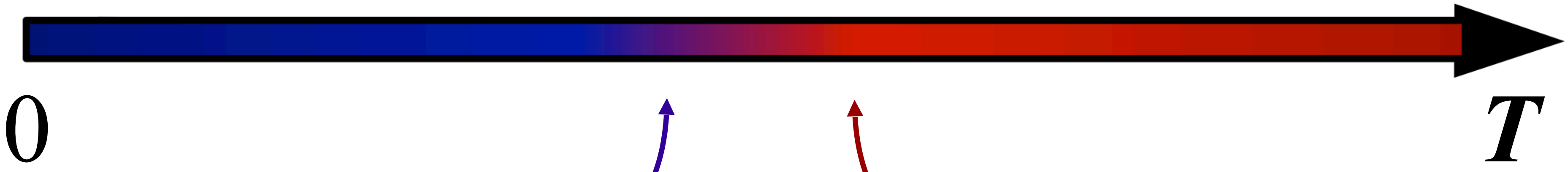


From a BNL webpage

3) The LHC and RHIC experiments probe low baryon density physics. One can safely take $\mu = 0$ in further discussions.

Phase diagram; $\mu = 0$

wide and smooth crossover

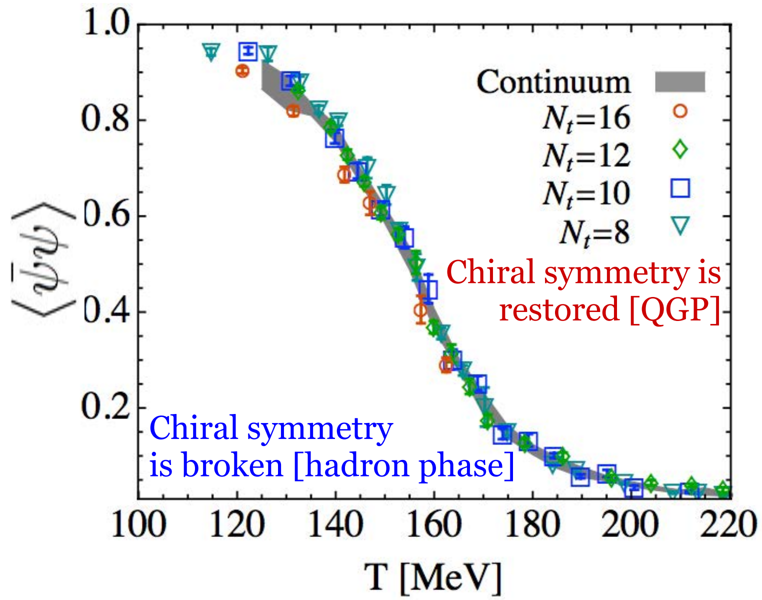


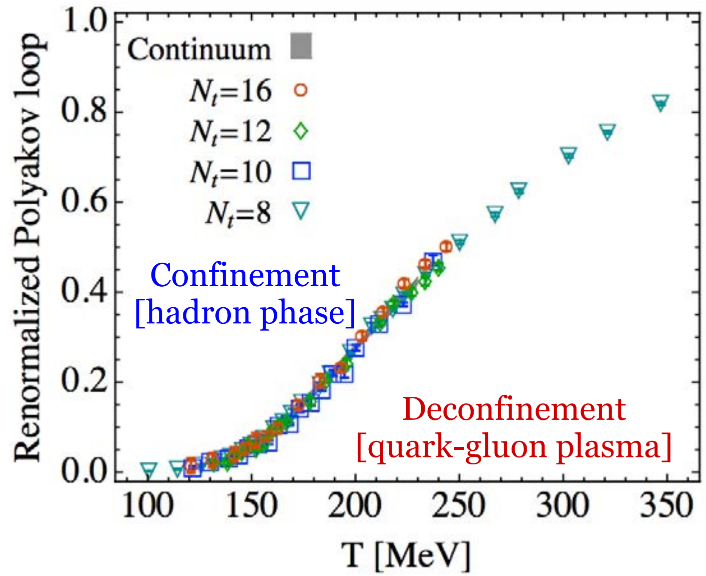
Restoration of the chiral symmetry

Deconfinement (via Polyakov loop)

$T_{\text{chiral}} \approx 155\,\text{MeV}$

$T_{\text{deconf}} \approx 170\,\text{MeV}$





Borsanyi et al, JHEP 1009 (2010) 073, ArXiv:1005.3508]

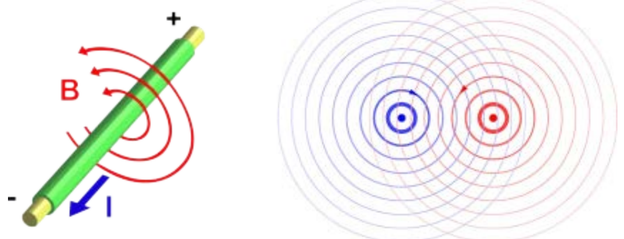
What happens with this picture in a rotating plasma?

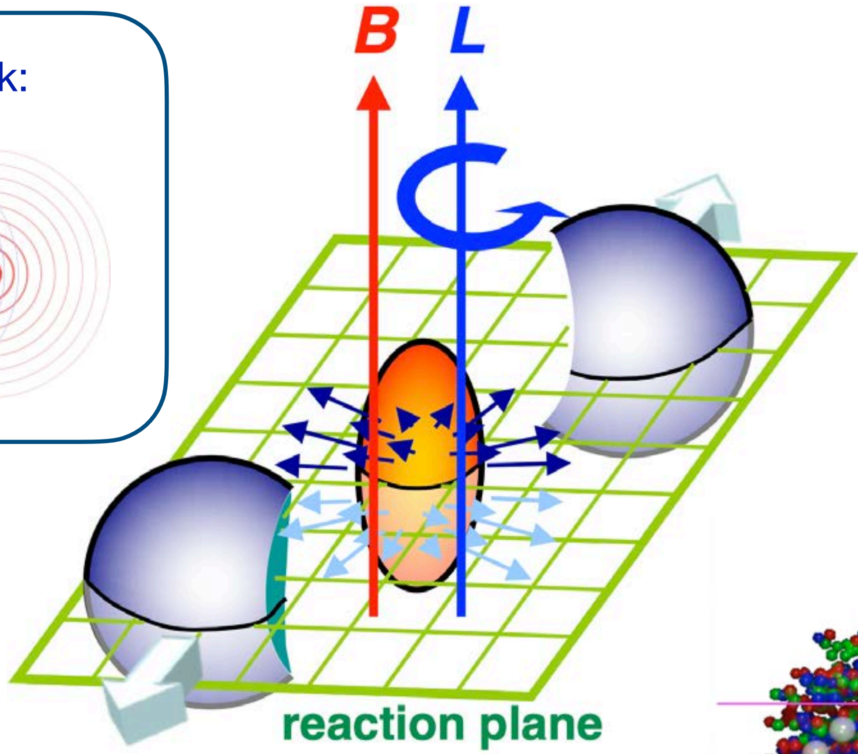
Why we need to study the rotating plasma?

Noncentral collisions

generate magnetic field and angular momentum

Electromagnetism at work:





Classical mechanics at work

$$\mathbf{L} = \mathbf{r} \times \mathbf{p}$$
$$\sim 10^6 \hbar$$

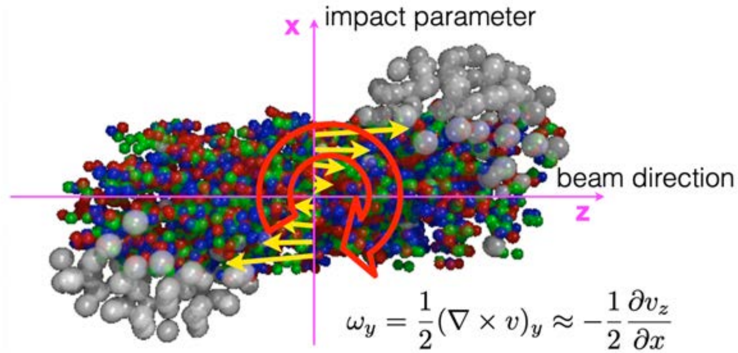
Large orbital angular momentum

Strong magnetic field

$$B \sim 10^{13}\ \mathrm{T}$$

$$eB \sim m_\pi^2 \quad (\tau \sim 0.2\ \mathrm{fm})$$

et early times of the collision



strong vorticity

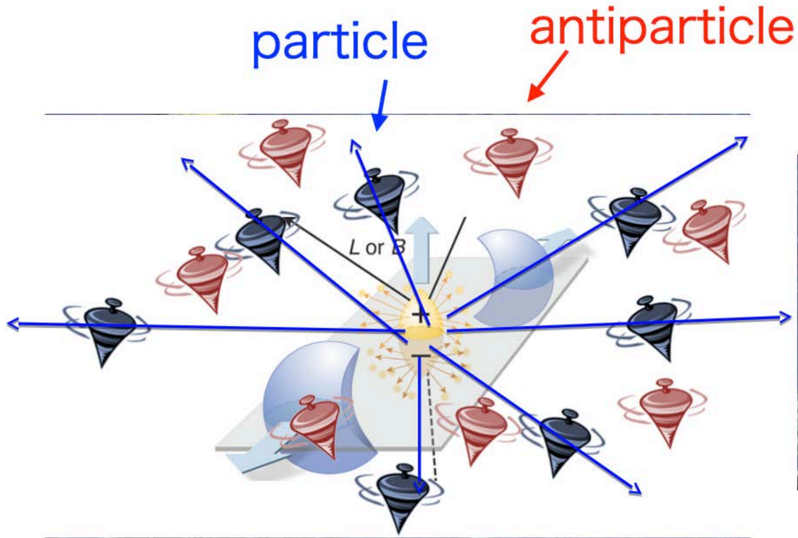
the effects of magnetic fields may be small (under discussion)

D. Kharzeev, L. McLerran, and H. Warringa, Nucl.Phys.A803, 227 (2008);
McLerran and Skokov, Nucl. Phys. A929, 184 (2014)

Z.-T. Liang and X.-N. Wang, PRL94, 102301 (2005);
S. Voloshin, nucl-th/0410089 (2004)

How to measure the vorticity?

the vorticity could be probed via quark's spin polarization



The mechanism:

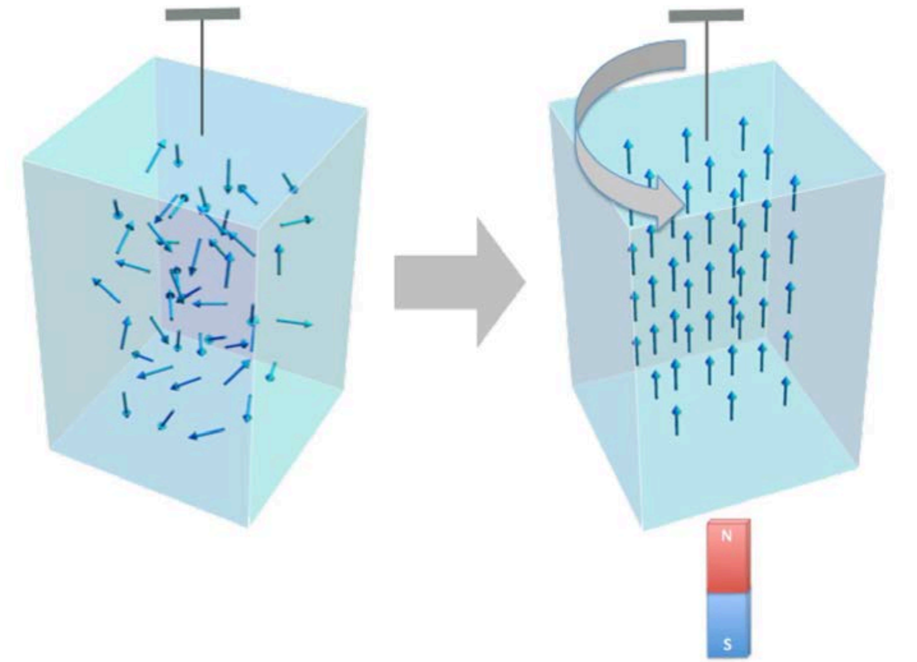
1) **orbital angular momentum of the rotating quark-gluon plasma is transferred to the particle spin**

The mechanism is similar to the Barnett effect (found in 1915)

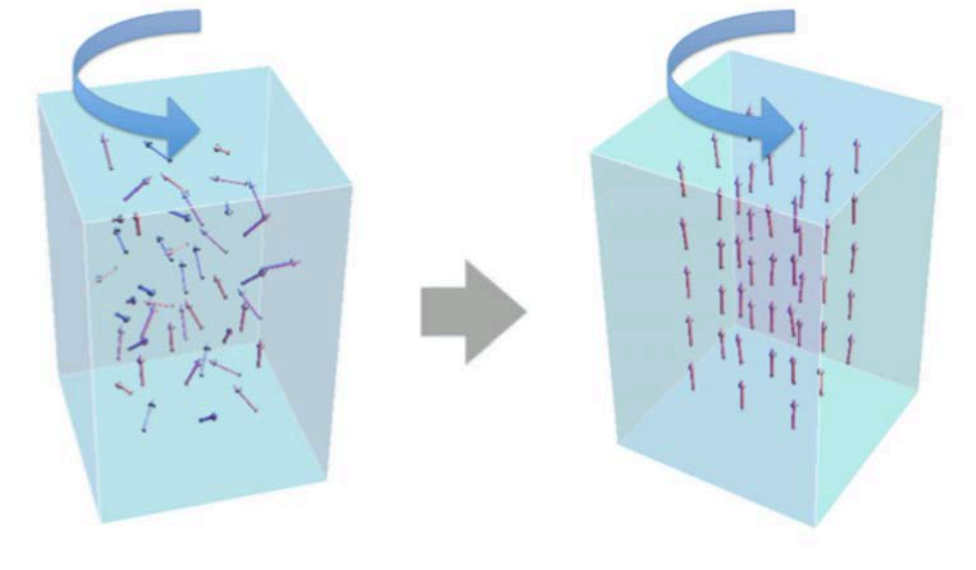
Spin, magnetic field and rotation

The Barnett effect

Coupling between mechanical rotation and spin orientation



Einstein-de Haas effect. Modulation of magnetization of iron by applying the external magnetic filed, magnetic angular momentum is changed. As a result, the mechanical angular momentum is induced for compensating the modulation of the annular momentum.



Barnett effect. Magnetization is induced by applying mechanical rotation since an effective magnetic field, emerges in a rotating body.

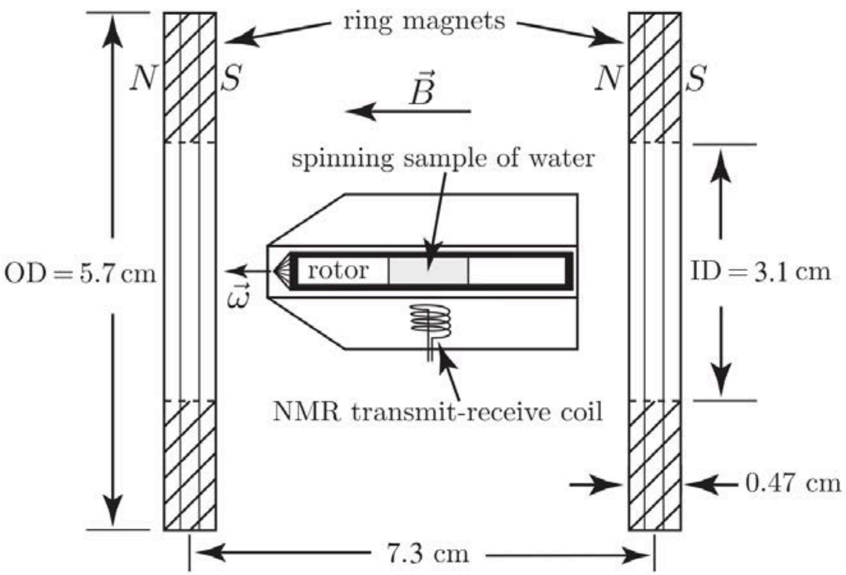
Magnetization due to rotation: $M = \chi \Omega / \gamma$

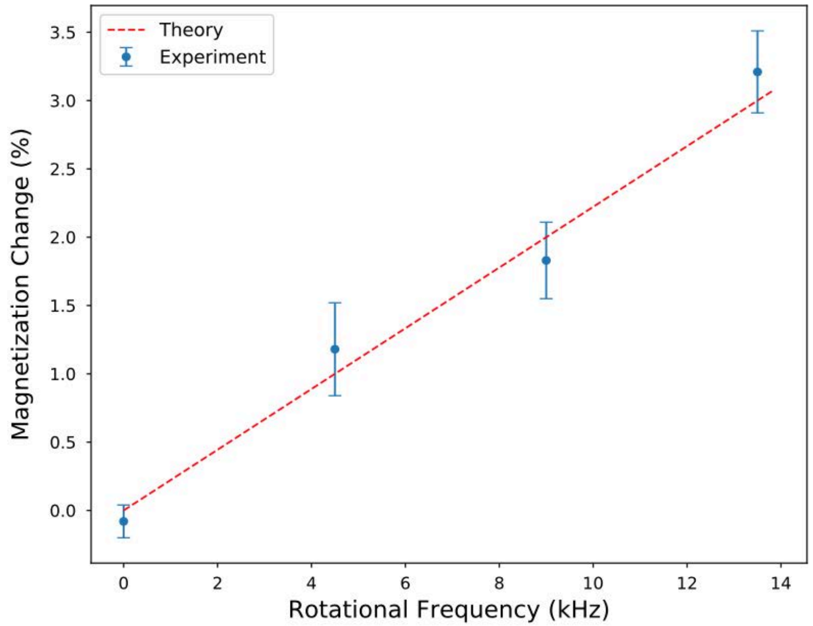
Effective magnetic field: $B_\Omega = \Omega / \gamma$

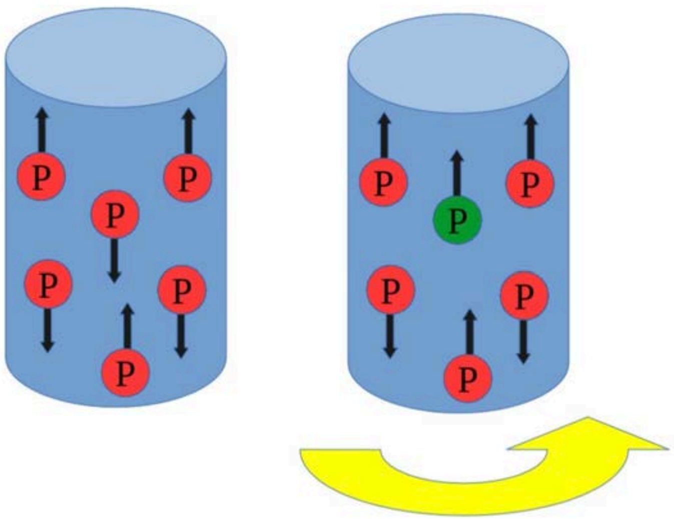
χ is the magnetization susceptibility of the medium

Figures from Matsuo, Ieda, Maekawa, Frontiers in Physics 3, 54 (2015)

Nuclear Barnett Effect found in water





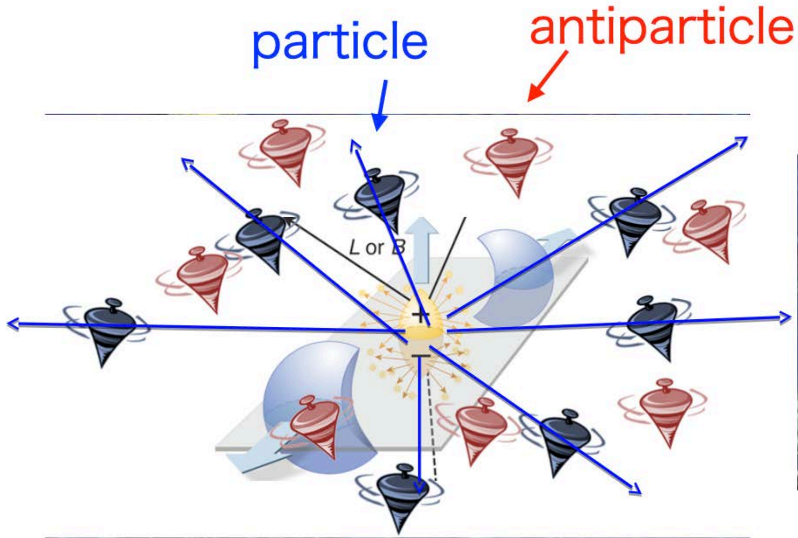


Measured the nuclear Barnett effect by rotating a sample of water at rotational speeds up to 13.5 kHz in a weak magnetic field and observed a change in the polarization of the protons in the sample that is proportional to the frequency of rotation.

Arabgol and Sleator, PRL 122, 177202 (2019)

How to measure the vorticity?

the vorticity could be probed via quark's spin polarization



The mechanism:

1) orbital angular momentum
   of the rotating quark-gluon plasma
   is transferred to the particle spin

The mechanism is similar to
the Barnett effect (found in 1915)

2) both particles and anti-particles
   are polarized in the same way
   (spin polarization is not sensitive to the particle charge)

3) The vorticity may be measured via the polarization of the produced particles

Which particles? Hyperons! (and other particles with a nonzero spin like vector mesons)

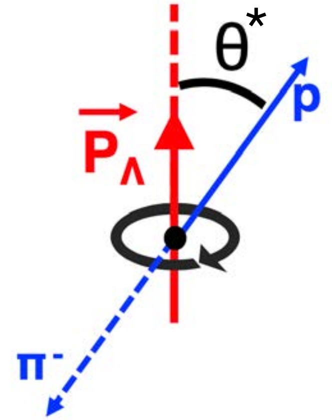
“Self-analysis” of hyperons

Daughter baryon is predominantly emitted in the direction of hyperon’s spin (opposite for anti-particle)

$$\Lambda \rightarrow p + \pi^-$$

(BR: 63.9%, cτ~7.9 cm)

$$\frac{dN}{d\cos\theta^*} \propto 1 + \alpha_H \mathrm{P_H} \cos\theta^*$$



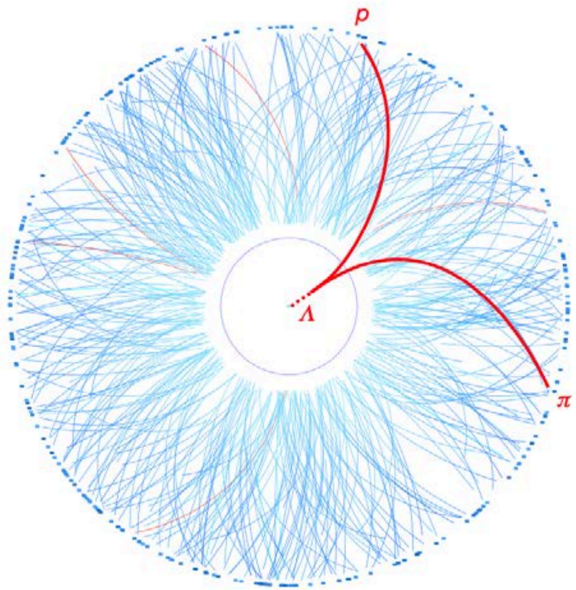
P_H: hyperon polarization

θ^*: polar angle of daughter relative to the polarization directior in hyperon rest frame

α_H: hyperon decay parameter

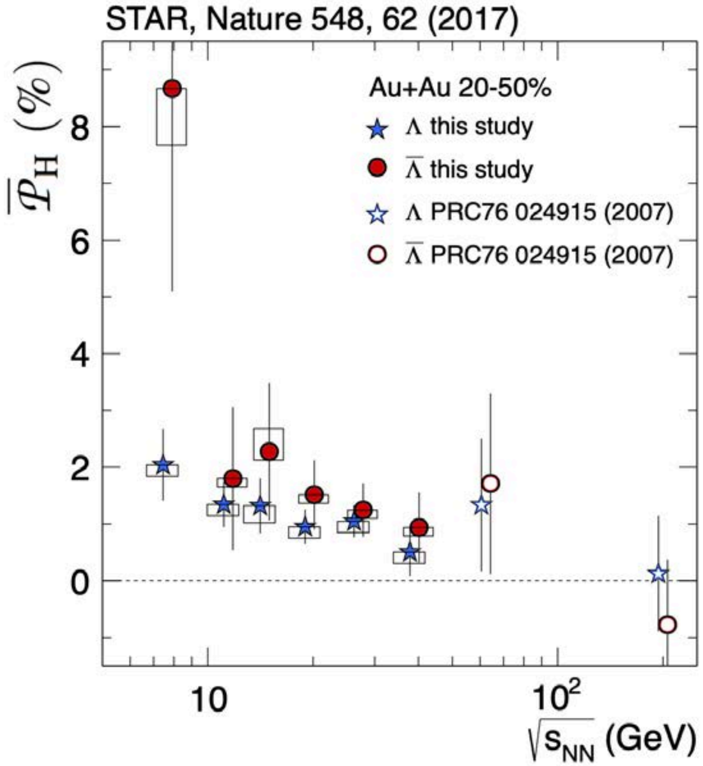
Note: α_H for Λ recently updated (BESIII and CLAS)
α_Λ=0.732±0.014, $\alpha_{\bar{\Lambda}}$=-0.758±0.012

P.A. Zyla et al. (PDG), Prog.Theor.Exp.Phys.2020.083C01

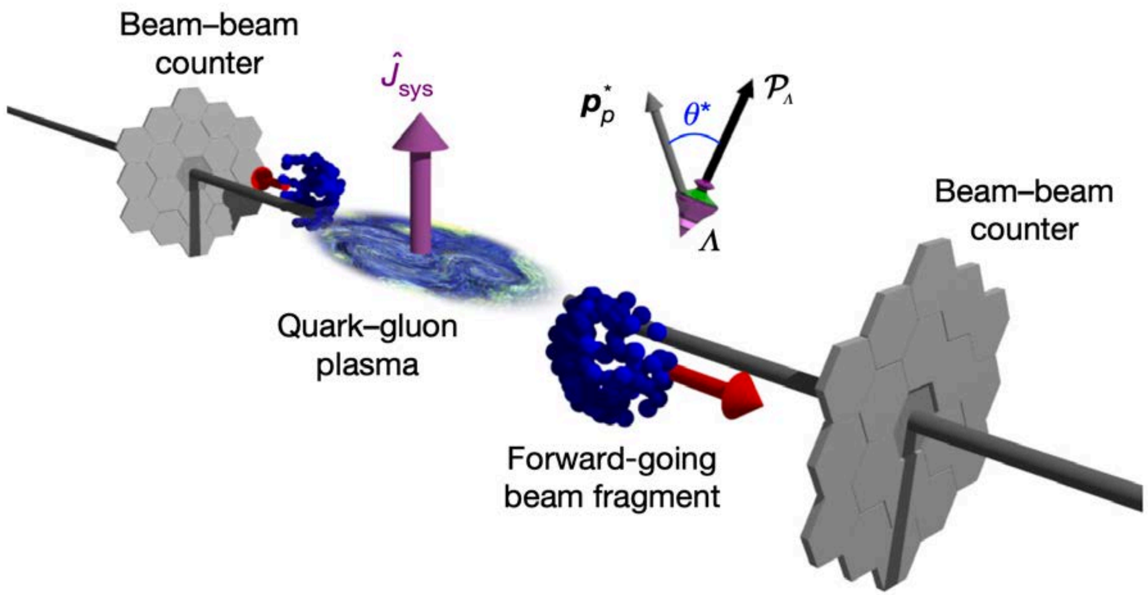


adapted from the talk of T. Niida, ECT* Spin/hydro in heavy-ion collisions 2020

How to measure the polarization?



The observed asymmetry in the hyperon spin polarization ignited much interest.



Overview of the experimental situation: T. Niida, talk at the workshop "Spin and hydrodynamics in relativistic nuclear collisions" ECT*, Trento, Italy, Oct. 05-16, 2020.

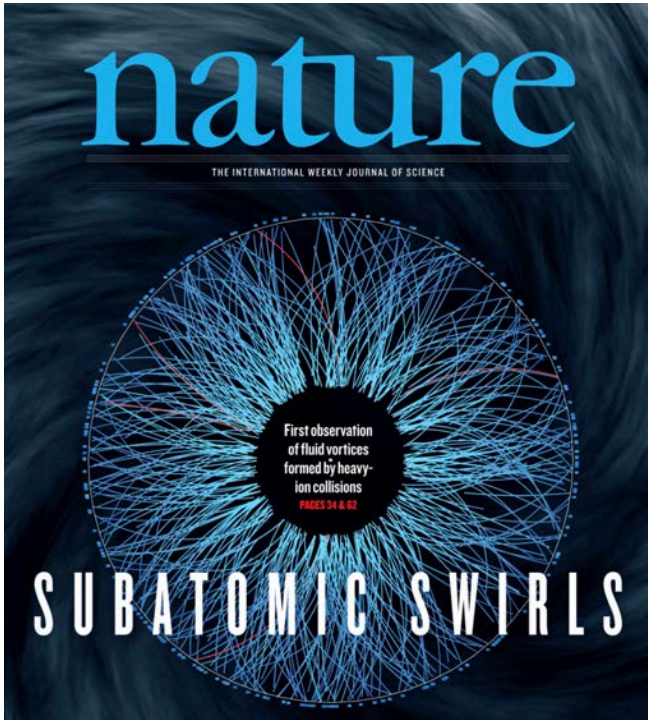
Overview of the theoretical situation: "Vorticity and Spin Polarization in Heavy Ion Collisions: Transport Models", X.-G. Huang, J. Liao, Q. Wang, X.-L. Xia, arXiv:2010.08937

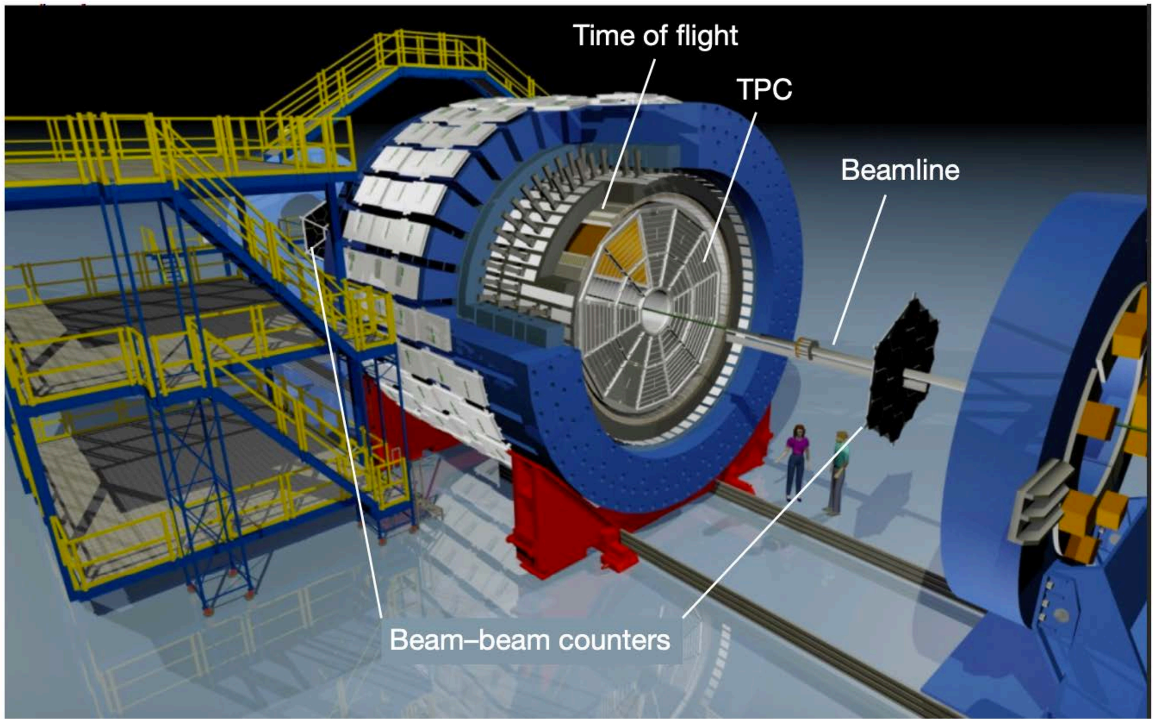
Baznat, Gudima, Sorin, Teryaev, Phys. Rev. C 88, 061901(R) (2013); Sorin and Teryaev, Phys. Rev. C 95, no.1, 011902(R) (2017).
Becattini, Karpenko, Lisa, Upsal, Voloshin, Phys.Rev.C 95 (2017) 5, 054902; Teryaev, Zakharov, Phys.Rev.D 96 (2017) 9, 096023.
Baznat, Gudima, Sorin and Teryaev, Phys. Rev. C 97, no.4, 041902(R) (2018); Csernai, Kapusta, and Welle, Phys. Rev. C 99, no.2, 021901(R) (2019);
D-Xian Wei, Wei-Tian Deng, and Xu-Guang Huang, Phys. Rev. C 99, 014905 (2019); Vitiuk, Bravina and Zabrodin, Phys. Lett. B 803, 135298 (2020)
B. Fu, K. Xu, X.-G. Huang, H.Song, ArXiv:2011.03740; V. E. Ambrus, M.N. Chernodub ArXiv:2010.05831, and others

The most vortical fluid ever observed

The experimental result for the vorticity:

$$\omega \approx (9 \pm 1) \times 10^{21}\,\mathrm{s}^{-1}$$





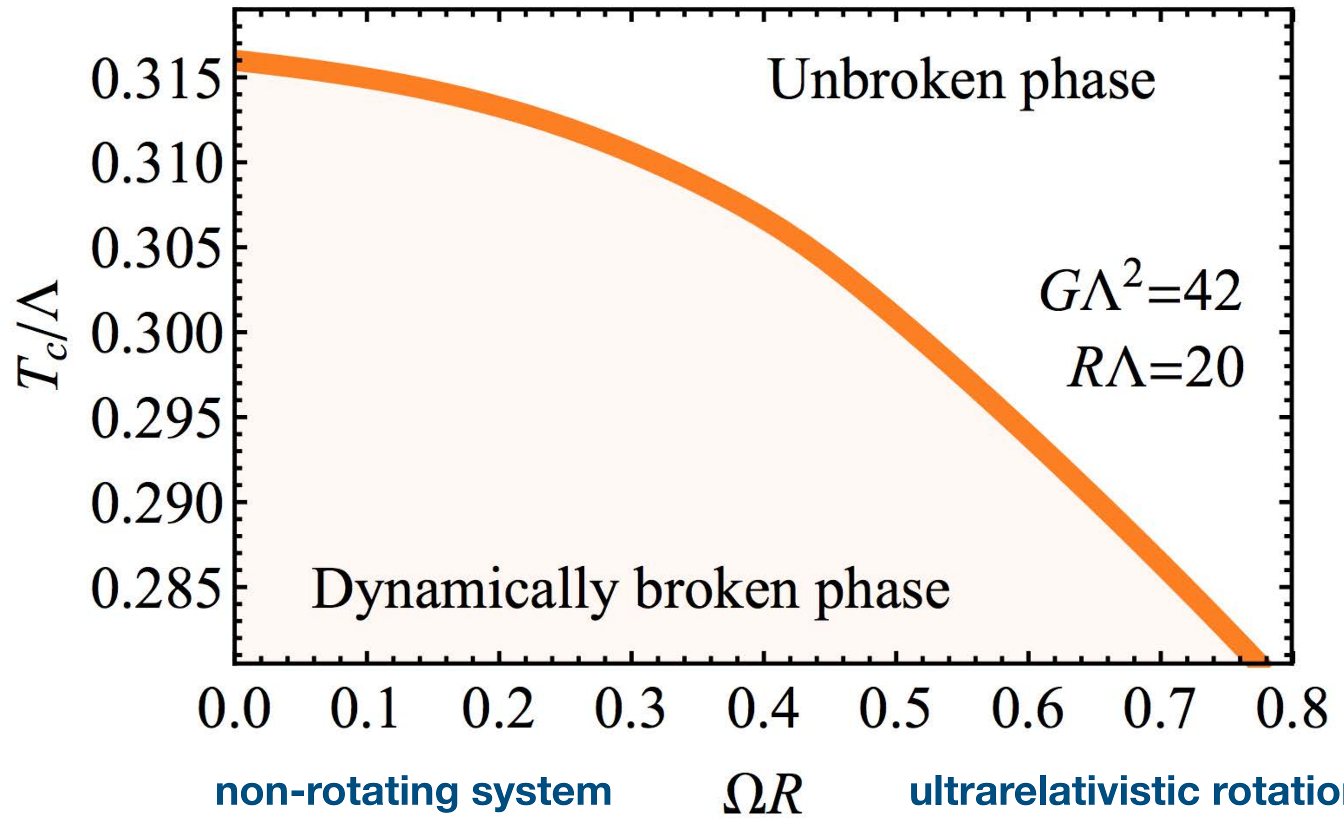
The STAR Collaboration, Nature 62, 548 (2017)

Phase diagram at finite temperature

Rotation decreases the critical temperature of the chiral phase transition

The critical temperature of the chiral symmetry breaking transition

Uniform rotation restores the chiral symmetry



non-rotating system — ultrarelativistic rotation

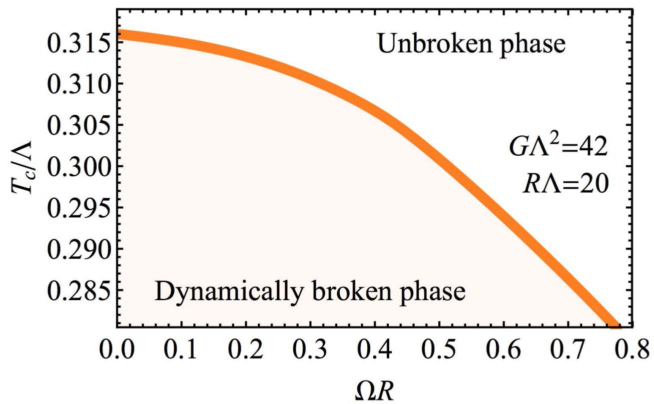
Holographic approaches [B. McInnes, Nucl.Phys. B911 (2016) 173],
Nambu—Jona-Lasinio models [H.-L. Chen, K. Fukushima, X.-G. Huang, K. Mameda, Phys.Rev. D93 (2016) 104052],
[Y. Jiang, J. Liao, Phys.Rev.Lett. 117 (2016), 192302]; M.Ch. and Shinya Gongyo, JHEP 01, 136 (2017)

What is the mechanism?

The “Barnett coupling” in QCD

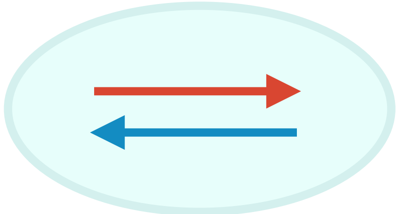
Uniform rotation restores the chiral symmetry

What is the mechanism?

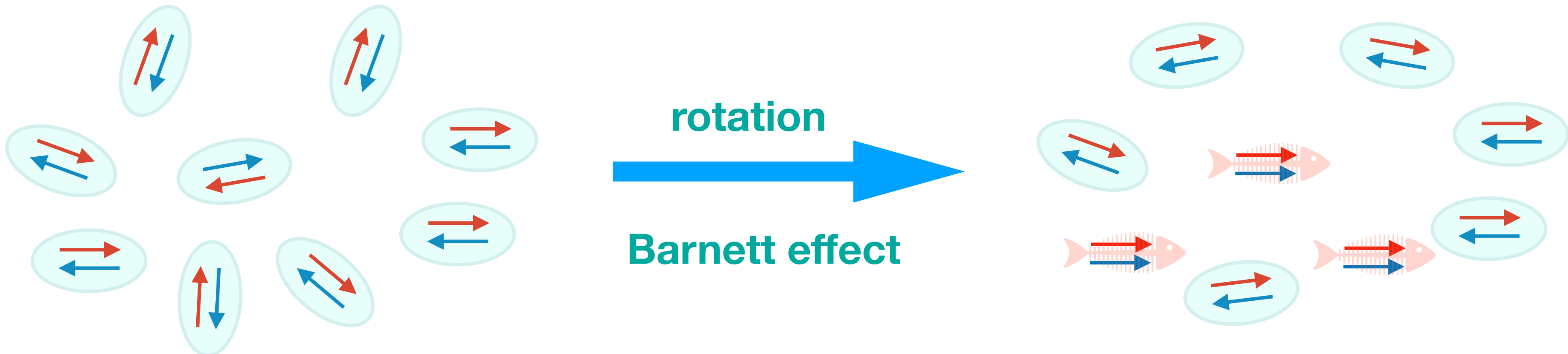


The chiral condensate is a spin-0 object

$$\langle \bar{\psi}\psi \rangle = -\frac{\sigma}{2G}$$



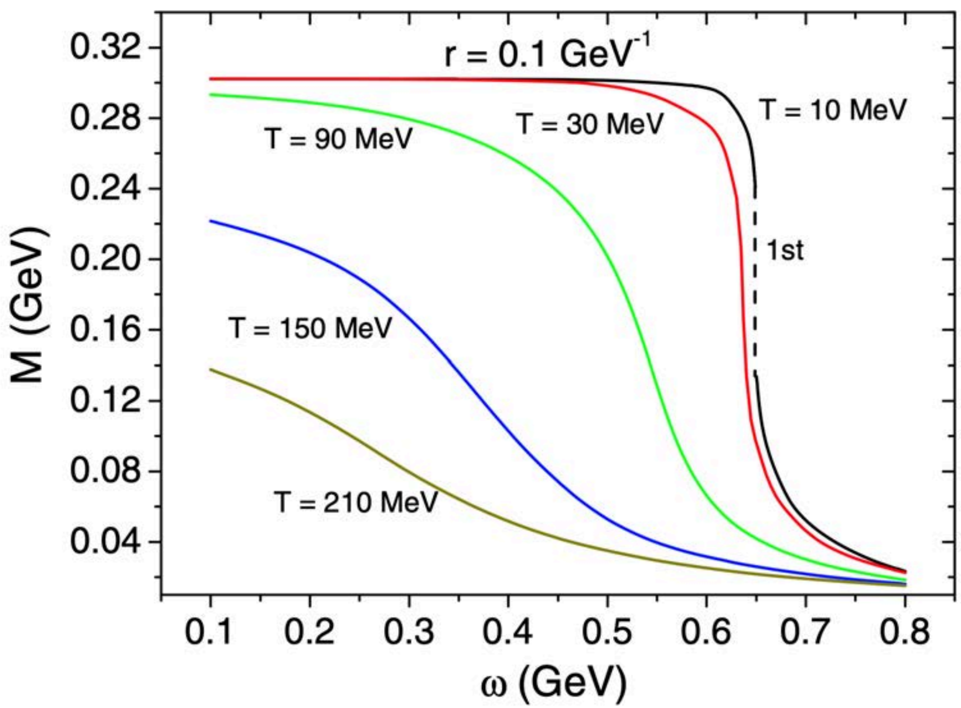
The Barnett effect polarized both the spin of a quark and the spin of an anti-quark along the axis of rotation



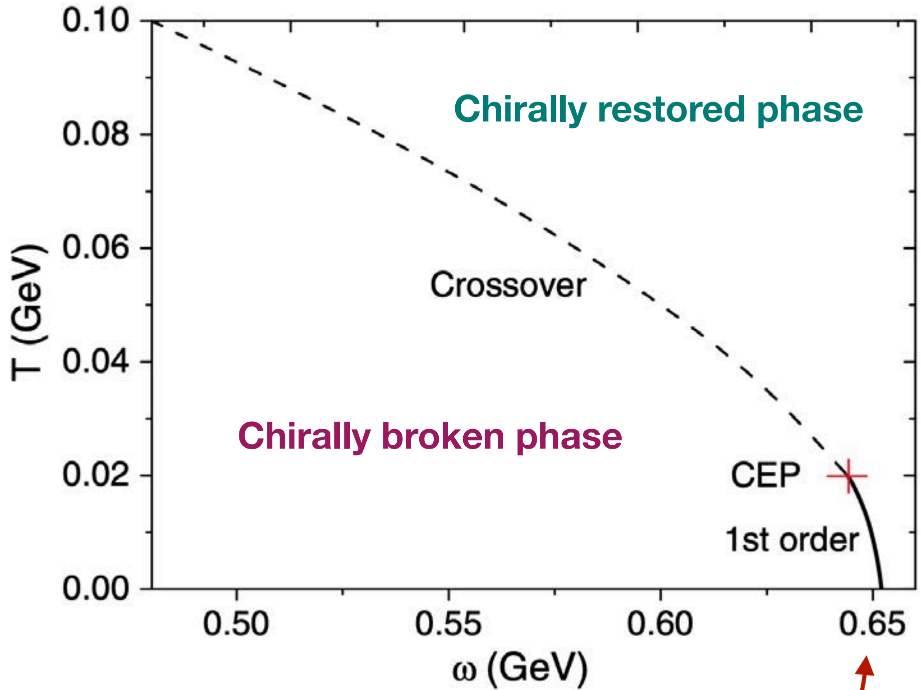
The chiral condensate is destroyed by rotation due to an analogue of the Barnett effect

Chiral symmetry and rotation in QCD

chiral condensate vs. rotation frequency



the phase diagram



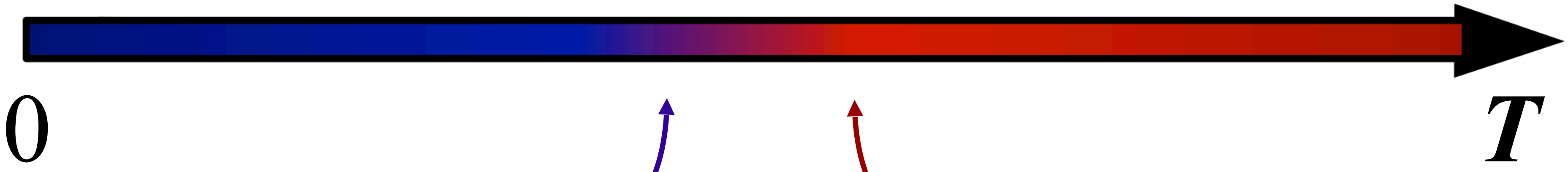
Finite-size effects are expected to be strong.

$$R_{\max}\Omega = 1 \longrightarrow R_{\max} \simeq 0.3\,\mathrm{fm}$$

Small transverse size close to the perturbative regime

Juang and Liao, PRL 117, 192302 (2016)

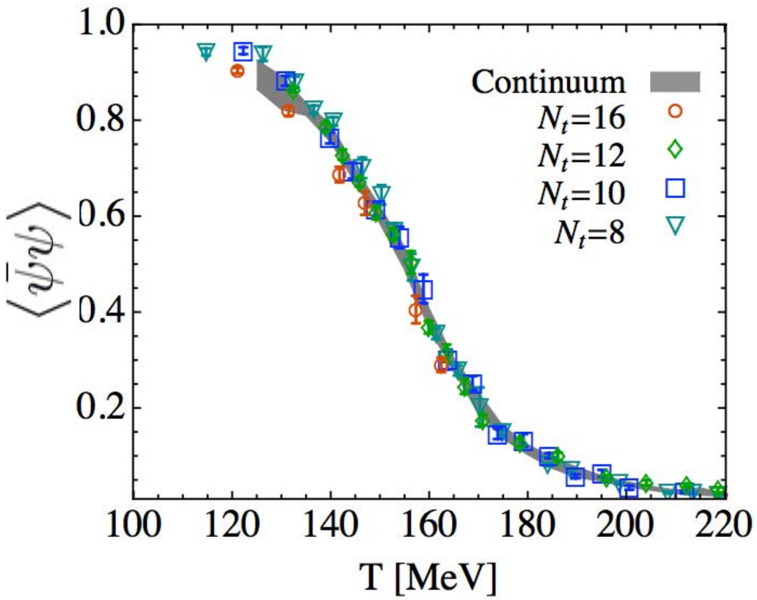
We have also a deconfining property of the QCD crossover



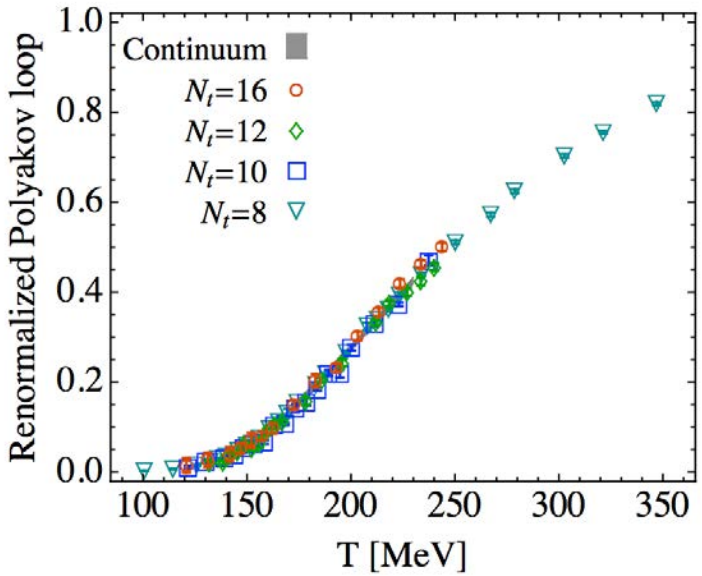
Restoration of the chiral symmetry

Deconfinement transition

$$T_{\text{chiral}} \approx 155\,\text{MeV}$$



$$T_{\text{deconf}} \approx 170\,\text{MeV}$$



What happens with the confining properties in the rotating plasma?

What is the effect of rotation on confinement?

Disclaimer: we don't know for sure. But let's talk about it anyway.

Papers on the subject (exhaustive list, in order of appearance):

1. V. Braguta, A. Kotov, D. Kuznedelev, and A. Roenko, JETP Lett. 112, 6 (2020)
   first-principles lattice calculation;

2. X. Chen, L. Zhang, D. Li, D. Hou, and M. Huang, arXiv:2010.14478
   holographic approach;

3. M. Chernodub, arXiv:2012.04924
   toy model analysis

4. Y. Fujimoto, K. Fukushima, and Y. Hidaka, arXiv:2101.09173
   hadron resonance gas model

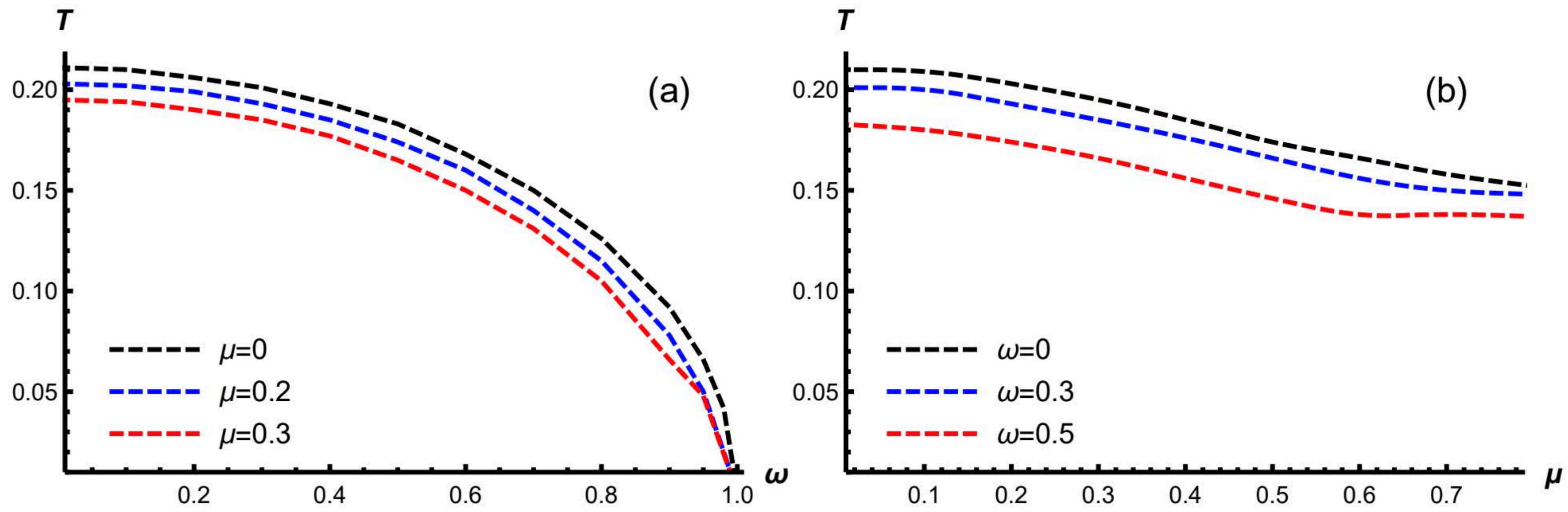
5. V. Braguta, A. Kotov, D. Kuznedelev, and A. Roenko, arXiv:2102.05084
   more detailed first-principles analysis

The confusion is a solid signature that the situation is far from trivial: three independent theoretical papers [2,3,4] based on three different approaches agree with each other and they together **contradict** qualitatively (!) the first-principles simulations [1,5].

Spoiler: no hope, this talk will probably deepen the confusion

Rotation effect from holography

phase diagram



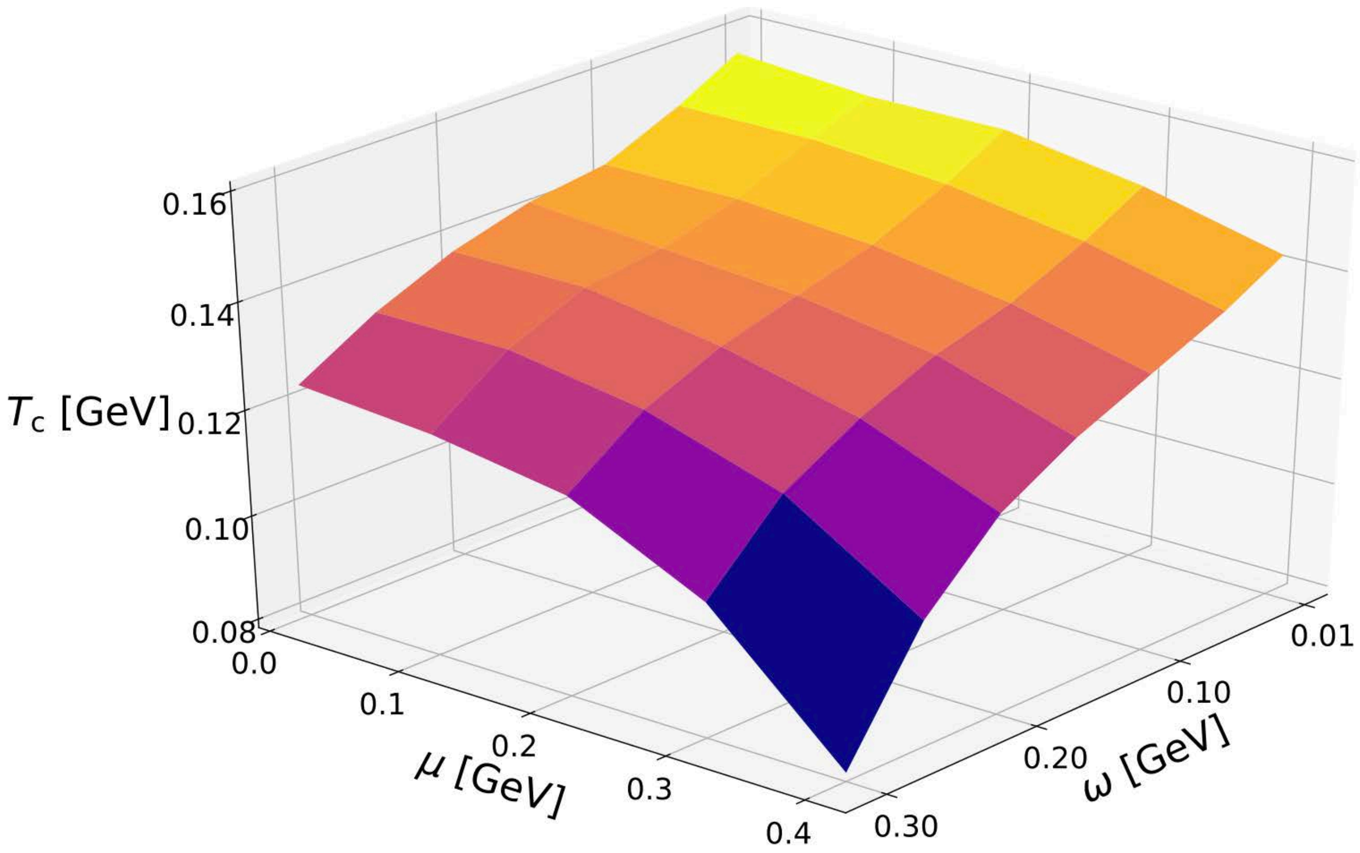
dense rotating quark-gluon matter at high-temperature

→ rotation decreases deconfinement temperature

X. Chen, L. Zhang, D. Li, D. Hou, and M. Huang, arXiv:2010.14478

Deconfinement due to rotation in HGR

The phase diagram of rotating hadron resonance gas



Y. Fujimoto, K. Fukushima, and Y. Hidaka, arXiv:2101.09173

Deconfinement due to rotation: General arguments

Gluons and quarks are living in the co-rotating frame, which rotates together with the plasma.

→ The laboratory system is the flat Minkowski spacetime
→ The co-rotating system corresponds to the curvilinear reference system with the following metric tensor

$$g_{\mu\nu} = \begin{pmatrix} 1-(x^2+y^2)\Omega^2 & y\Omega & -x\Omega & 0 \\ y\Omega & -1 & 0 & 0 \\ -x\Omega & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

corresponding to the line element of the curved space-time:

$$ds^2 \equiv g_{\mu\nu}dx^\mu dx^\nu = \left(1-\rho^2\Omega^2\right)dt^2 - 2\rho^2\Omega dt d\varphi - d\rho^2 - \rho^2 d\varphi^2 - dz^2$$

next following M. Ch. arXiv:2012.04924

Tolman-Ehrenfest law

In a static background gravitational field, the temperature of a system in a thermal equilibrium is not constant:

$$T(\boldsymbol{x})\sqrt{g_{00}(\boldsymbol{x})} = T_0$$

Metric in rotating frame:

$$g_{\mu\nu} = \begin{pmatrix} 1-(x^2+y^2)\Omega^2 & y\Omega & -x\Omega & 0 \\ y\Omega & -1 & 0 & 0 \\ -x\Omega & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

local temperature on the axis of rotation

$$T_0 \equiv T(0)$$

in cylindrical coordinates:

$$g_{00} = 1 - \rho^2\Omega^2$$

distance from the axis

Temperature rises as the distance from the axis of rotation increases:

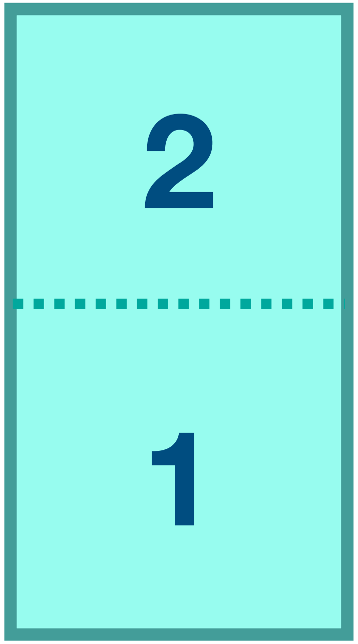
$$T(\rho) = \frac{T(0)}{\sqrt{1-\rho^2\Omega^2}}$$

R. C. Tolman, “On the Weight of Heat and Thermal Equilibrium in General Relativity,” Phys. Rev. 35, 904 (1930);
R. Tolman and P. Ehrenfest, “Temperature Equilibrium in a Static Gravitational Field,” Phys. Rev. 36, 1791 (1930).

Thermal equilibrium

- Consider a closed system divided arbitrarily into two subsystems
- Thermal equilibrium happens when the total entropy reaches its maximum

$$S = S_1 + S_2 \qquad dS_1 + dS_2 = 0$$



- Assume that we have no gravitational field
- If the quantity of heat leaves the first subsystem, it always enters the second subsystem:

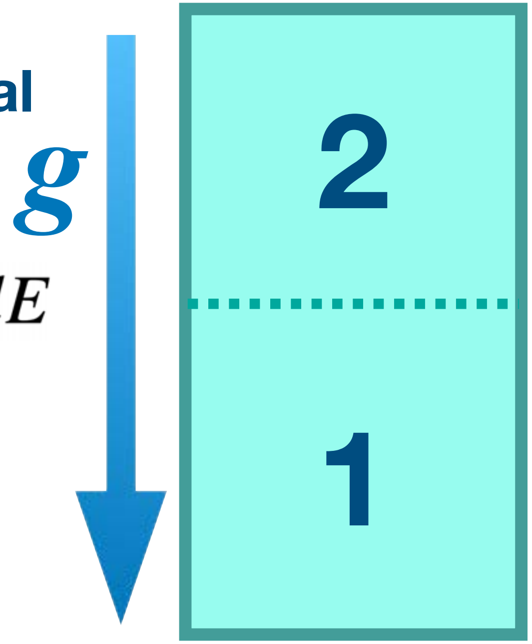
$$dE_1 = -dE \rightarrow dE_2 = dE \rightarrow dS_1/dE_1 = dS_2/dE_2 \rightarrow T_1 = T_2$$

definition of temperature $1/T = \partial S/\partial E$

In the absence of gravitational field, the temperature is constant

How to understand the Tolman-Ehrenfest law?

- In a static gravitational field Φ, the heat quantity dE possesses an inertial mass $$dm = dE/c^2$$
- the equivalence between inertial and gravitational masses: a quantity of heat has a weight



- When heat leaves the first subsystem, $dE_1 = -dE$ it enters the second subsystem, and performs work agains the gravity (heat = mass):

$$dE_2 = dE + (\Phi_2 - \Phi_1)dm = dE_2(1 + \Delta\Phi/c^2)$$

$$\Delta\Phi = \Phi_2 - \Phi_1$$

change of the gravitational potential

$$T_1 = T_2$$

- Entropy maximum

$$dS_1 + dS_2 = 0$$

- Local temperature

$$g_{00} = 1 + \frac{2\Phi}{c^2}$$

$$T_2 = T_1(1 + \Delta\Phi/c^2)$$

Tolman-Ehrenfest law

$$T(x) = T_0/\sqrt{g_{00}(x)}$$

C. Rovelli and M. Smerlak, “Thermal time and Tolman-Ehrenfest effect: temperature as the speed of time” Class. Quant. Grav. 28, 4 (2010)

Thermal equilibrium in rotating QGP

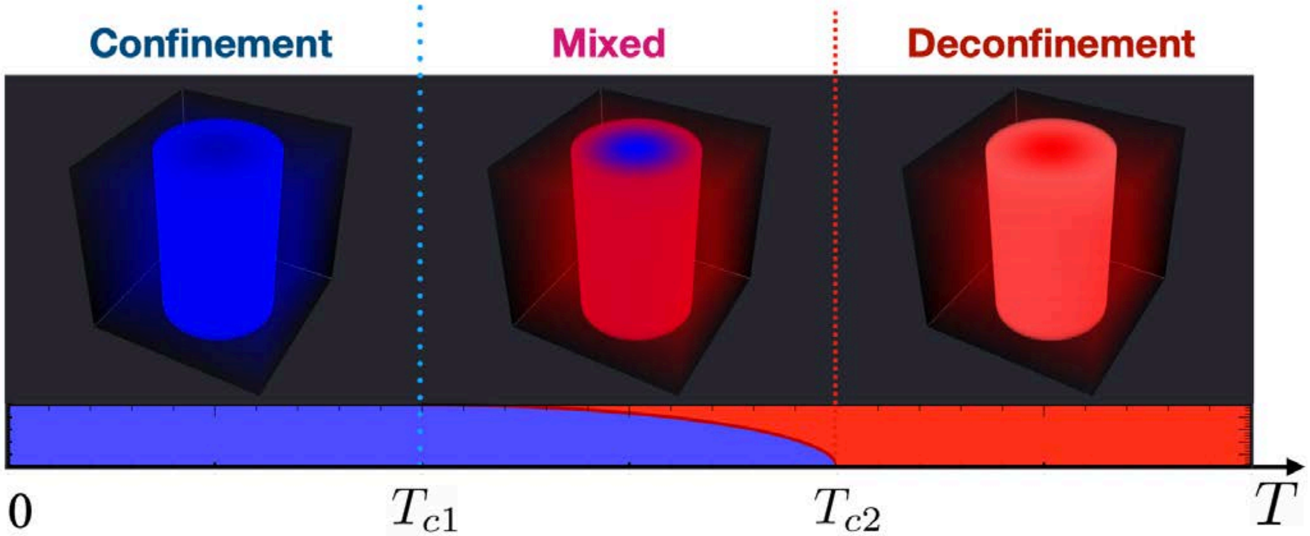
Temperature is colder in the center and higher at the edges of the system: $T(\rho) = \frac{T(0)}{\sqrt{1 - \rho^2 \Omega^2}}$

$$T_\Omega(\rho) < T_{c,\infty} \qquad \text{(confinement)},$$
$$T_\Omega(\rho) > T_{c,\infty} \qquad \text{(deconfinement)}$$

$T_{c,\infty}$ the critical temperature in a thermodynamically large, non-rotating system

The phase structure:

New mixed phase



Two critical temperatures: $T_{c1} = T_{c,\infty}\sqrt{1 - \Omega^2 R^2}, \qquad T_{c2} = T_{c,\infty}$

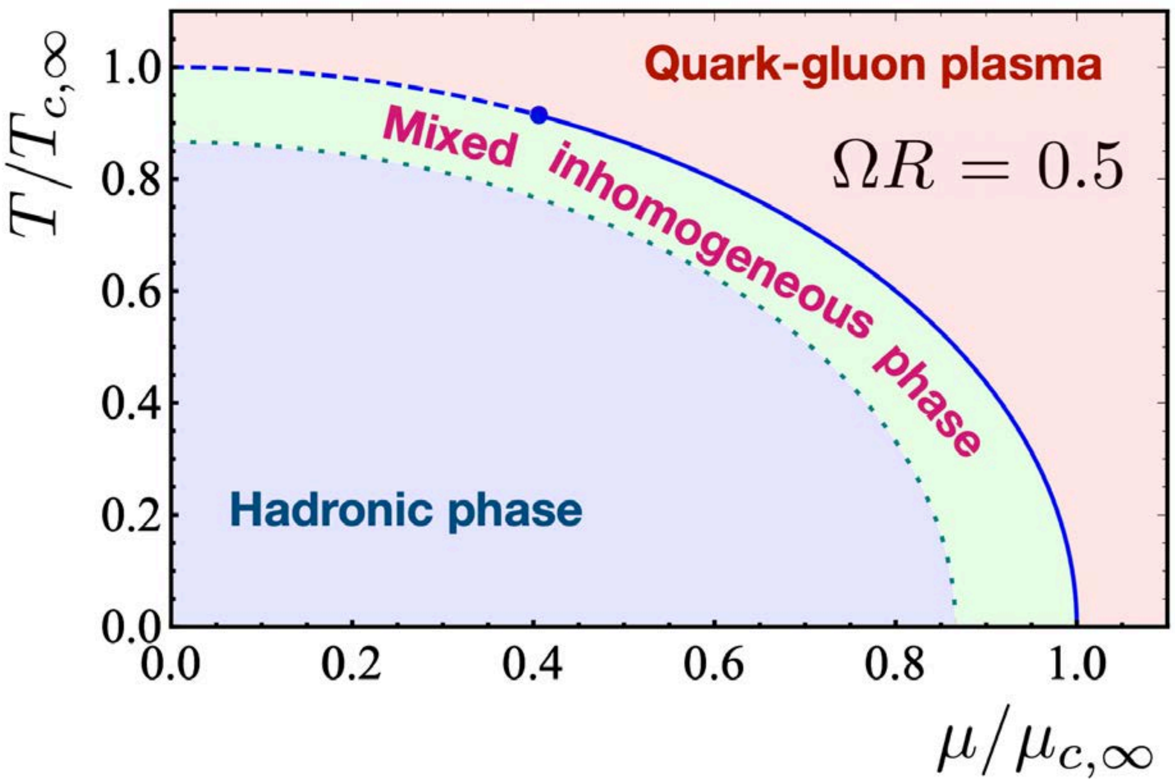
Inverse harmonization effect: plasma cools from inside!

M. Ch. arXiv:2012.04924

Hot dense rotating quark-gluon plasma

The Tolman-Ehrenfest law for temperature and chemical potential

$$T(\boldsymbol{x})\sqrt{g_{00}(\boldsymbol{x})} = T_0, \qquad \mu_B(\boldsymbol{x})\sqrt{g_{00}(\boldsymbol{x})} = \mu_{B0}$$



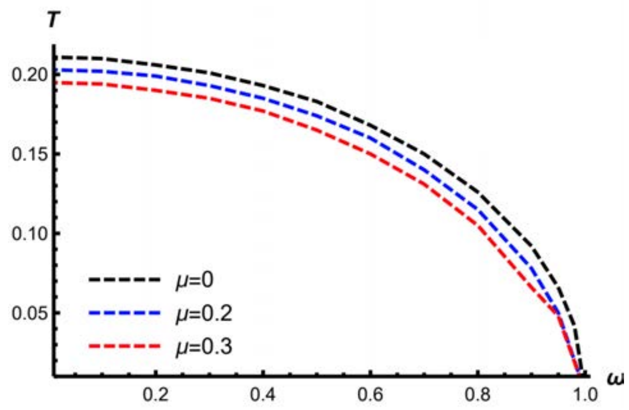
The Tolman-Ehrenfest effect of rotation and confinement?
One can demonstrate it in an effective model for confinement (cQED)!

M. Ch. arXiv:2012.04924

Head-on collision of analytics and numerics ?

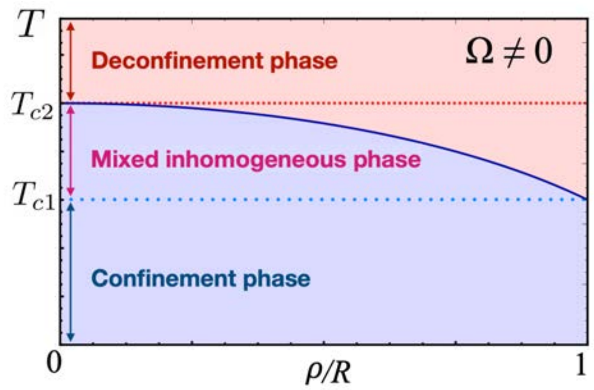
Analytical results: rotation decreases deconfinement temperature

holography



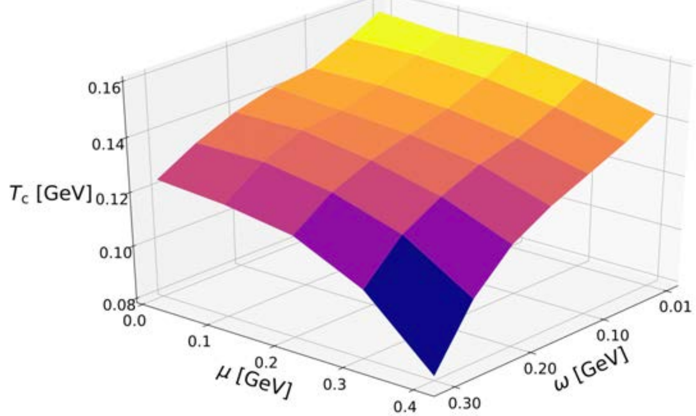
Chen, Zhang, Li, Hou, Huang (arxiv:2010.14478)

Tolman-Ehrenfest



M. Ch. (arxiv: 2012.04924)

hadron resonance gas



Fujimoto, Fukushima, Hidaka (arxiv:2101.09173)

First-principle numerical results in lattice Yang-Mills theory (imaginary rotation in Euclidean + analytical continuation to Minkowski):
Rotation increases deconfinement temperature (!):

$$T_c(\Omega)/T_c(0) = 1 + C_2\Omega^2 \text{ with } C_2 > 0$$

Braguta, Kotov, Kuznedelev, Roenko (Phys.Rev.D 103 (2021) 9, 094515; arxiv:2102.05084)

Details on lattice results

PHYSICAL REVIEW D **103**, 094515 (2021)

Influence of relativistic rotation on the confinement-deconfinement transition in gluodynamics

V. V. Braguta,[1,2,3,*] A. Yu. Kotov,[4,†] D. D. Kuznedelev,[3,‡] and A. A. Roenko[1,§]

Lattice Yang-Mills in curved Euclidean spacetime

$$S_G = \frac{1}{4g^2}\int d^4x\,\sqrt{g_E}\,g_E^{\mu\nu}\,g_E^{\alpha\beta}\,F^a_{\mu\alpha}F^a_{\nu\beta}$$

Going to Euclidean with

$$\begin{aligned} S_G = \frac{1}{2g^2}\int d^4x\,[&(1-r^2\Omega^2)F^a_{xy}F^a_{xy} + (1-y^2\Omega^2)F^a_{xz}F^a_{xz} \\ &+ (1-x^2\Omega^2)F^a_{yz}F^a_{yz} + F^a_{x\tau}F^a_{x\tau} + F^a_{y\tau}F^a_{y\tau} \\ &+ F^a_{z\tau}F^a_{z\tau} - 2iy\Omega(F^a_{xy}F^a_{y\tau} + F^a_{xz}F^a_{z\tau}) \\ &+ 2ix\Omega(F^a_{yx}F^a_{x\tau} + F^a_{yz}F^a_{z\tau}) - 2xy\Omega^2 F^a_{xz}F^a_{zy}]. \end{aligned}$$

A need for imaginary rotation:

$$\Omega = i\Omega_I$$

Metric in Minkowski

$$g_{\mu\nu} = \begin{pmatrix} 1-r^2\Omega^2 & \Omega y & -\Omega x & 0 \\ \Omega y & -1 & 0 & 0 \\ -\Omega x & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

First academic attempt:

Lattice QCD in Rotating Frames

Arata Yamamoto and Yuji Hirono

Phys. Rev. Lett. **111**, 081601 – Published 22 August 2013

Analytic continuation $\Omega_I \to -i\Omega$

Lattice result for critical deconfining temperature:

$$\frac{T_c(\Omega_I)}{T_c(0)} = 1 - C_2\Omega_I^2$$

imaginary rotation

$$\frac{T_c(v_I)}{T_c(0)} = 1 - B_2\frac{v_I^2}{c^2}$$

$\Omega_I = -i\Omega$ $\qquad$ $v_I = -iv$

$$\frac{T_c(\Omega)}{T_c(0)} = 1 + C_2\Omega^2$$

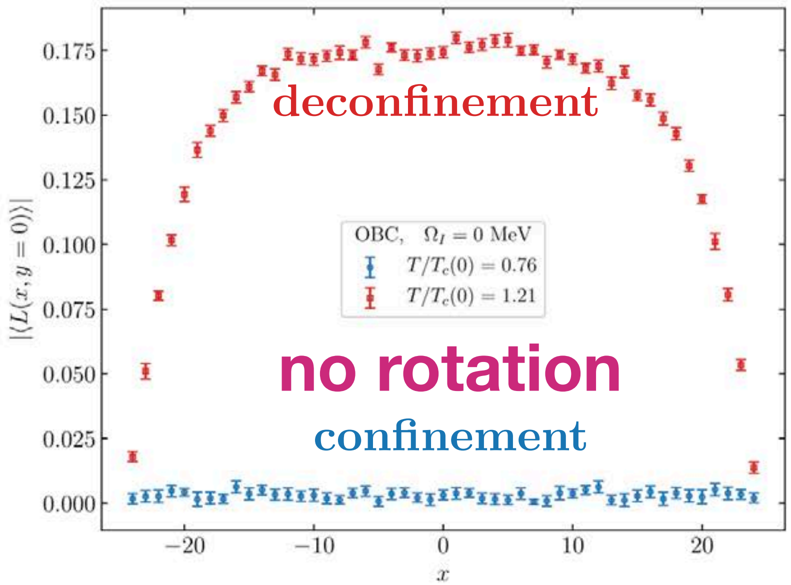
real rotation

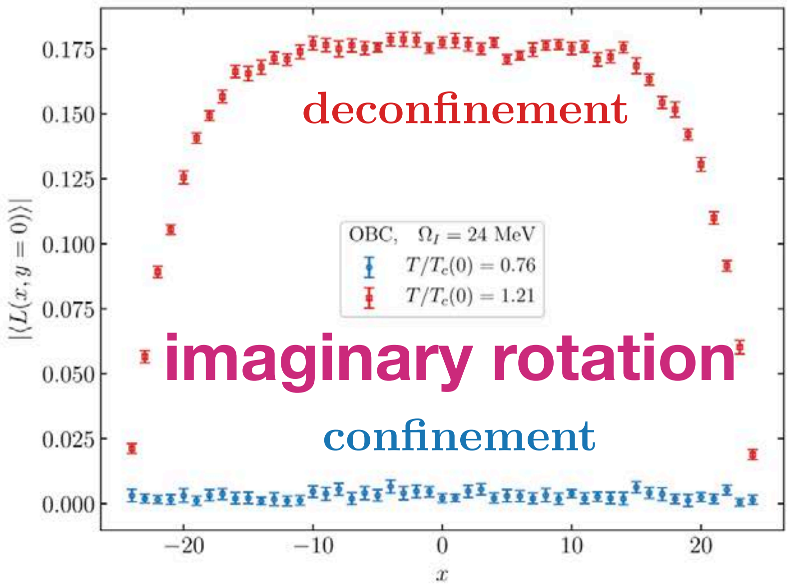
$$\frac{T_c(v)}{T_c(0)} = 1 + B_2\frac{v^2}{c^2}$$

as a function of … angular frequency $\qquad$ … linear velocity on the boundary

---

effect of imaginary rotation on spatial inhomogeneity: no effect





This talk: Imaginary rotation

Following works:

Fractal thermodynamics and ninionic statistics of coherent rotational states: realization via imaginary angular rotation in imaginary time formalism

M.N. Chernodub (IDP, Tours) (Oct 11, 2022)

e-Print: 2210.05651 [quant-ph]

Inhomogeneity of rotating gluon plasma and Tolman-Ehrenfest law in imaginary time: lattice results for fast imaginary rotation

M.N. Chernodub (IDP, Tours), V.A. Goy (Far Eastern Natl. U.), A.V. Molochkov (Far Eastern Natl. U.) (Sep 30, 2022)

e-Print: 2209.15534 [hep-lat]

Instantons in rotating finite-temperature Yang-Mills gas

M.N. Chernodub (IDP, Tours) (Aug 9, 2022)

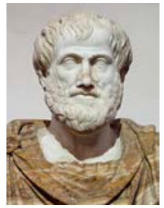
e-Print: 2208.04808 [hep-th]

Imaginary rotation

We work in Euclidean spacetime ($\to$ time is imaginary)

"Time is the measurable unit of movement" **—Aristotle.**

(~ 2400 years ago)

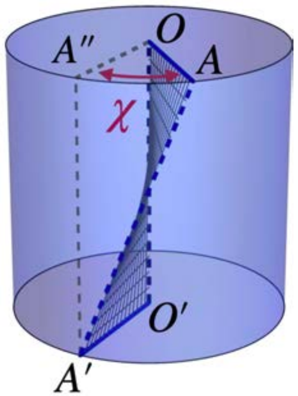


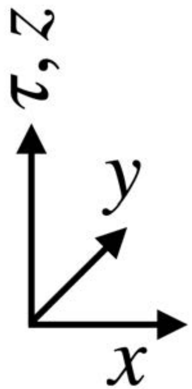
"In the imaginary time formalism, the movement (rotation) is also imaginary"

— this talk (Tuesday, year 2022 AC)

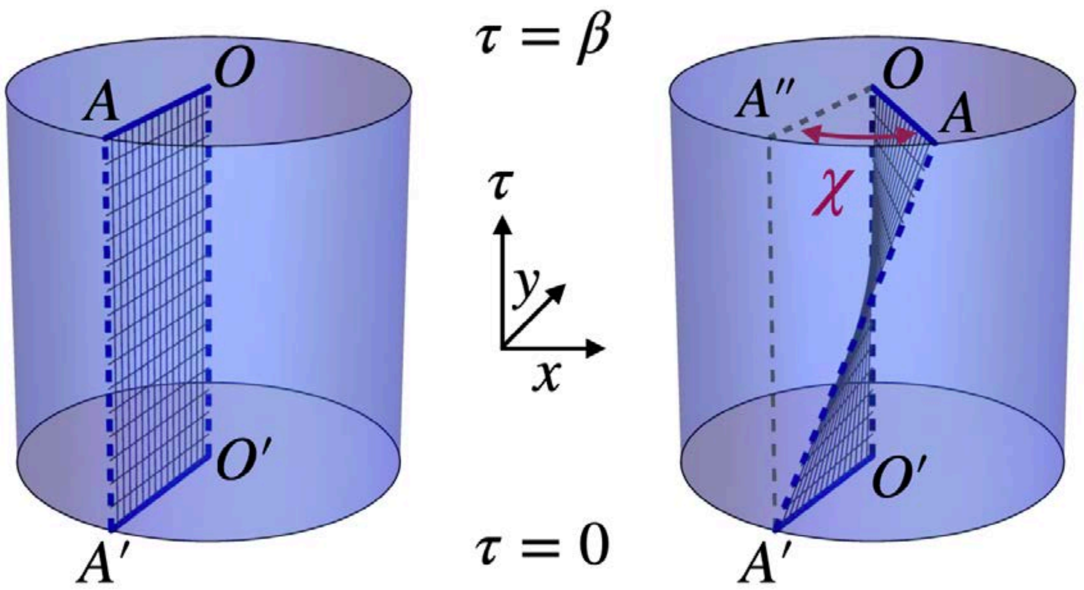
From Minkowski to Euclidean spacetime: $t \to -i\tau$ and $\Omega \to i\Omega_I$

$$\Omega = \frac{d\theta}{dt} \to i\frac{d\theta}{d\tau} = \Omega_I$$





Imaginary rotation



spatial rotation

$$\boldsymbol{x} \to \boldsymbol{x}' = \hat{R}_{\boldsymbol{\chi}} \boldsymbol{x}$$

by the angle

$$\boldsymbol{\chi} = \beta \boldsymbol{\Omega}_I$$

about the axis

$$\boldsymbol{n} = \boldsymbol{\chi}/\chi = \boldsymbol{\Omega}_I/\Omega_I$$

no imaginary rotation

$$\phi(\boldsymbol{x},\tau) = +\phi(\boldsymbol{x},\tau+\beta)$$
$$\psi(\boldsymbol{x},\tau) = -\psi(\boldsymbol{x},\tau+\beta)$$

imaginary rotation $\Omega_I = \chi/\beta$

$$\phi(\boldsymbol{x},\tau) = +\phi\left(\hat{R}_{\boldsymbol{\chi}}\boldsymbol{x},\tau+\beta\right),$$ **bosons**

$$\psi(\boldsymbol{x},\tau) = -\hat{\Lambda}_{\boldsymbol{\chi}}\psi\left(\hat{R}_{\boldsymbol{\chi}}\boldsymbol{x},\tau+\beta\right)$$ **fermions**

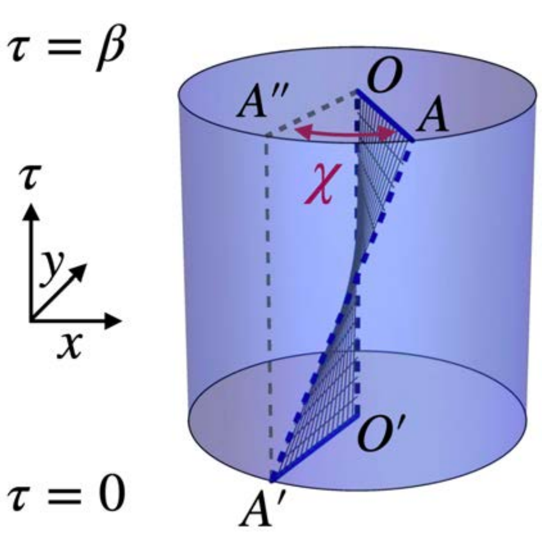
Rotwisted boundary conditions

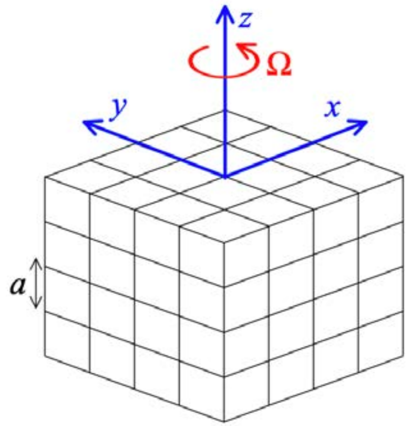
for bosons: $\phi(\rho,\varphi,z,\tau) = \phi(\rho,\varphi-\beta\Omega_I,z,\tau+\beta)$

periodicity: $\mathcal{O}(\Omega_I) = \mathcal{O}(\Omega_I + 2\pi n/\beta)\,, \quad \text{for} \quad n \in \mathbb{Z}$

Inhomogeneous confining-deconfining phases in rotating plasmas
M.N. Chernodub (IDP, Tours and Far Eastern Natl. U.) (Dec 9, 2020)
Published in: *Phys.Rev.D* 103 (2021) 5, 054027 • e-Print: 2012.04924 [hep-ph]

Imaginary rotation: two approaches





Lattice QCD in rotating frames
Arata Yamamoto (Nishina Ctr., RIKEN), Yuji Hirono
Published in: *Phys.Rev.Lett.* 111 (2013) 081601 •

Study of the Confinement/Deconfinement Phase Transition in Rotating Lattice SU(3) Gluodynamics
V.V. Braguta (Natl. U. Sci. Tech., Moscow and Dubna, JINR), A.Yu. Kotov (Natl. U. Sci. Tech., Moscow and Dubna, JINR and Moscow, ITEP), D.D. Kuznedelev (Moscow, MIPT), A.A. Roenko (Dubna, JINR) (2020)
Published in: *Pisma Zh.Eksp.Teor.Fiz.* 112 (2020) 1, 9-16, *JETP Lett.* 112 (2020) 1, 6-12

Inhomogeneous confining-deconfining phases in rotating plasmas
M.N. Chernodub (IDP, Tours and Far Eastern Natl. U.) (Dec 9, 2020)
Published in: *Phys.Rev.D* 103 (2021) 5, 054027 • e-Print: 2012.04924 [hep-ph]

Influence of relativistic rotation on the confinement-deconfinement transition in gluodynamics
V.V. Braguta (Dubna, JINR and Natl. U. Sci. Tech., Moscow and Moscow, MIPT), A.Yu. Kotov (Julich, NIC), D.D. Kuznedelev (Moscow, MIPT), A.A. Roenko (Dubna, JINR) (Feb 9, 2021)
Published in: *Phys.Rev.D* 103 (2021) 9, 094515 • e-Print: 2102.05084 [hep-lat]

$$(\rho, \varphi, z, \tau) \to (\rho, \varphi - \Omega_I \beta, z, \tau + \beta)$$

$$ds_E^2 = g^E_{\mu\nu} dx^\mu dx^\nu = \left(1 + \Omega_I^2 \rho^2\right) d\tau^2 + 2\Omega_I \rho^2 d\tau d\varphi + d\rho^2 + \rho^2 d\varphi^2 + dz^2 \,.$$

Rotwisted boundary conditions | **Lattice action in curved space time**

The same conditions for slow imaginary rotations, $|\Omega_I| \ll 2\pi/\beta$

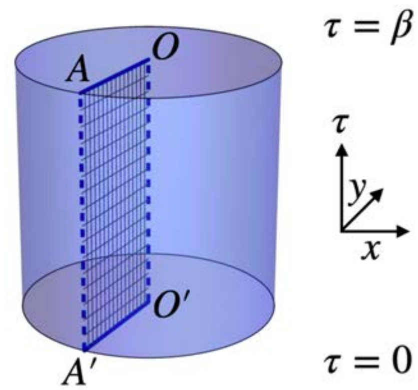
Substantial difference for fast imaginary rotations, $|\Omega_I| \sim 2\pi/\beta$

for example, periodicity: $\mathcal{O}(\Omega_I) = \mathcal{O}(\Omega_I + 2\pi n/\beta)\,, \quad \text{for} \quad n \in \mathbb{Z}$

Imaginary rotation: confinement parameter

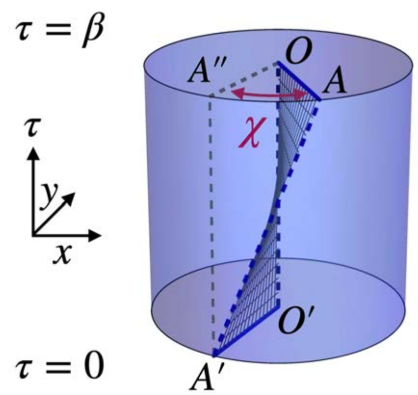
In the absence of rotation, $\Omega_I = 0$,
the order parameter is Polyakov loop:

$$\mathcal{P}(\boldsymbol{x}) = \mathrm{Tr}\,\mathcal{P}\exp\left\{ i \oint_{\mathcal{C}} \hat{A}^a_\tau(\boldsymbol{x},\tau)d\tau \right\}$$



For imaginary rotation, $\Omega_I \neq 0 \bmod 2\pi$,
the Polyakov loop should be modified to
respect the rotwisted boundary conditions:

$$(\rho, \varphi, z, \tau) \to (\rho, \varphi - \Omega_I \beta, z, \tau + \beta)$$



Purely <u>kinematic</u> effect: the length of the Polyakov loop increases,
we increase the distance from the axis of rotation!

Further from axis → longer Polyakov loop → closer to confinement!

(A Euclidean version of the Tolman Ehrenfest effect!)

Tolman-Ehrenfest effect

redshift/blueshift of thermal length: $T(\boldsymbol{x})\sqrt{g_{00}(\boldsymbol{x})} = T_0$

→ **temperature is not uniform in thermal equilibrium in gravitation field**

Real rotation, Minkowski

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = \left(1 - \Omega^2\rho^2\right)dt^2 - 2\Omega\rho^2 dt d\varphi$$
$$-d\rho^2 - \rho^2 d\varphi^2 - dz^2 \,.$$

Imaginary rotation, Euclidean

$$ds_E^2 = g^E_{\mu\nu}dx^\mu dx^\nu = \left(1 + \Omega_I^2\rho^2\right)d\tau^2 + 2\Omega_I\rho^2 d\tau d\varphi$$
$$+d\rho^2 + \rho^2 d\varphi^2 + dz^2 \,.$$

The equilibrium temperature:

$$T_{\rm TE}(\rho) = \frac{T_0}{\sqrt{1 - \Omega^2\rho^2}}$$

$$T^E_{\rm TE}(\rho, \Omega_I) = \frac{T_0}{\sqrt{1 + \rho^2 T_0^2 [\Omega_I/T_0]^2_{2\pi}}}$$

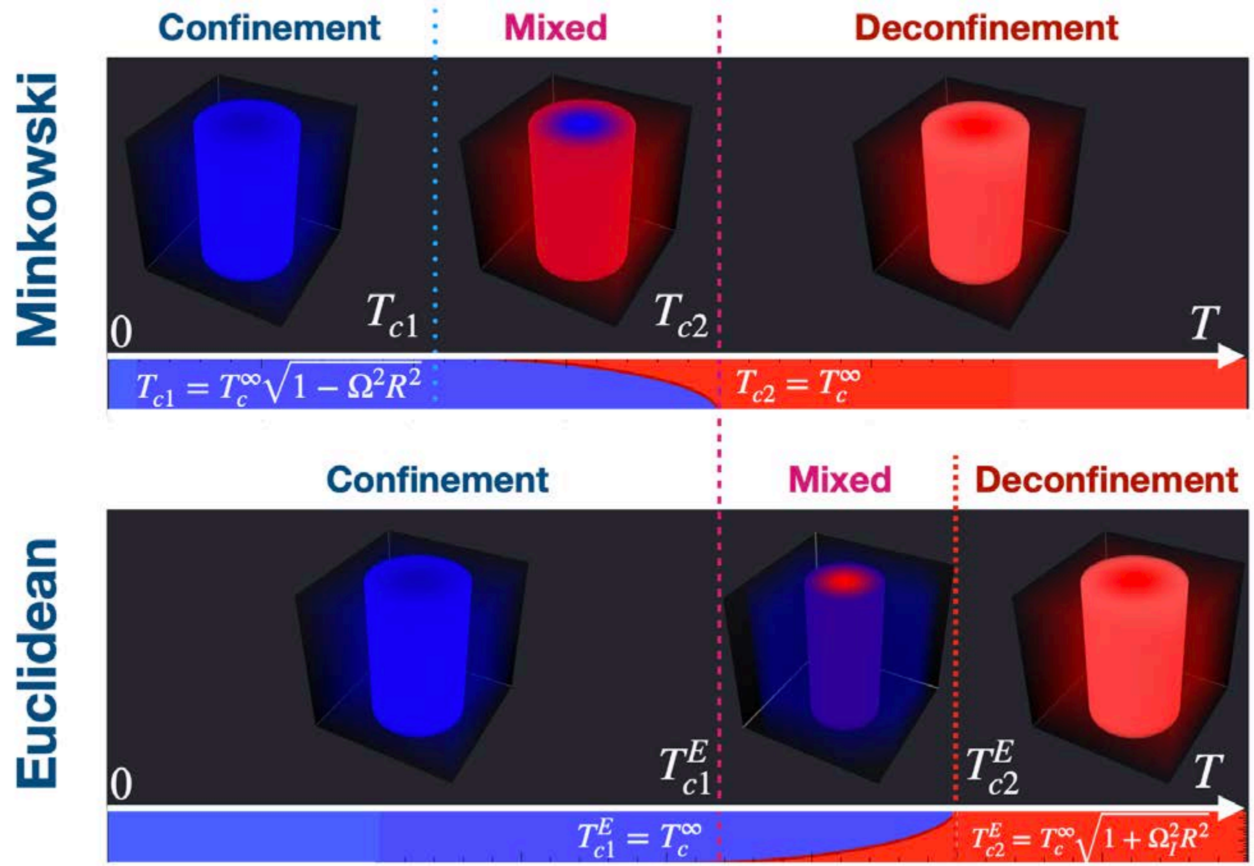
For small frequencies, $\Omega_I \sim \Omega \ll 2\pi T_0$,
the analytical continuation works: $\Omega^2 \Leftrightarrow -\Omega_I^2$

Globally, there are differences (seen in the periodicity in imaginary time)

$$[x]_{2\pi} = x + 2\pi k \in [-\pi, \pi), \qquad k \in \mathbb{Z}$$

Minkowski vs Euclidean rotation

Equilibrium gluon plasma in a cylinder of a radius R.



$$T_{\rm TE}(\rho) = \frac{T_0}{\sqrt{1-\Omega^2\rho^2}}$$

$$T_{c1} = T_{c,\infty}\sqrt{1-\Omega^2R^2}$$

$$T_{c2} = T_{c,\infty}$$

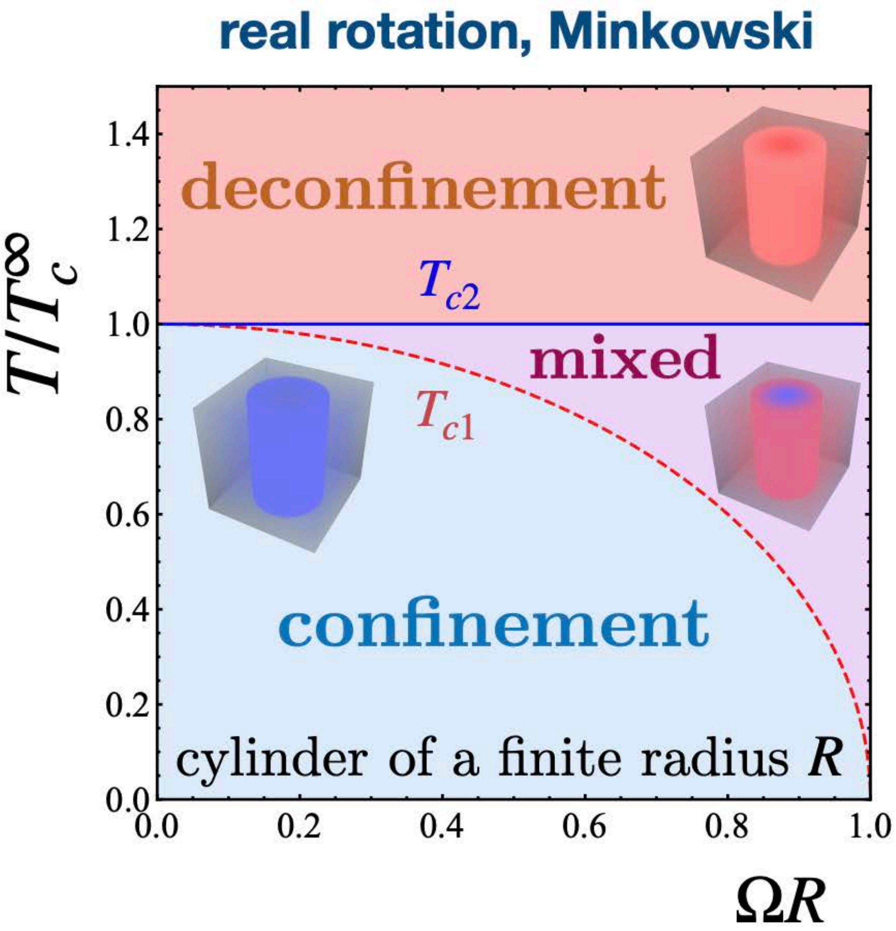
$$T^E_{\rm TE}(\rho,\Omega_I) = \frac{T_0}{\sqrt{1+\rho^2\Omega_I^2}} \quad \text{for} \quad |\Omega_I| < \pi T_0$$

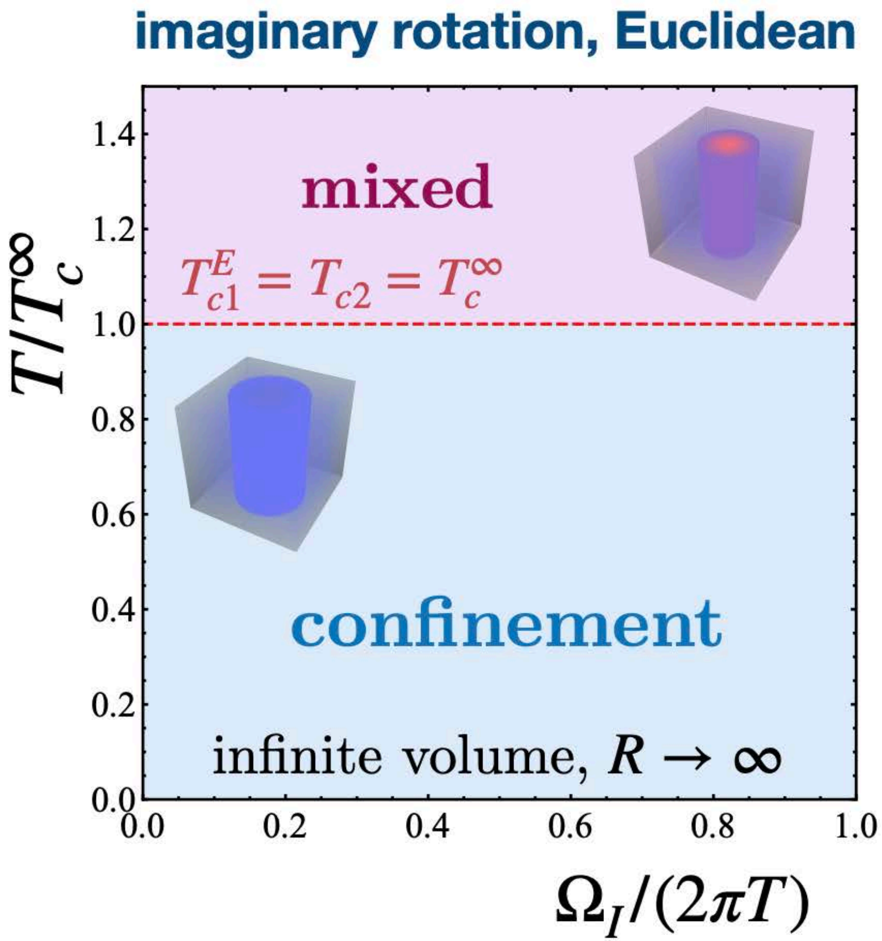
$$T^E_{c1} = T_{c,\infty}$$

$$T^E_{c2} = T_{c,\infty}\sqrt{1+\Omega_I^2R^2}\,, \quad \text{for} \quad -\pi \leqslant \Omega_I\beta < \pi.$$

Two different critical temperatures, signatures of the same (kinematic) Tolman-Ehrenfest effect. New, inhomogeneous phase in QCD.

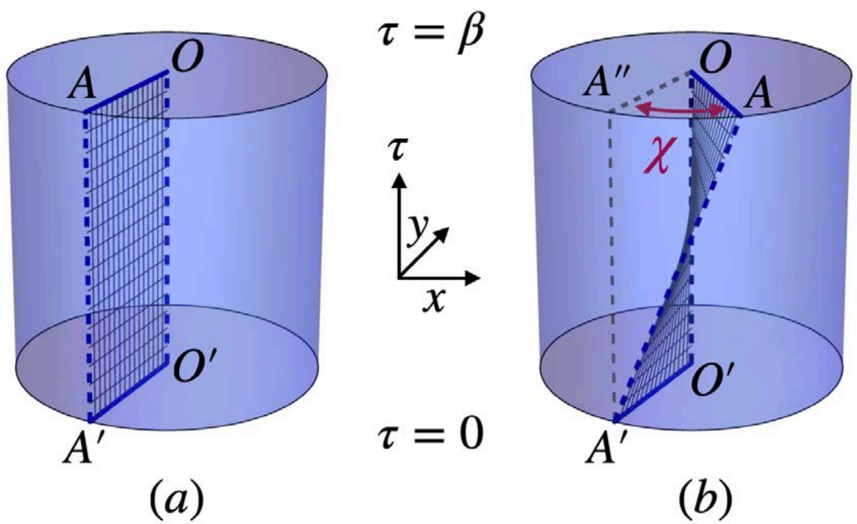
Minkowski vs Euclidean: phase structure



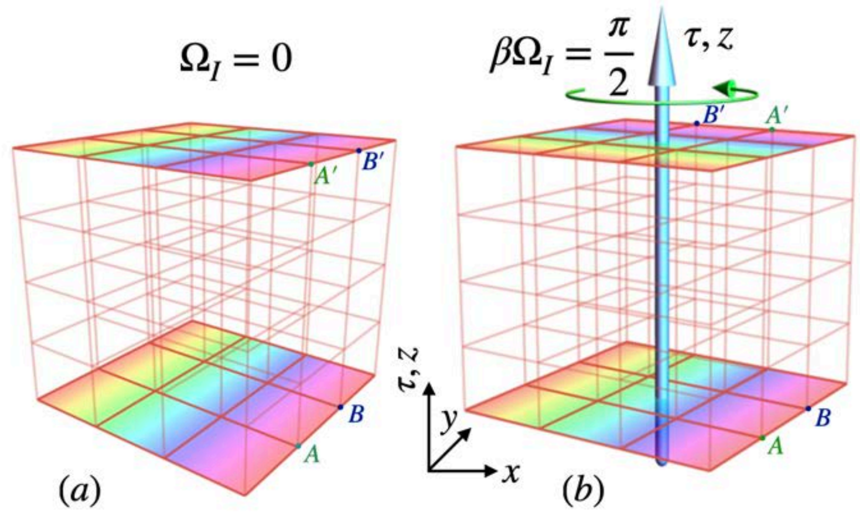


Imaginary rotation on the lattice

Continuum limit (any angle χ)



Hypercubic lattice $(\chi = 0, \pi/2, \pi)$



Lattice:

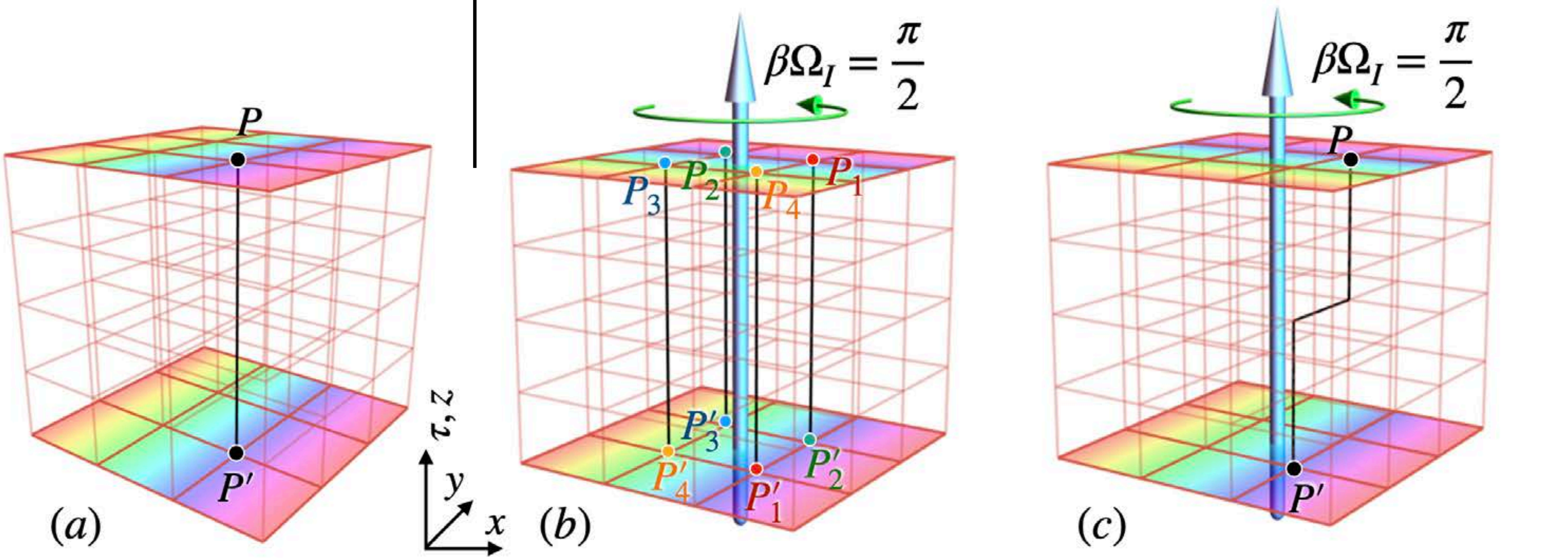
$$\Omega_I = \frac{\pi}{2} T \qquad (x, y, z, \tau) \to (-y, x, z, \tau + \beta)$$

$$\Omega_I = \pi T \qquad (x, y, z, \tau) \to (-x, -y, z, \tau + \beta)$$

(if one takes the convention $\beta = 1/T$)

Polyakov loops

no rotation | $\pi/2$ **rotation:** $(x, y, z, \tau) \to (-y, x, z, \tau + \beta)$



usual loop

4-times winding loop

$\mathcal{P}_4 = \mathrm{Tr}\, U_{P_1' P_1} U_{P_2' P_2} U_{P_3' P_3} U_{\mathcal{P}_4' \mathcal{P}_4}$

laboratory frame

loop with jumper

$P = \mathrm{Tr}\, U_{P' P}$

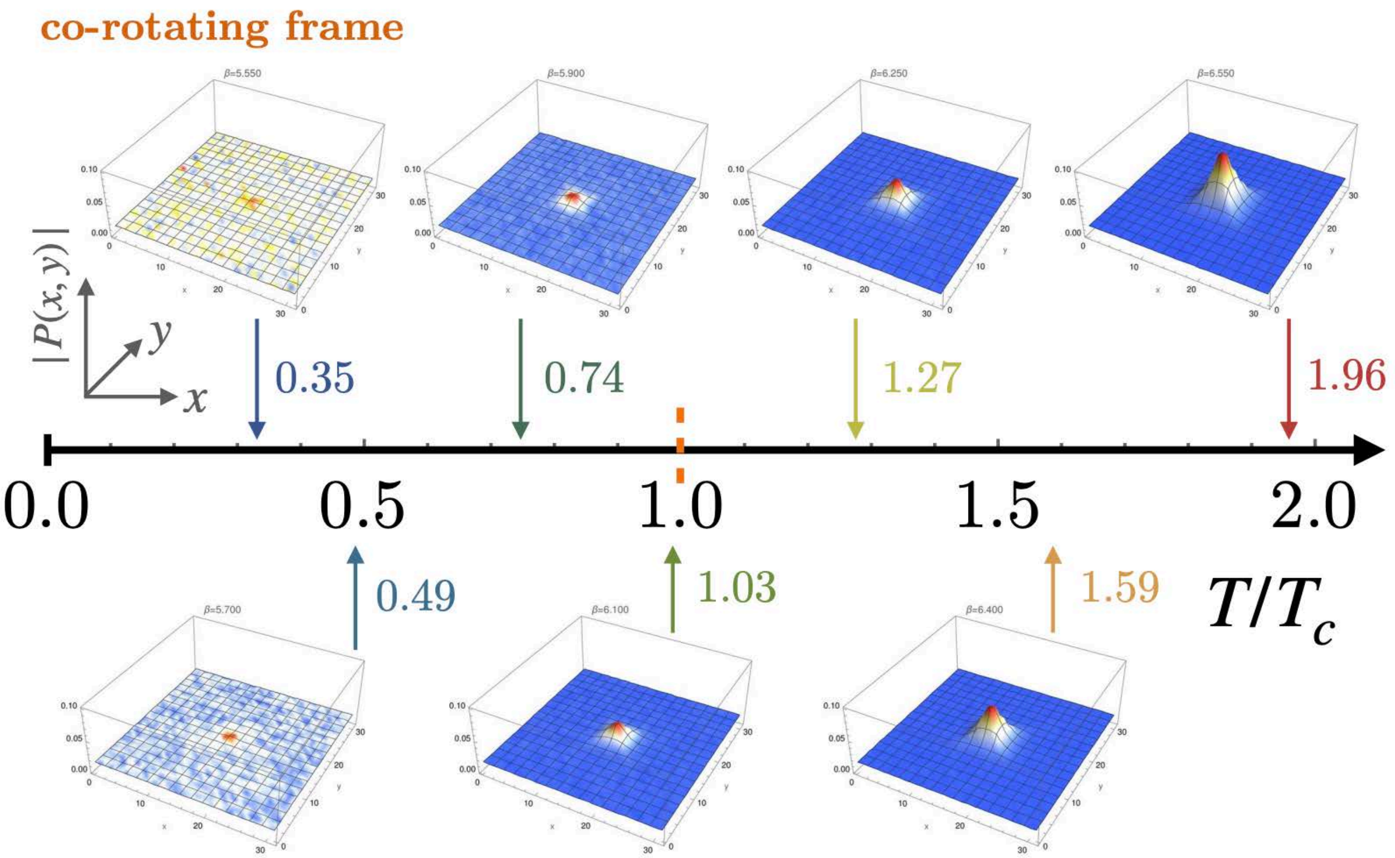
co-rotating frame

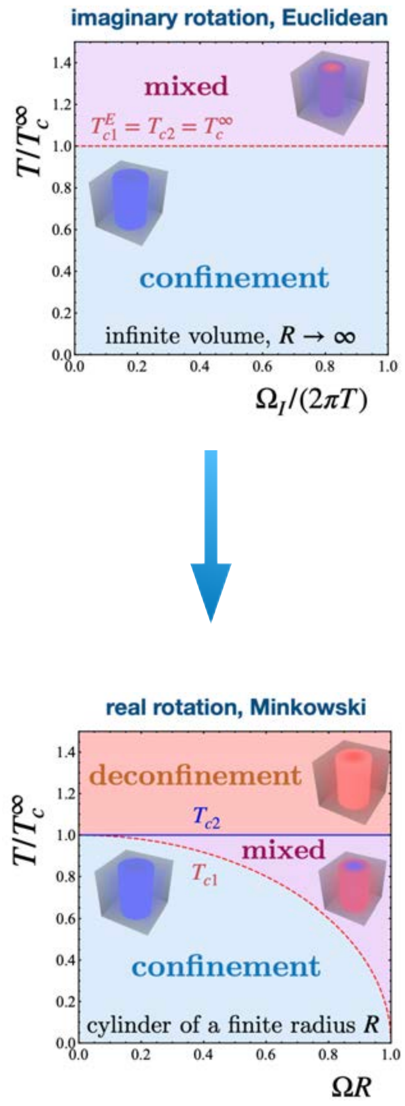
all loops are sensitive to center symmetry:

$$\mathcal{P} \to e^{\frac{2\pi i}{3} n} \mathcal{P}\,, \qquad \mathcal{P}_4 \to e^{4 \frac{2\pi i}{3} n} \mathcal{P}_4\,, \qquad n = 0, 1, 2$$

Polyakov loop in co-rotating frame

SU(3) lattice gauge theory at finite temperature



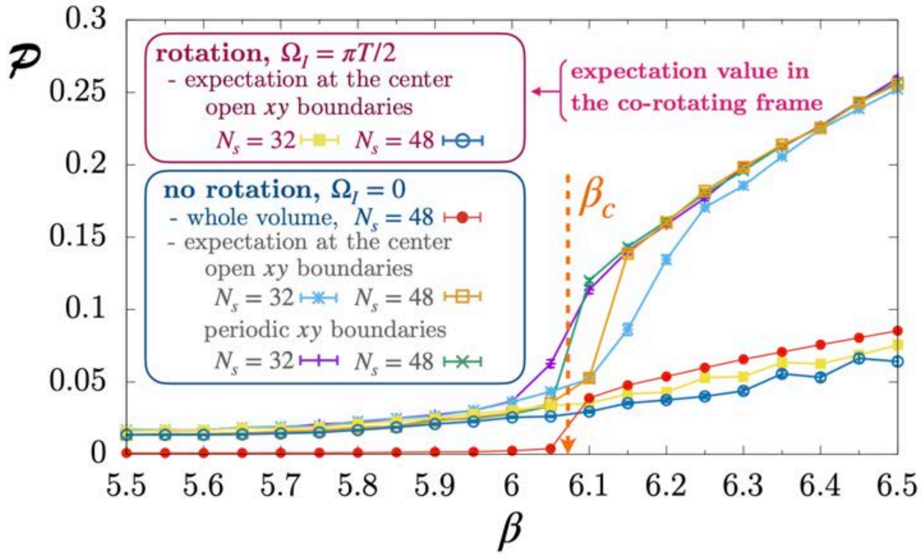


Expected from theory: confirmation of the formation of the new inhomogeneous confining-deconfining phase of quark-gluon plasma subjected to rigid rotation.

Co-rotating vs laboratory frames

co-rotating frame

in the center of rotating plasma



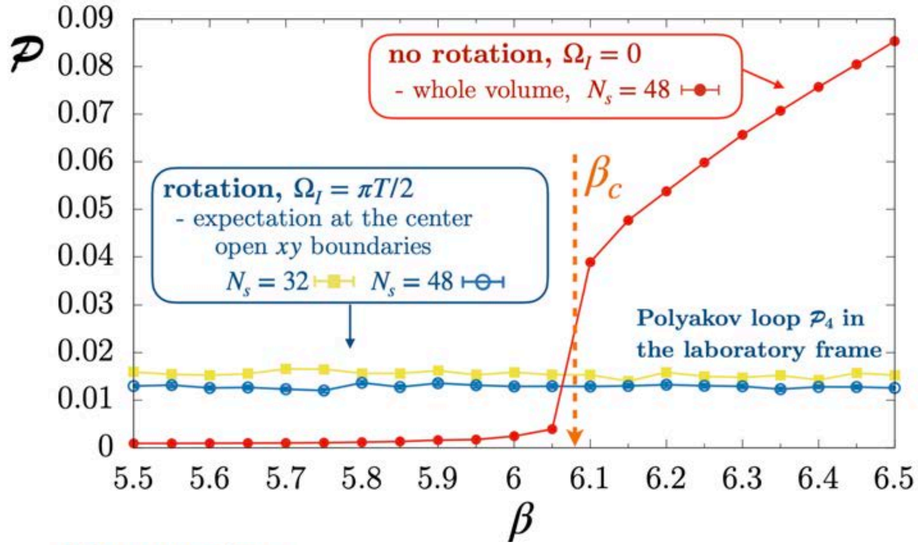
the global rotation drastically softens the confinement-deconfinement phase transition at the rotation center (1st order becomes a very weak crossover)

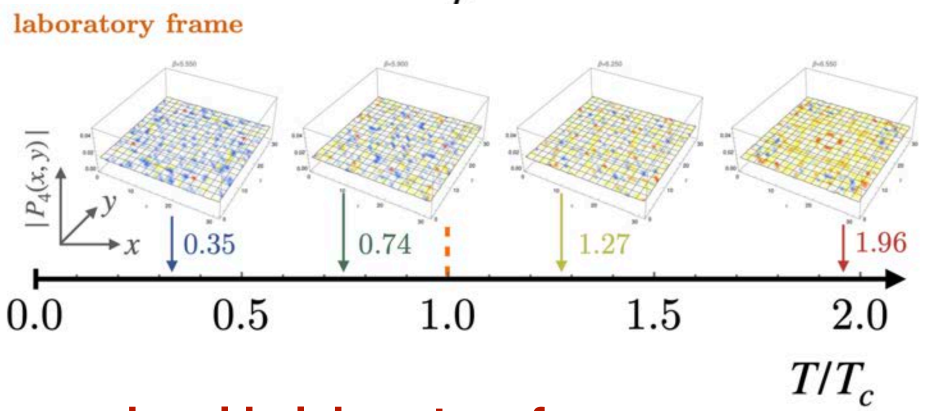
somewhat contradicts the perturbative analysis

Confinement in thermal Yang-Mills theories with imaginary and real rotation
Shi Chen (Tokyo U.), Kenji Fukushima (Tokyo U.), Yusuke Shimada (Tokyo U.) (Jul 26, 2022)
e-Print: 2207.12665 [hep-ph]

laboratory frame

in the center of rotating plasma



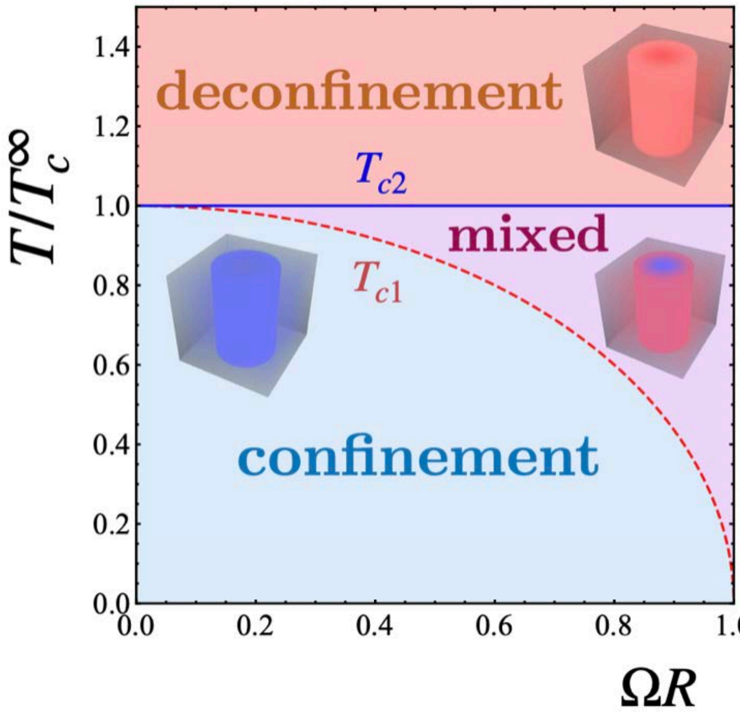


no signal in laboratory frame

Conclusions

— **Current state of the art:**
the phase diagram of quark-gluon plasma is not understood.

— **Indications from the lattice that the global rotation of quark-gluon plasma leads to the formation of the new inhomogeneous phase**



Not in the talk:

— **Imaginary rotation leads to fractal properties of thermodynamics.**

— **No-go for analytical continuation between imaginary and real rotations in thermodynamic limit.**
Good news: at finite volume, the no-go theorem does not work.