SIGNATURES OF THE YANG-MILLS DECONFINEMENT TRANSITION FROM THE GLUON TWO-POINT CORRELATOR

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arXiv:2206.03841 [hep-ph]

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- Background field methods
- Background effective action
- Center-symmetric effective action
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INTRODUCTION

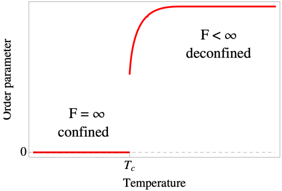
- In general: insight in the low-energy regime of QCD, especially in the confinement/deconfinement transition.
- Most results are coming from non-perturbative, numerical (lattice) or semi-analytical (FRG and SD) methods.
- From this, we know that:
  - At some very high temperature T_c, hadrons become free quarks and gluons $\rightarrow$ quark-gluon plasma.
  - This transition is related to the breaking of the center symmetry of a gauge group when using pure Yang-Mills theory (infinitely heavy quarks):

$$\mathcal{L} = \frac{1}{4}(F^a_{\mu\nu})^2$$

CENTER SYMMETRY

- The order parameter for the confinement/deconfinement transition is the Polyakov loop:

$$\mathcal{P} \propto \langle P e^{i\int_0^\beta d\tau A_0(\tau,x)} \rangle \propto e^{-\beta F}$$



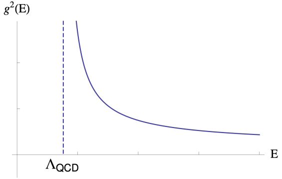
- In the confined phase, F is infinite $\rightarrow \mathcal{P} = 0$. In the deconfined phase, F is finite $\rightarrow \mathcal{P} \neq 0$.
- Under center symmetry $\mathcal{P} \rightarrow Z_N \mathcal{P}$, with Z_N the center elements of the gauge group. So, breaking of the center symmetry signals deconfinement.
- Confirmed by lattice data: second order transition for $SU(2)$, first order for $SU(3)$.

ANALYTICAL RESULTS

- At high energies, gluon dynamics are well described by an $SU(3)$ pure Yang-Mills action with a Faddeev-Popov gauge fixing:

$$\mathcal{L} = \frac{1}{4}(F^a_{\mu\nu})^2 + \partial_\mu \bar{c}^a (D_\mu c)^a + i b^a \partial_\mu A^a_\mu$$

- At high energies, the coupling constant decreases: Asymptotic freedom.
- At low energies, the coupling constant increases, and diverges: Landau pole.



- Does this mean we have an infinite coupling at low energies? Probably not!

GRIBOV PROBLEM

- Gribov: for high values of the coupling constant, the FP gauge fixing does not uniquely fix the gauge field

$$A'^a_\mu = A^a_\mu - D^{ab}_\mu \alpha^b \qquad \partial_\mu A^a_\mu = \partial_\mu {A^a}'_\mu = 0$$

  so that

$$\partial_\mu D^{ab}_\mu \alpha^b = (\partial^2 \delta^{ab} - g f^{abc} \partial_\mu A^c_\mu)\alpha^b = 0$$

- An analytic model of the IR regime should restrict the number of Gribov copies:
  - Gribov-Zwanziger model:

$$\mathcal{L} = \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \bar{c}^a \partial_\mu D^{ab}_\mu c^b + i b^a \partial_\mu A^a_\mu - \bar{\omega}^{ae}_\nu \partial_\mu D^{ab}_\mu \omega^{be}_\nu$$
$$+ \bar{\varphi}^{ae}_\nu \partial_\mu D^{ab}_\mu \varphi^{be}_\nu - g\gamma^{1/2} f^{abc} A^a_\mu \left(\varphi^{bc}_\mu + \bar{\varphi}^{bc}_\mu\right) - \gamma d d_G$$

  - Curci-Ferrari model:

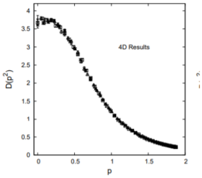
$$\mathcal{L} = \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \bar{c}^a \partial_\mu D^{ab}_\mu c^b + i b^a \partial_\mu A^a_\mu + m^2 A^a_\mu A^a_\mu$$

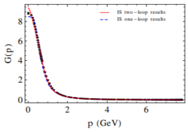
CURCI-FERRARI MODEL

- $\mathcal{L} = \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \bar{c}^a \partial_\mu D^{ab}_\mu c^b + i b^a \partial_\mu A^a_\mu + m^2 A^a_\mu A^a_\mu$
- Gluons do not have a mass perturbatively
- BRST-symmetry that defines the physical space is broken
- However, for $p \gg m$, $m \approx 0$
- Confined gluons do not have a physical interpretation, so BRST symmetry might be broken non-perturbatively

CURCI-FERRARI MODEL

- $\mathcal{L} = \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \bar{c}^a \partial_\mu D^{ab}_\mu c^b + i b^a \partial_\mu A^a_\mu + m^2 A^a_\mu A^a_\mu$
- Lattice results show that the gluon propagator saturates in the IR





- The CF model accounts very nicely for the lattice result, is IR safe and higher order terms are surpressed by the mass term: perturbation theory in the non-perturbative region.
- Can we use the CF (or GZ) model to describe the confinement/deconfinement transition? In principle: Yes. In practice: Maybe.

ENCODING OF THE TRANSITION

Polyakov loop:

$$\mathcal{P} \sim \left\langle P e^{i \int_0^\beta d\tau A_0(\tau, x)} \right\rangle \sim e^{-\beta F}.$$

Because the Polyakov loop is related to A_0, it is expected that the transition is encoded in (the tower of)

$$\langle A_0 \rangle, \langle A_0 A_0 \rangle, \ldots, \langle A_0^n \rangle.$$

For the appropriate choice of gauge, can the transition be reflected in the lowest order correlators?

LANDAU GAUGE CORRELATOR

In principle:

- $\langle A \rangle$ is found by minimizing the effective action $\Gamma[A]$. It represents the state of the system . $\langle A_0 \rangle \rightarrow$ order parameter.
- The two-point correlator derives from the effective action

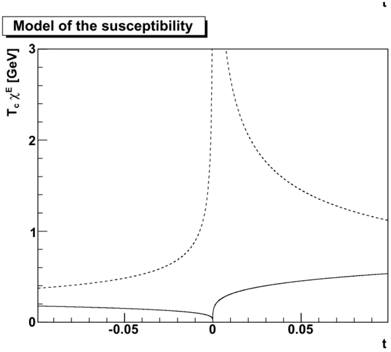
$$1 \bigg/ \left. \frac{\partial^2 \Gamma}{\partial A^2} \right|_{A=\langle A \rangle} = \langle AA \rangle_c,$$

so for $SU(2)$, $\langle A_0 A_0 \rangle$ should diverge at T_c.

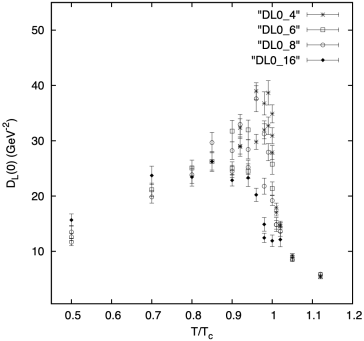
In practice:

- In the Landau gauge, $\partial_\mu A_\mu = 0$, then $\langle A_0 \rangle = 0$. $\rightarrow$ no order parameter.
- No evidence of divergence of $\langle A_0 A_0 \rangle$ was found on the (gauge-fixed) lattice and in the continuum.

$SU(2)$ LANDAU GAUGE CORRELATORS

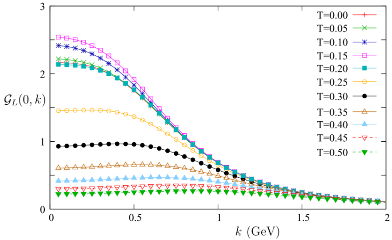


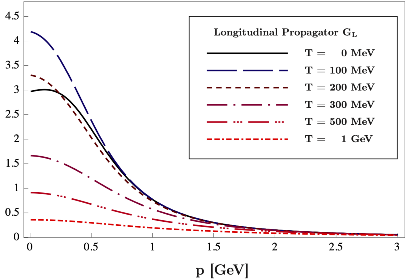
Electric susceptibility (zero momentum longitudinal propagator)



*T. Mendes and A. Cucchieri, PoS LATTICE2014, 183 (2015).

$SU(2)$ LANDAU GAUGE CORRELATORS





Longitudinal gluon propagator

*U. Reinosa, J. Serreau, M. Tissier, N. Wschebor, Phys. Rev. D, (2015)

*L. Fister and J. M. Pawlowski, [arXiv:1112.5440 [hep- ph]](2012).

BACKGROUND FIELD GAUGES

- A priori there is no reason to believe that the Landau gauge will provide the right environment to keep track of the centersymmetry breaking.
- In the Landau gauge the effective action is not explicitly center-symmetric $\Gamma[A] \neq \Gamma[A^U]$.
- This should not alter the physical results in principle, but it can lead to problems when approximations (loop calculations) are involved.
- To regain gauge invariance, one solution is to work with the Background Field Gauges.

BACKGROUND FIELD GAUGES

To retain the center symmetry, we introduce the the Landau-DeWitt gauge

$$\overline{D}_\mu(A_\mu - \overline{A}_\mu) = 0,$$

with $\overline{A}_\mu$ an arbitrary background field and $\bar{D}_\mu \equiv \partial_\mu - \left[\bar{A}_\mu, \cdot\right]$. We consider the action

$$S[A,\overline{A}] = \int_x \left\{ \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \overline{D}_\mu \overline{c}^a D_\mu c^a + i b^a \overline{D}_\mu (A^a_\mu - \overline{A}^a_\mu) + m^2 (A^a_\mu - \overline{A}^a_\mu) \right\},$$

which is invariant under the simultaneous $SU(N)$ transformation of the fields A^a_μ and $\overline{A}^a_\mu$

$$S[A^U, \overline{A}^U] = S[A,\overline{A}],$$

and center symmetry is preserved.

BACKGROUND FIELD GAUGES

$$\begin{aligned} S[A^U, \bar{A}^U] &= S[A, \bar{A}] \\ \Gamma[A^U, \bar{A}^U] &= \Gamma[A, \bar{A}] \end{aligned}$$

Checking the invariance under center transformations of $A_{\mathrm{min}}[\bar{A}]$ is ambiguous:

$$\Gamma[A_{min}(\bar{A}), \bar{A}] = \Gamma[A^U_{min}(\bar{A}), \bar{A}^U].$$

Transforming the minimizing state transforms it into the minimizing function of another potential, with another gauge fixing. Therefore, we cannot identify the center-symmetric states.

Background effective action

In the method of the BG effective action[1], we define a new object

$$\tilde{\Gamma}[\bar{A}] \equiv \Gamma[A = \bar{A}, \bar{A}],$$

and we can show that $\bar{A}_{min}$ such that

$$\tilde{\Gamma}[A_{min}(\bar{A})] \leq \tilde{\Gamma}[\bar{A}], \quad \forall \bar{A}$$

are alternative order parameters for center symmetry. There is no ambiguity anymore since:

$$\tilde{\Gamma}[\bar{A}] = \tilde{\Gamma}[\bar{A}^U].$$

The drawback of the BG method is that $\tilde{\Gamma}[\bar{A}]$ is a formal object an does not relate directly to gauge-fixed quantities such as propagators and vertices.

[1] J. Braun, H. Gies and J.M. Pawlowski, Phys. Lett. B 684,262-267 (2010)

Center-symmetric effective action

In the solution of the CS effective action , we define the center symmetric state A_c and make the gauge choice $\overline{A} = A_c$ and define

$$\Gamma_c[A] = \Gamma[A, \overline{A} = A_c].$$

Now

$$\Gamma_c[A] \neq \Gamma_c[A^U], \quad \forall U$$

but

$$\Gamma_c[A] = \Gamma_c[A^{U^c}], \quad \forall U^c$$

with U^c the center transformations.

There is no ambiguity since the background field $\overline{A}$ is fixed.

Since $\Gamma_c[A]$ is nothing but a particular gauge choice of $\Gamma[A, \overline{A}]$, we can directly reach gauge-fixed quantities such as propagators and vertices.

OUR SETUP

- We work in the Landau-deWitt gauge with a background field $\bar{A}_\mu$:

$$\bar{D}_\mu(A_\mu - \bar{A}_\mu) = 0, \text{ with } \bar{D}_\mu \equiv \partial_\mu - \left[\bar{A}_\mu, \cdot\right].$$

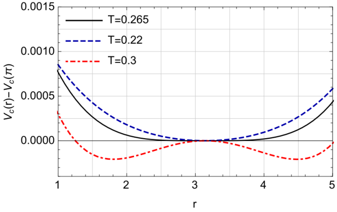
- We take $\bar{A}$ and $\langle A \rangle$ in the temporal direction, $\propto \delta_{\mu 0}$, and along the diagonal color directions (σ^3 for $SU(2)$, (λ^3, λ^8) for $SU(3)$), so that $\Gamma[A, \bar{A}] \propto V(A, \bar{A})$. We write $\langle A \rangle = \delta_{\mu 0} \frac{T}{g} r \frac{\sigma^3}{2}$ and $\bar{A} = \delta_{\mu 0} \frac{T}{g} \bar{r} \frac{\sigma^3}{2}$.
- For example in $SU(2)$, under center symmetry $r \to 2\pi - r$ so centersymmetric value is $r_c = \pi$. For $SU(3)$, we $r_c = (3/4\pi, 0)$
- We fix $\bar{r} = r_c$: Center-symmetric Landau gauge. Center-symmetric phase when $r = r_c$, $\to$ order parameter.

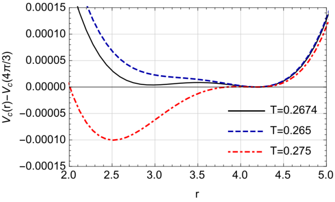
CURCI-FERRARI MODEL

We have computed $\langle A \rangle$ and $\langle A(0,p)A(0,-p) \rangle$ up to first loop order in the finite temperature Curci-Ferrari model:

$$S = S_{YM} + S_{gf} + \int_{x,\tau} \frac{m^2}{2}(A^a_\mu - \bar{A}^a_\mu)^2$$

We looked at the one-loop potential to find $\langle A \rangle$ and T_c





RESULTS - $T_c(MeV)$

	Lattice	FRG-BG[2]	CF-BG, 1-lp[3]	CF-BG, 2-lp[4]	CF-CS, 1-lp[5]
SU(2)	295	230	238	284	**265**
SU(3)	270	275	185	254	**267**

BG: Background effective action
CS: Centersymmetric Landau gauge
.

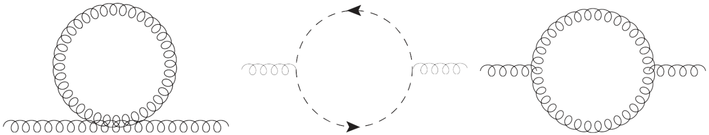
[2] L. Fister and J. M. Pawlowski, Phys.Rev. D88 (2013) 045010

[3] U. Reinosa, J. Serreau, M. Tissier and N. Wschebor, Phys.Lett. B742 (2015) 61-68.

[4] U. Reinosa, J. Serreau, M. Tissier and N. Wschebor, Phys.Rev. D93 (2016) 105002.

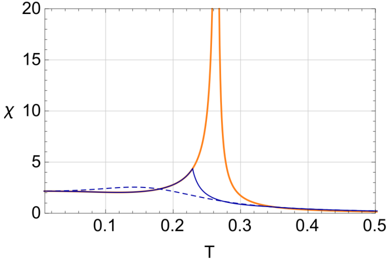
[5] **DvE, U. Reinosa, J. Serreau and M. Tissier, SciPost Phys. 12, 087 (2022)**

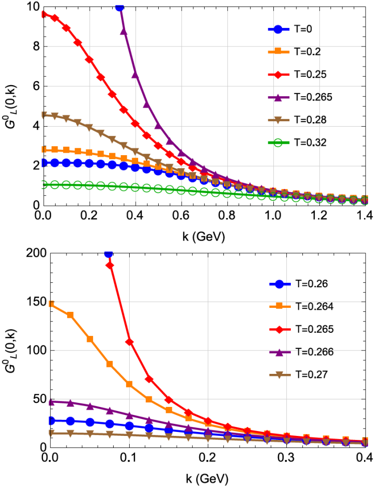
FEYNMAN DIAGRAMS GLUON PROPAGATOR



- Calculated in finite temperature trough Matsubara techniques: $\int d^dQ \rightarrow T\sum_q \int d^{d-1}q$
- We calculated the spatial integral with Feynman techniques, the Matsubara sums numerically.

RESULTS: SU(2) GLUON PROPAGATOR



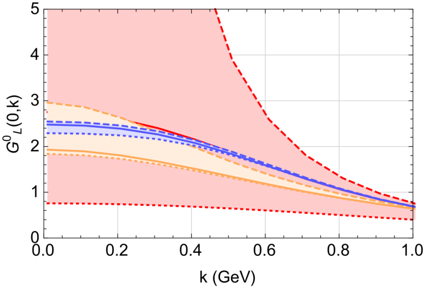


Landau gauge
Background field effective action
Centersymmetric Landau gauge

m=0.68 GeV, μ=1 GeV, g=7.5

*DvE, U. Reinosa, J. Serreau and M. Tissier, SciPost Phys. 12, 087 (2022).

RESULTS: SU(2) GLUON PROPAGATOR

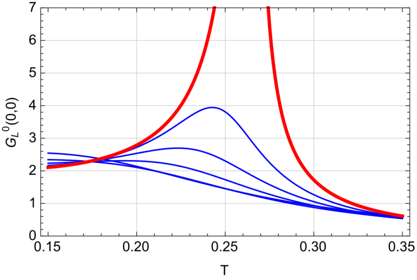


Landau gauge

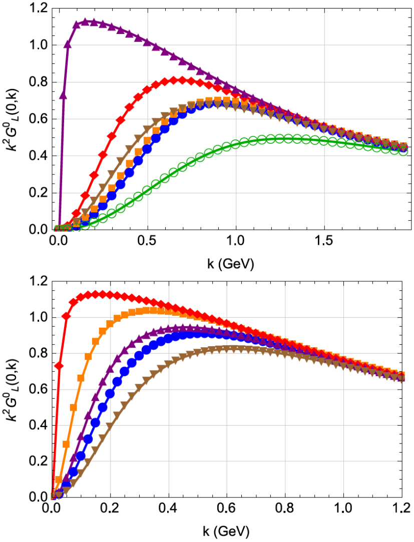
Background Field Effective action

Center-symmetric Landau Gauge

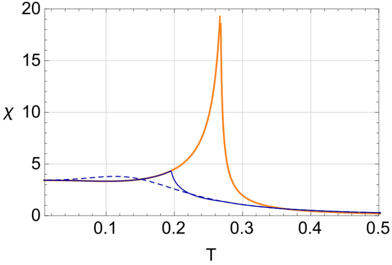
RESULTS: SU(2) GLUON PROPAGATOR

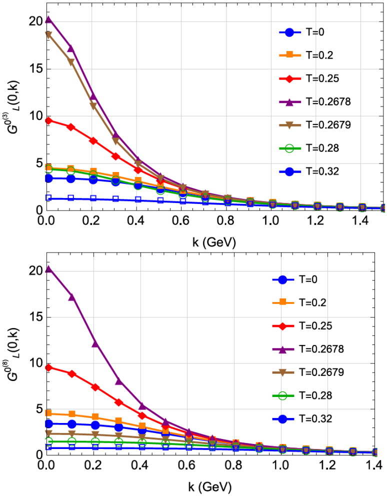


SU(2) DRESSING FUNCTION



RESULTS: SU(3) GLUON PROPAGATOR





Landau gauge

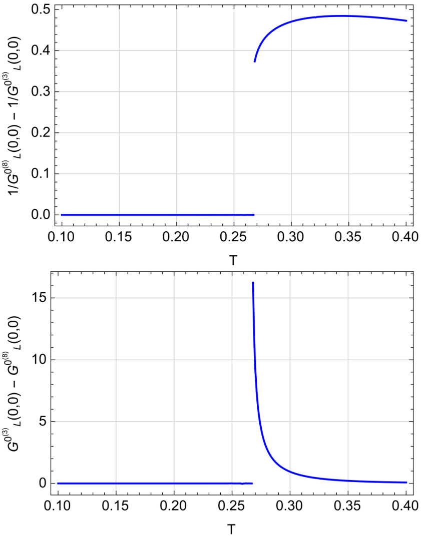
Background field effective action

Centersymmetric Landau gauge

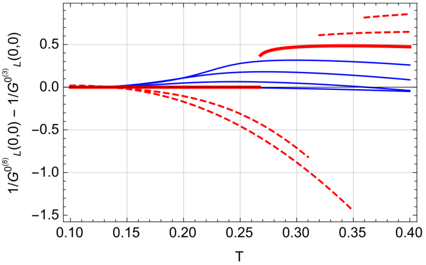
m=0.54 GeV, μ=1 GeV, g=4.9

*DvE, U. Reinosa, J. Serreau and M. Tissier, SciPost Phys. 12, 087 (2022).

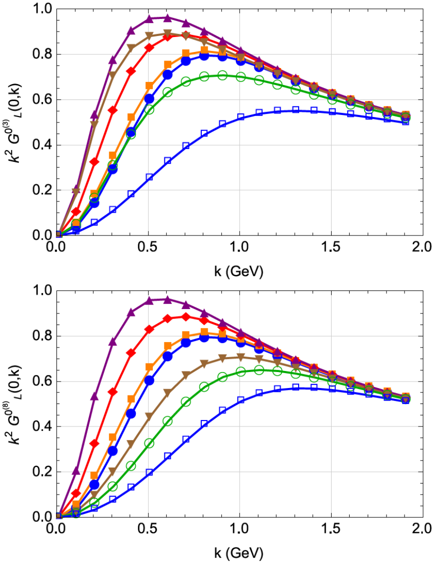
SU(3) PROPAGATOR DIFFERENCE



SU(3) PROPAGATOR DIFFERENCE



SU(3) DRESSING FUNCTION



CONCLUSION AND OUTLOOK

- We have performed, for the first time, calculations of the gluon one-and two-point correlator in the centersymmetric Landau gauge.
- We find a good agreement with lattice data for T_c.
- We find that for $SU(2)$, the deconfinement transition is signaled by a divergence of the longitudinal gluon propagator for $k \to 0$.
- For $SU(3)$, the difference between the propagators in the neutral color mode is an order parameter for the transition.
- This model can be tested on the lattice by changing the boundary conditions in the Landau gauge [with O. Oliveira and P. Silva].
- Ideas for future works: RG improvement, transversal propagator and dynamically generated mass [with D. Dudal and D. Vercauteren].