Jet quenching in evolving media

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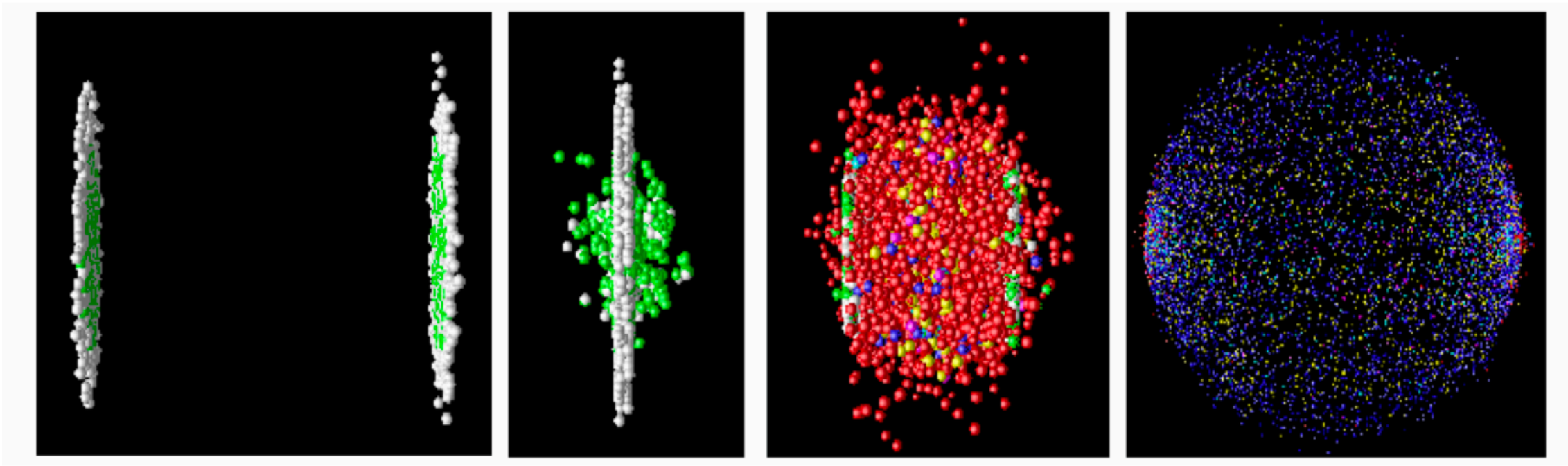
CPHT, École polytechnique
RPP2022, Tours, 5 - 7 December 2022

Heavy-ion collisions

- 20 years of HICs at RHIC and 10 years of HICs at the LHC



- Why? To study the QGP!

- How? Using probes sensitive to the QCD matter
- Under experimental control
- Theoretically modeled
- Connect theory and experiments

Such as jets/high-p_T particles

Why hard probes?

- Large energy or mass $\longrightarrow$ pQCD applies!

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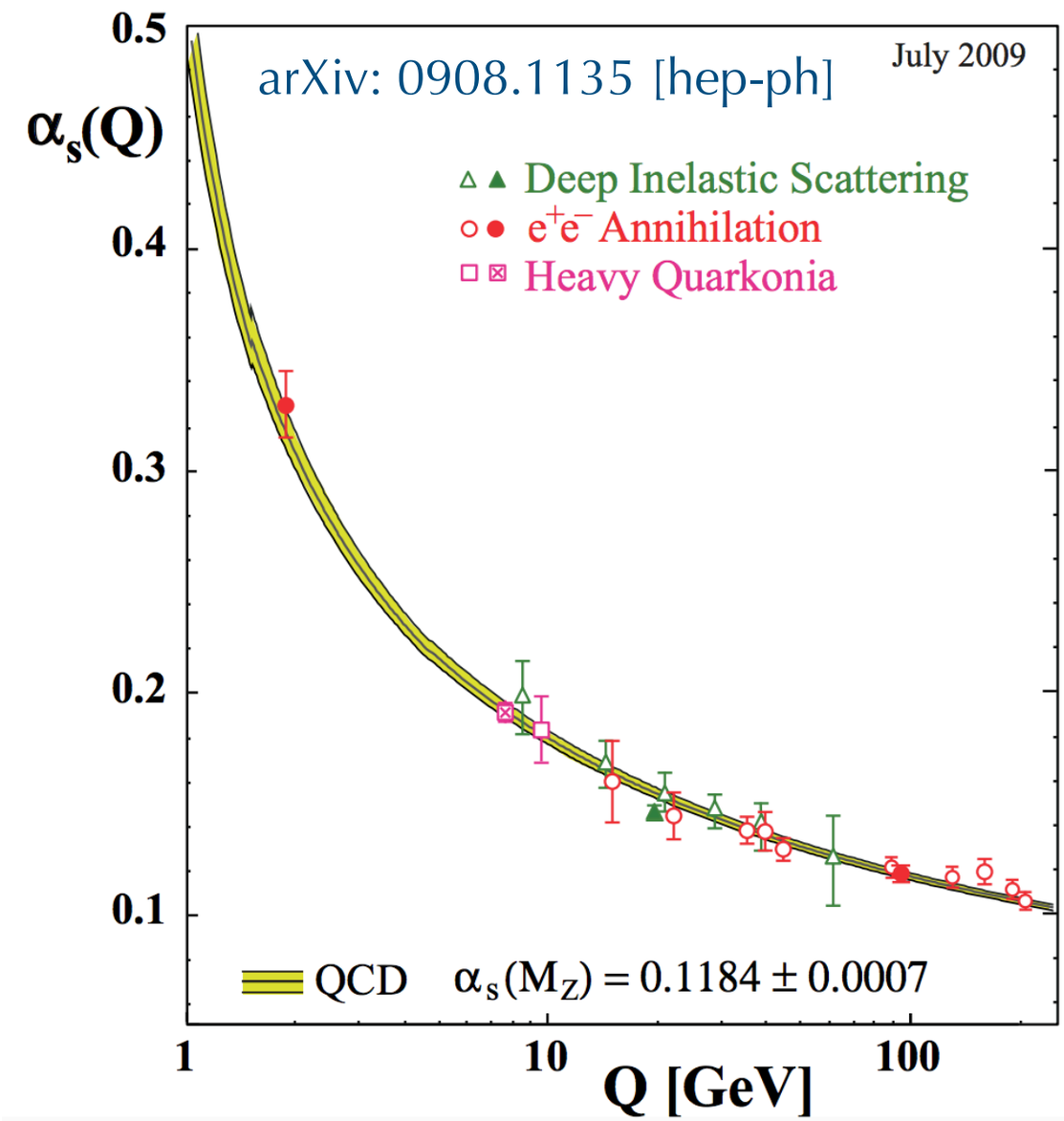

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- Created before the formation of the QGP ⟶ they traverse the QGP

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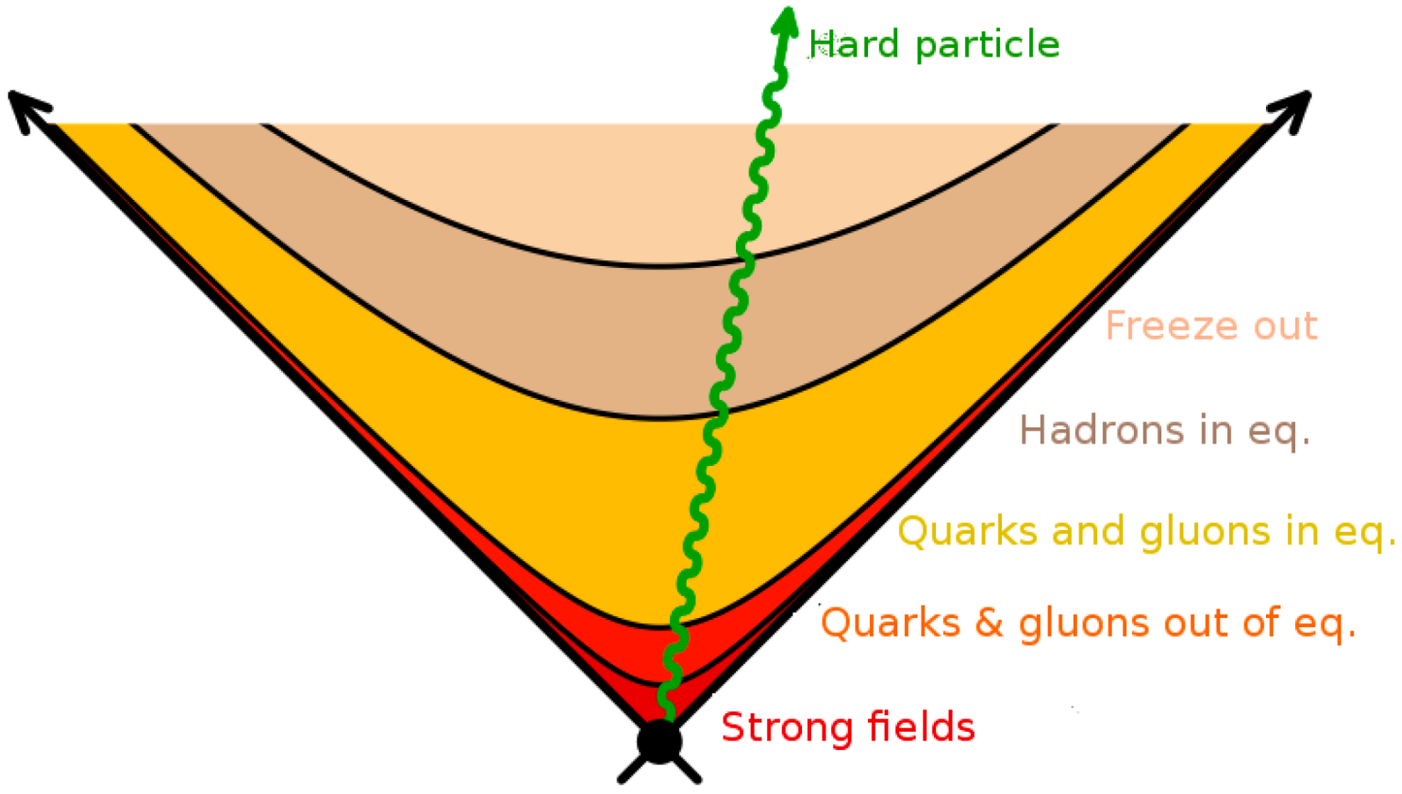
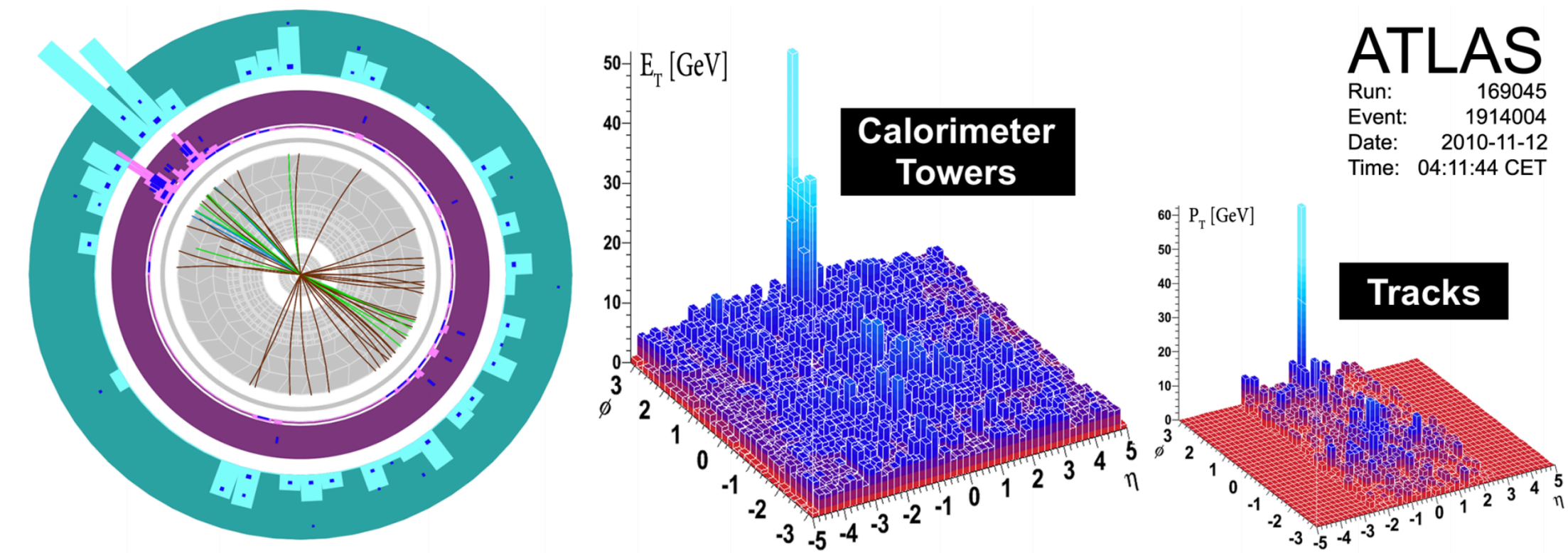
↓

Enable the extraction of properties of the medium!



Jet quenching

- Jet quenching: high-energy partons interact with the QGP losing energy

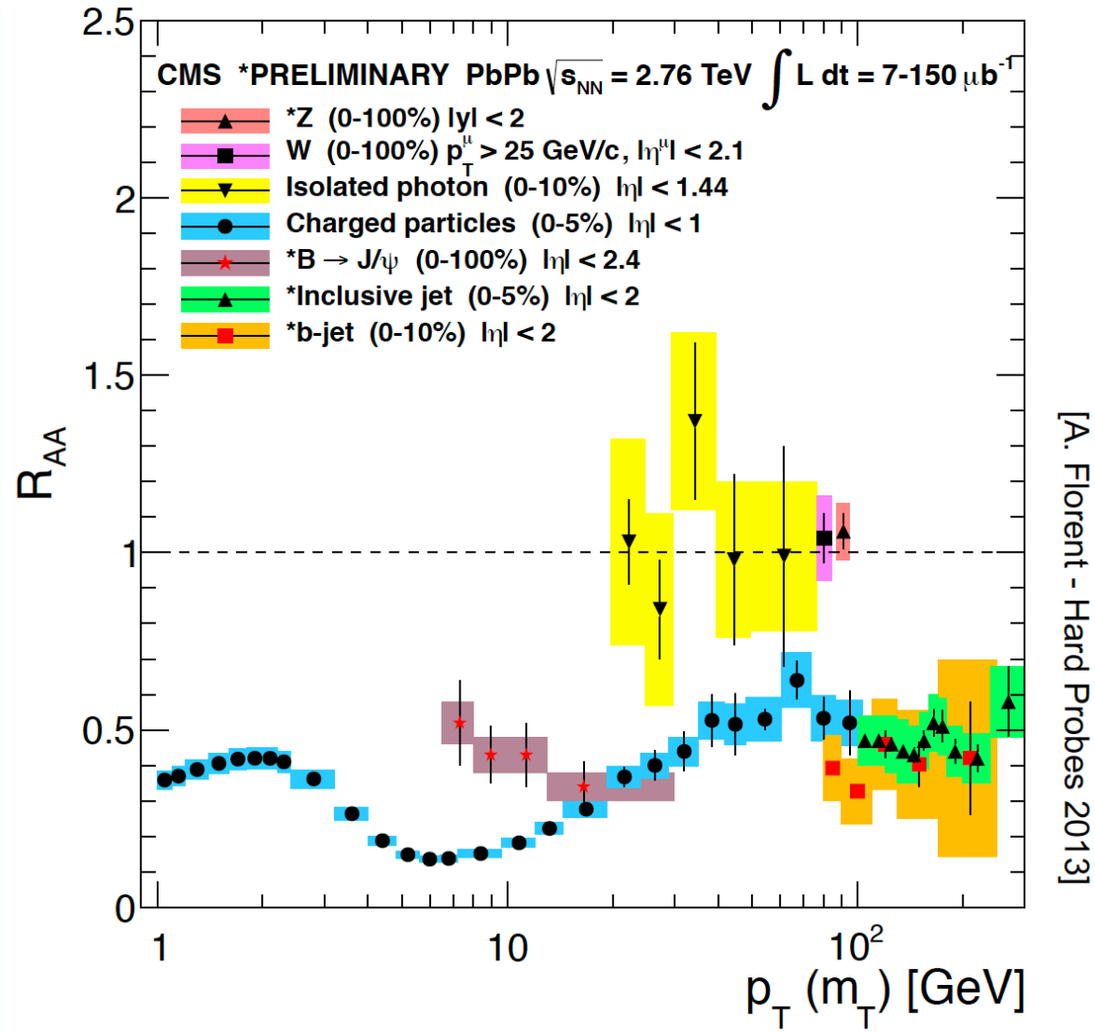


arXiv:1011.6182 [hep-ex]

Highly asymmetric dijet event

Jet quenching

$$R_{AA} = \frac{dN_{AA}/d^2p_T\,dy}{\langle N_{coll}\rangle\, dN_{pp}/d^2p_T\,dy}$$



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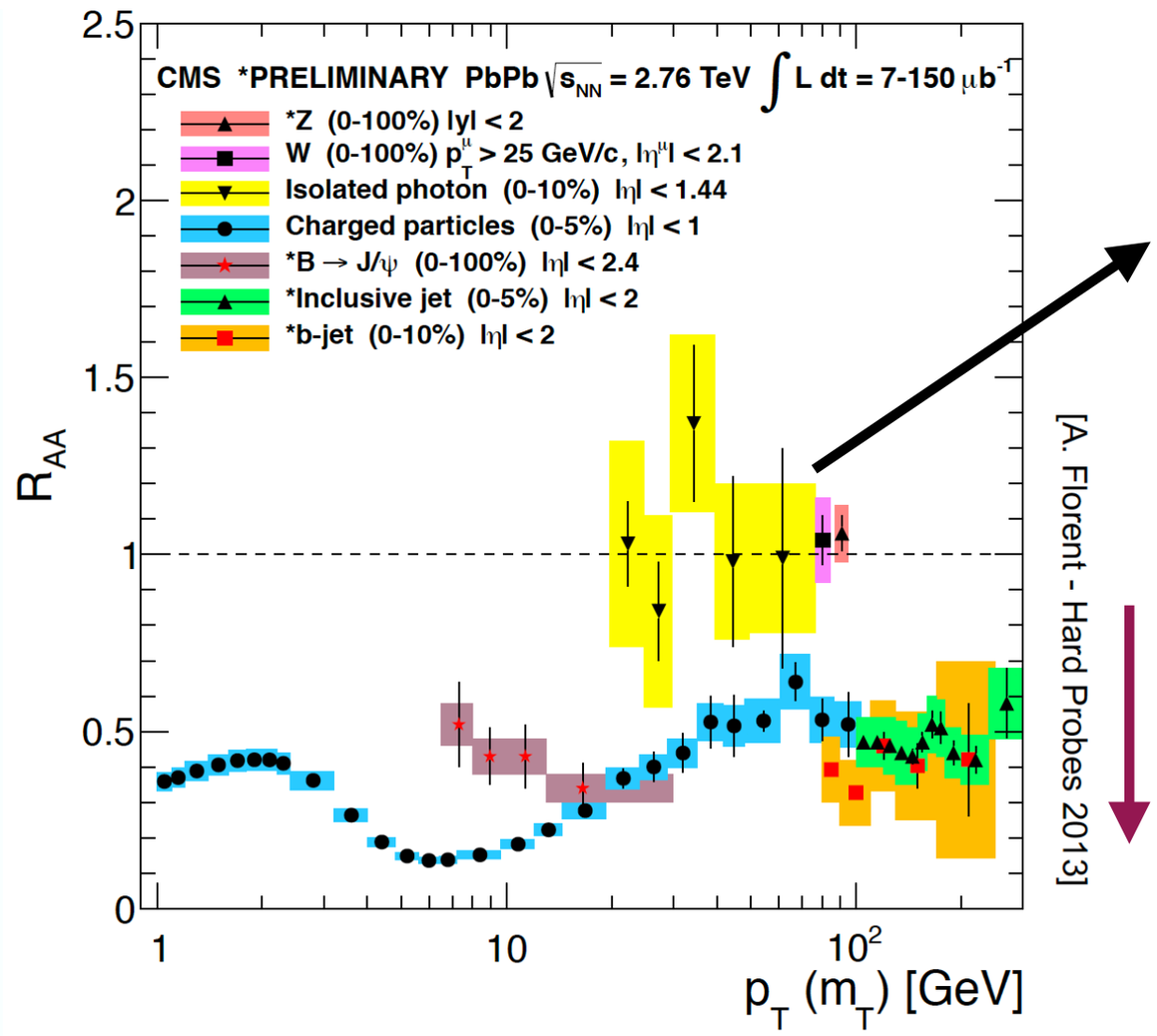

Colorless probes: no suppression

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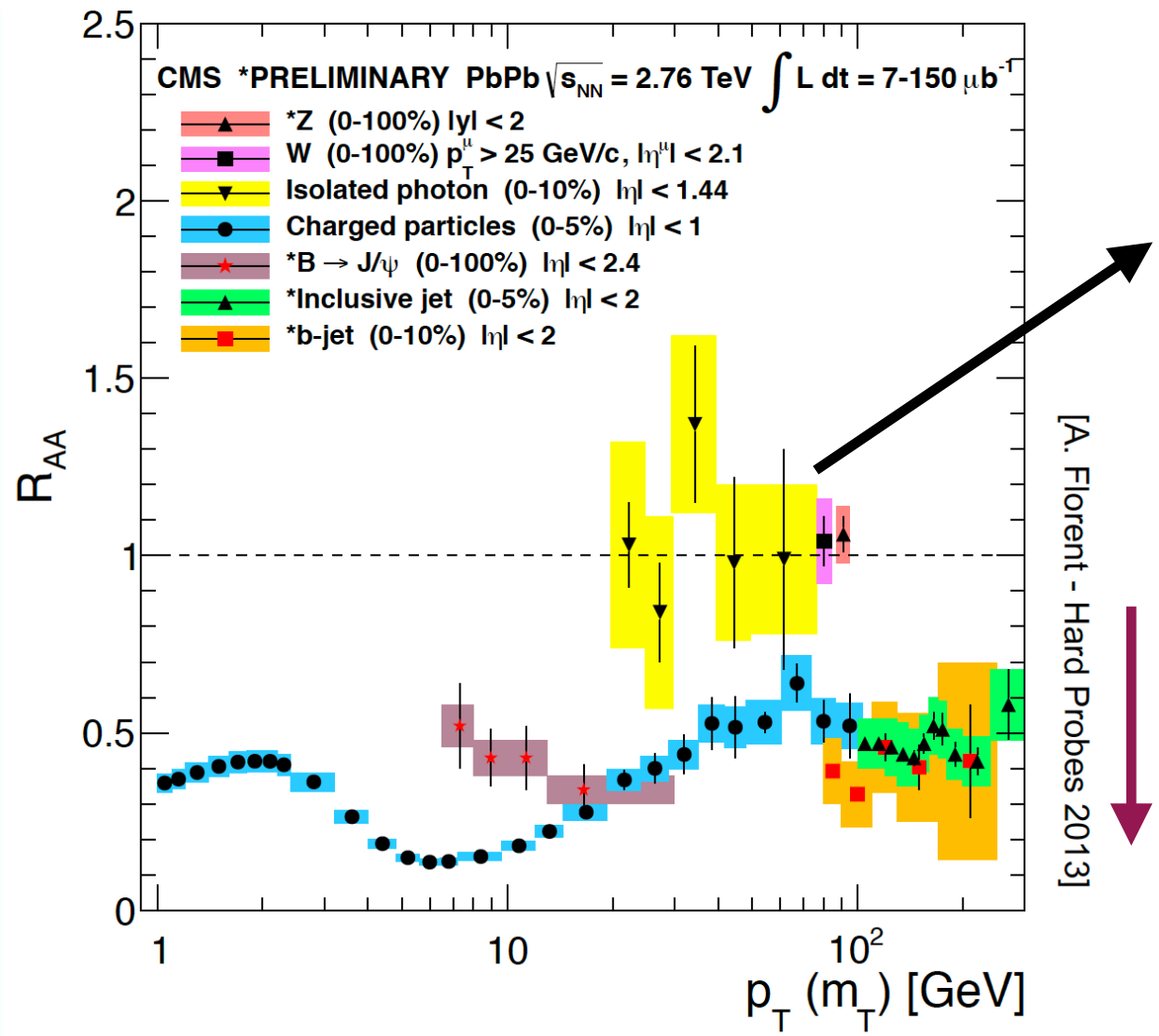
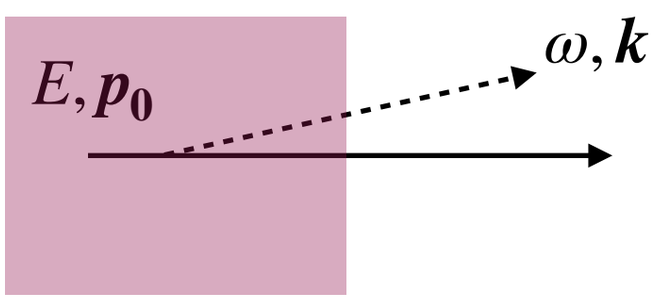
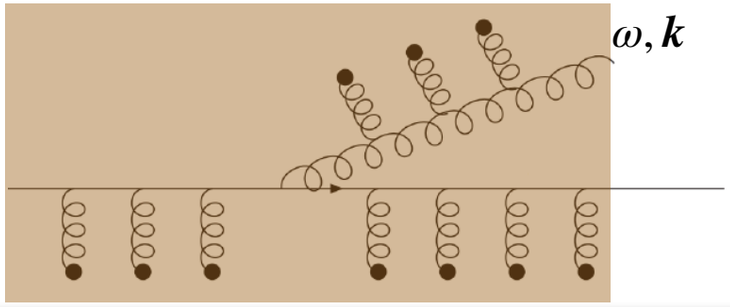


Colorless probes: no suppression

Jet quenching

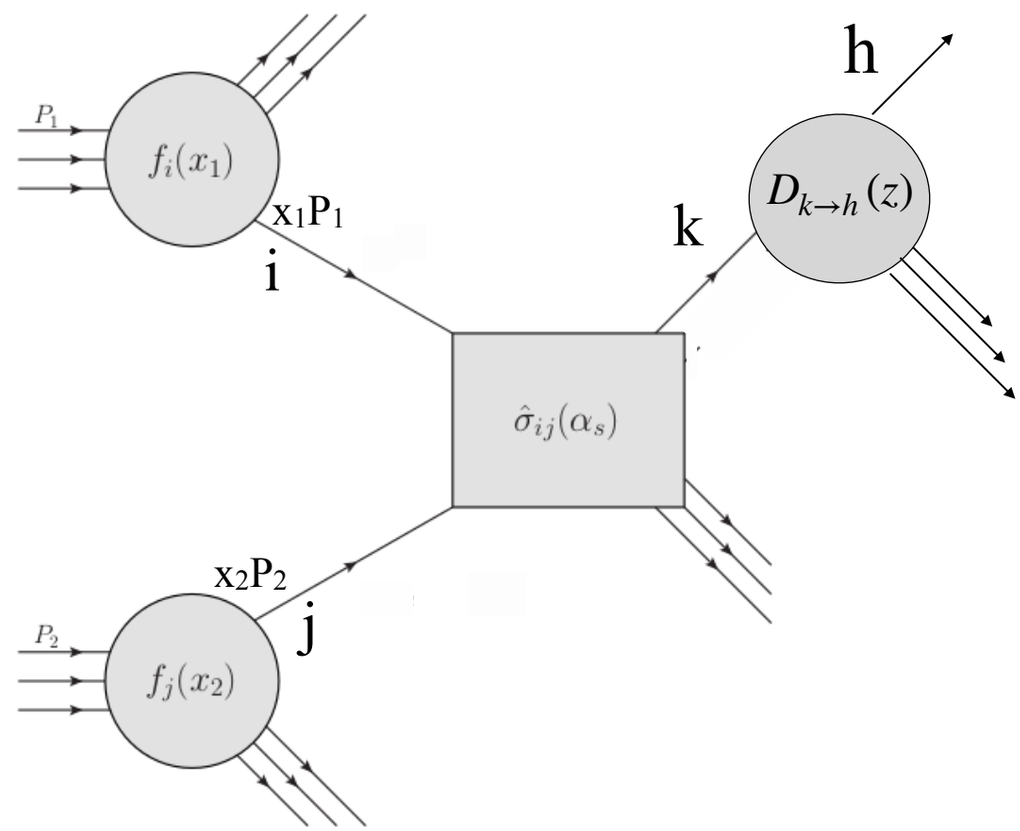
- The principal mechanism of energy loss is medium-induced radiation





How? Collinear Factorization

$$d\sigma^{A\,A\to h\,X} = \sum_{ijk} f_i(x_1,\mu_f^2) \otimes f_j(x_2,\mu_f^2) \otimes d\sigma_{ij\to k} \otimes D_{k\to h}(z,\mu_f^2) + \mathcal{O}(\mu_f^{-2n})$$



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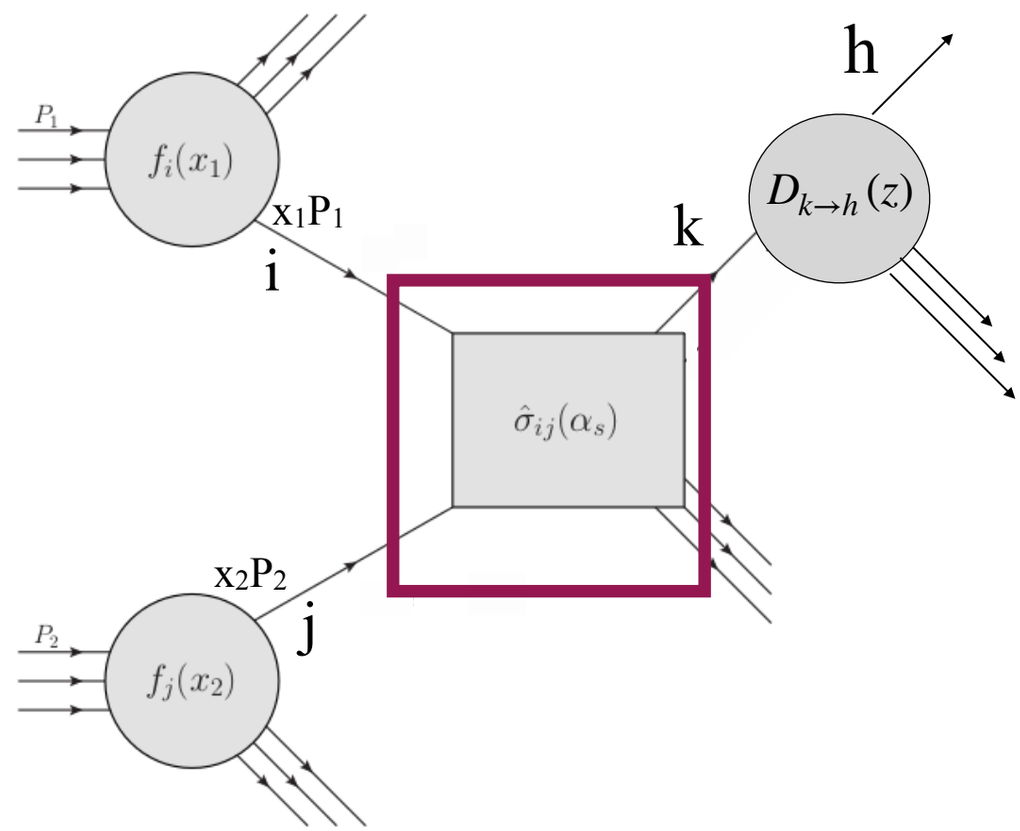

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(n)PDFs pQCD

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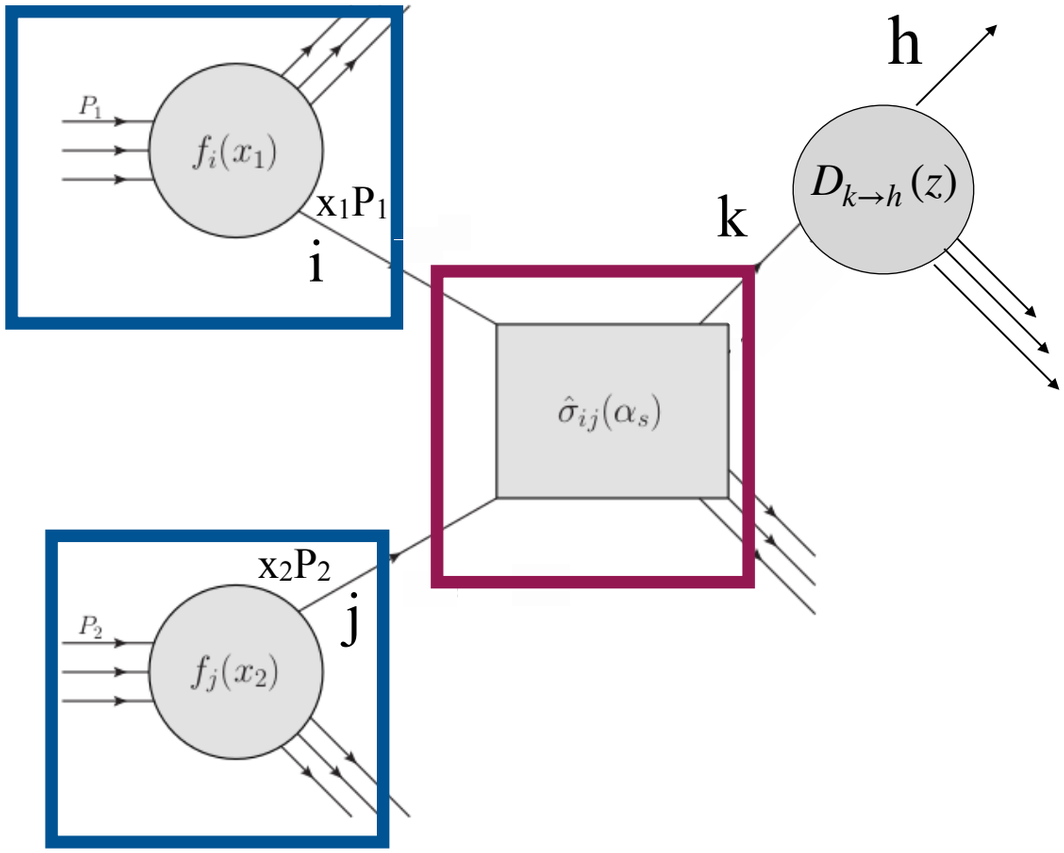

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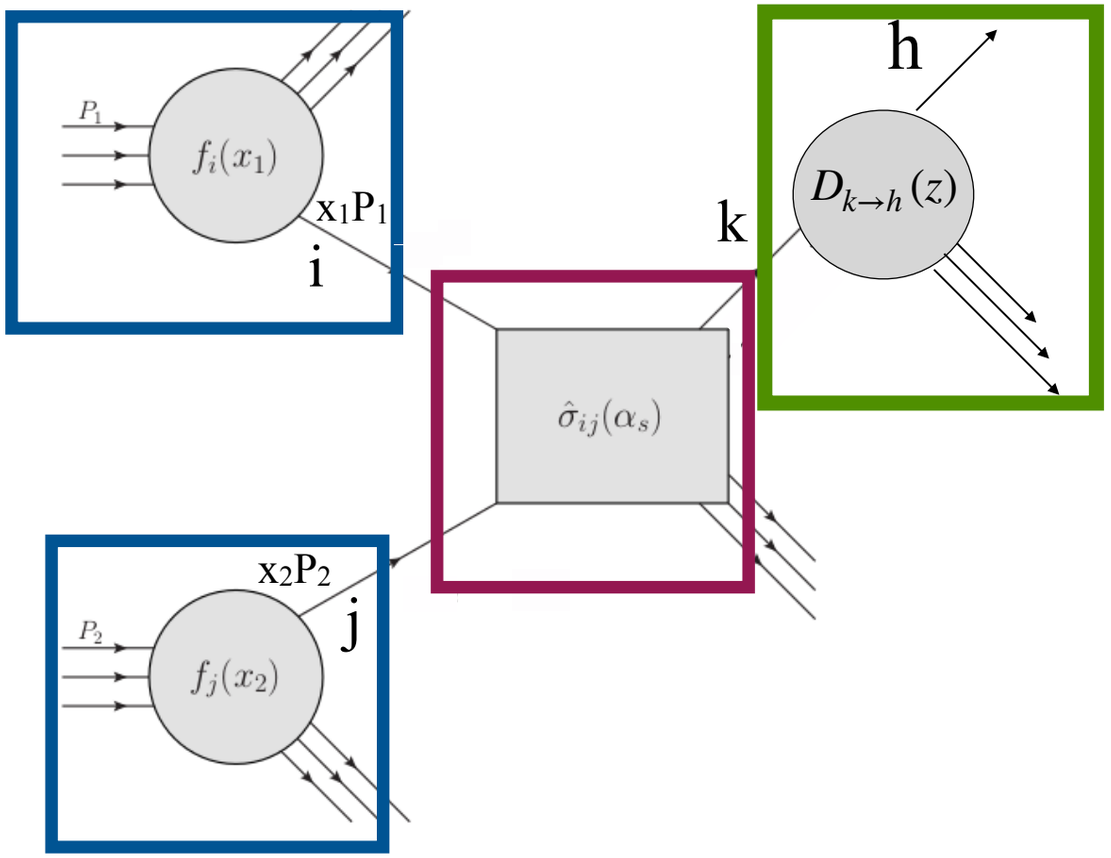

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- <u>Non perturbative</u>
  But universal

- Their evolution
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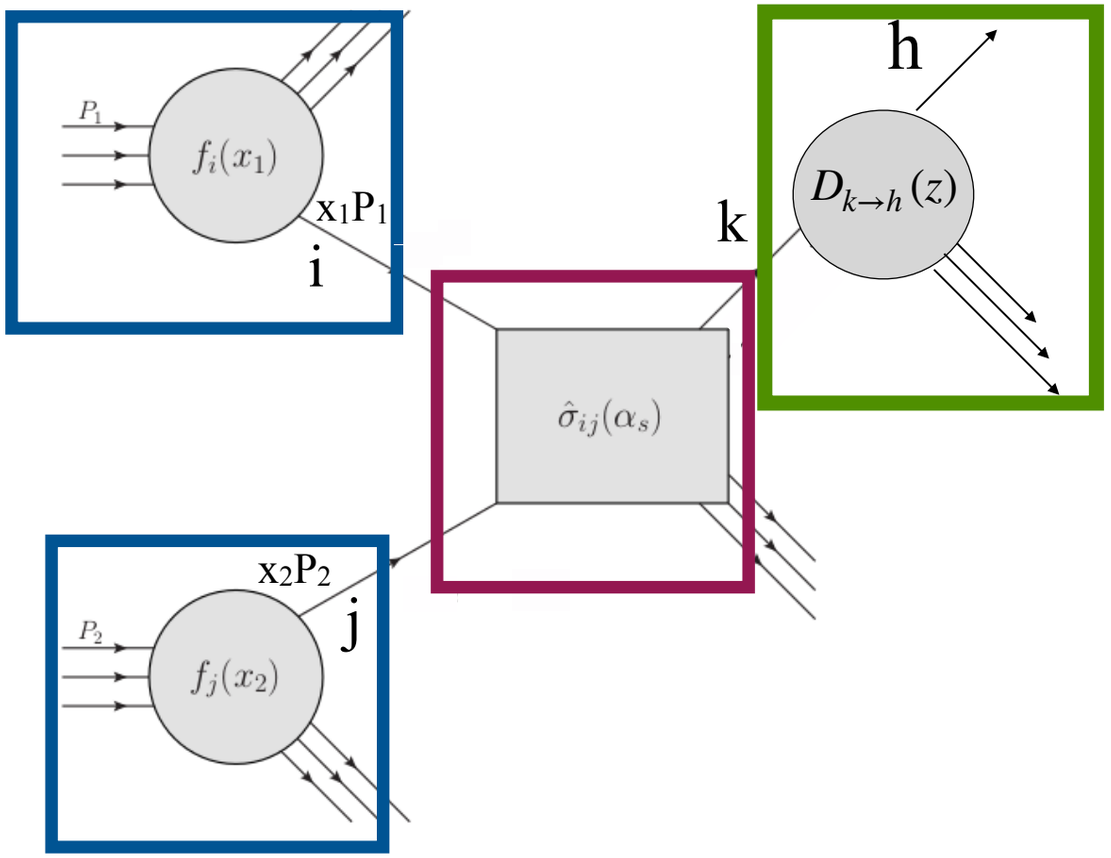

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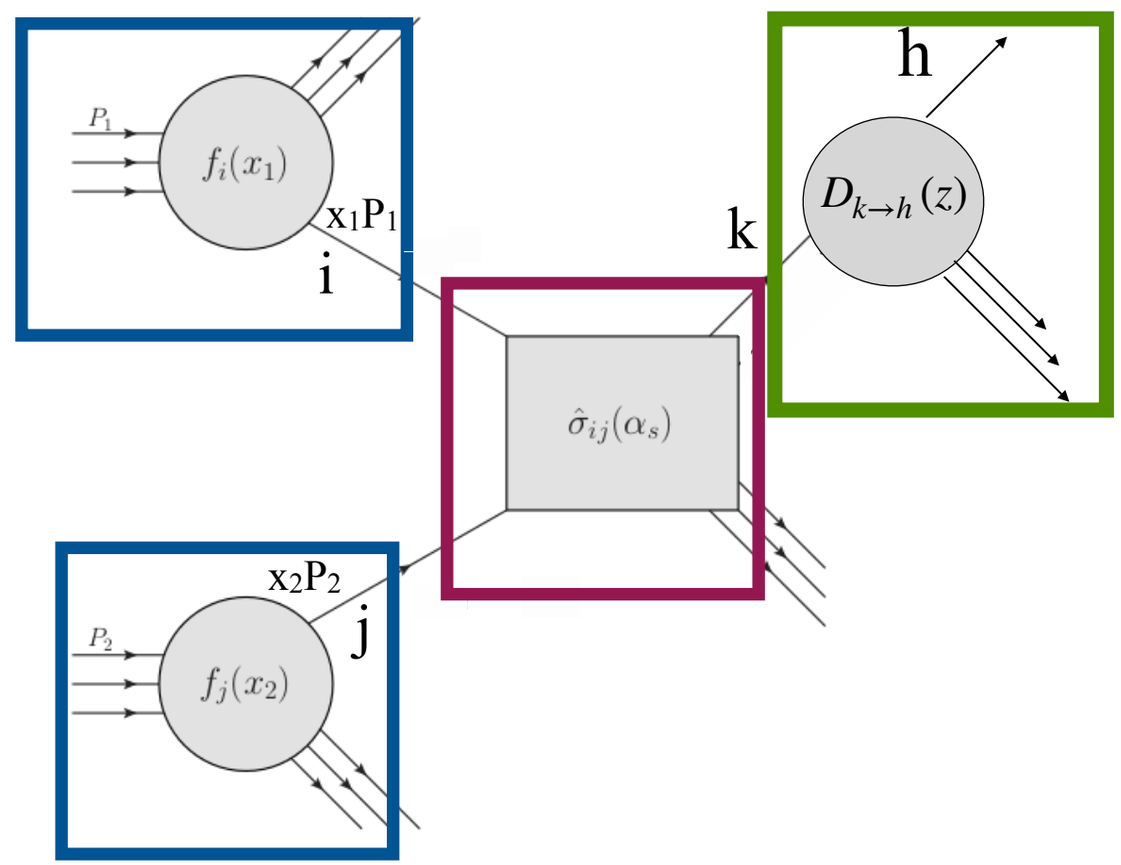
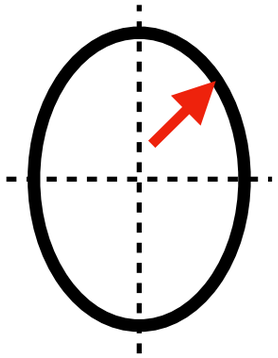
$$Q^2 \frac{\partial f_i(x,Q^2)}{\partial Q^2} = \sum_j \boxed{P_{ij}} \otimes f_j(x,Q^2)$$

Splitting functions (pQCD)



Where does the jet quenching appear?

Formalism



- Single-inclusive cross section:

$$\frac{d\sigma^{AA\to h+X}}{dp_T dy} = \int \frac{dx_1}{x_1}\frac{dx_2}{x_2}\frac{dz}{z}\sum_{i,j,k} x_1 f_{i/A}(x_1,Q^2)\, x_2 f_{j/A}(x_2,Q^2)\,\frac{d\hat{\sigma}^{ij\to k}}{d\hat{t}}\, D^{(med)}_{k\to h}(z,\mu_F^2)$$

nPDFs

- Fragmentation functions:

$$D^{(med)}_{k\to h}(z,\mu_F^2) = \int_0^1 d\epsilon\, P_E(\epsilon)\,\frac{1}{1-\epsilon}\, D^{(vac)}_{k\to h}\left(\frac{z}{1-\epsilon},\mu_F^2\right)$$

FFs

Local medium energy density

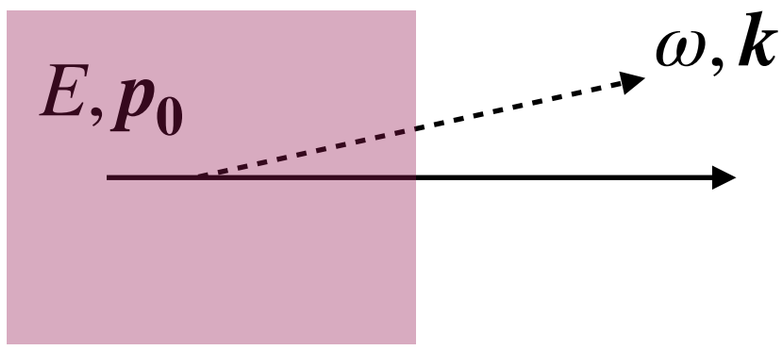
ENERGY LOSS: Quenching Weights (QWs)

$$\hat{q}(\xi) = K\cdot 2\epsilon^{3/4}(\xi)$$

Probability distribution of a fractional energy loss $\epsilon = \Delta E/E$ of the hard parton in the medium

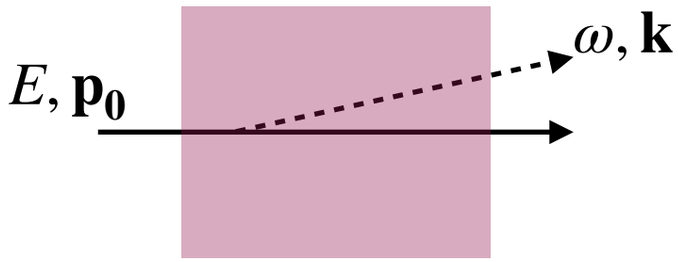
The building block

The building block



In-medium radiation: assumptions

- The gluon is emitted at small angles and $\mu \ll \mathbf{k}, \mathbf{p_0} \ll \omega, \mathbf{E}$



- The radiation is due to elastic scatterings mediated by gluons

- The interactions are instantaneous. The medium is seen as recoilless background field

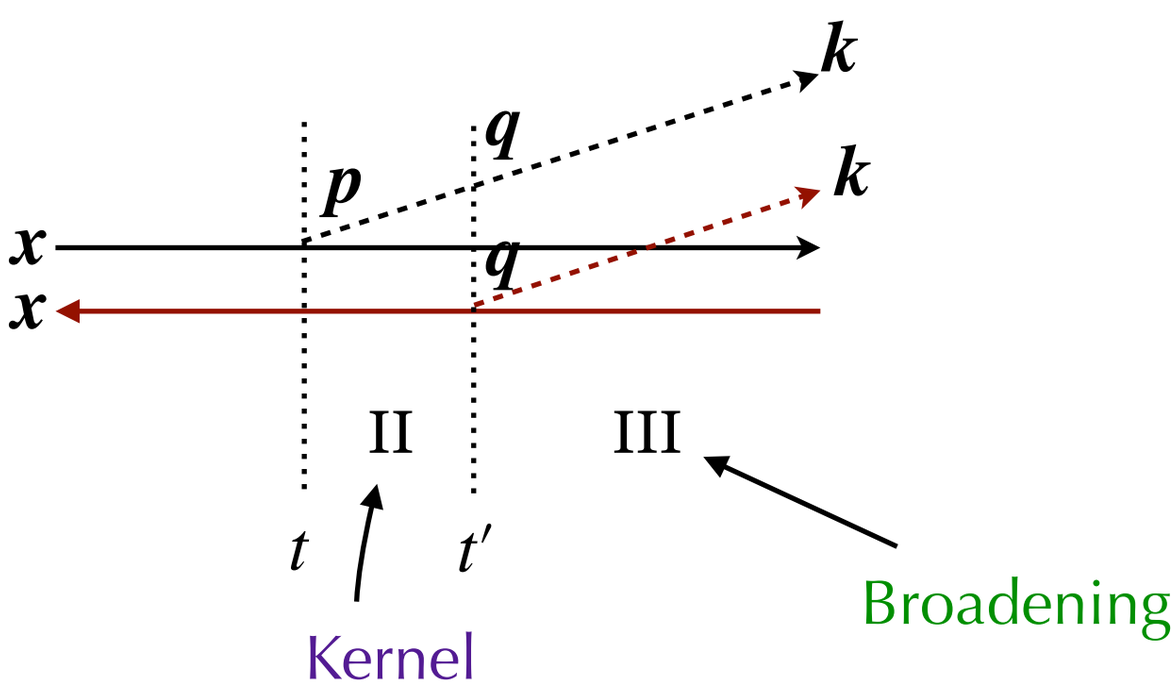
- The details of the parton-medium interaction are given by the dipole cross section

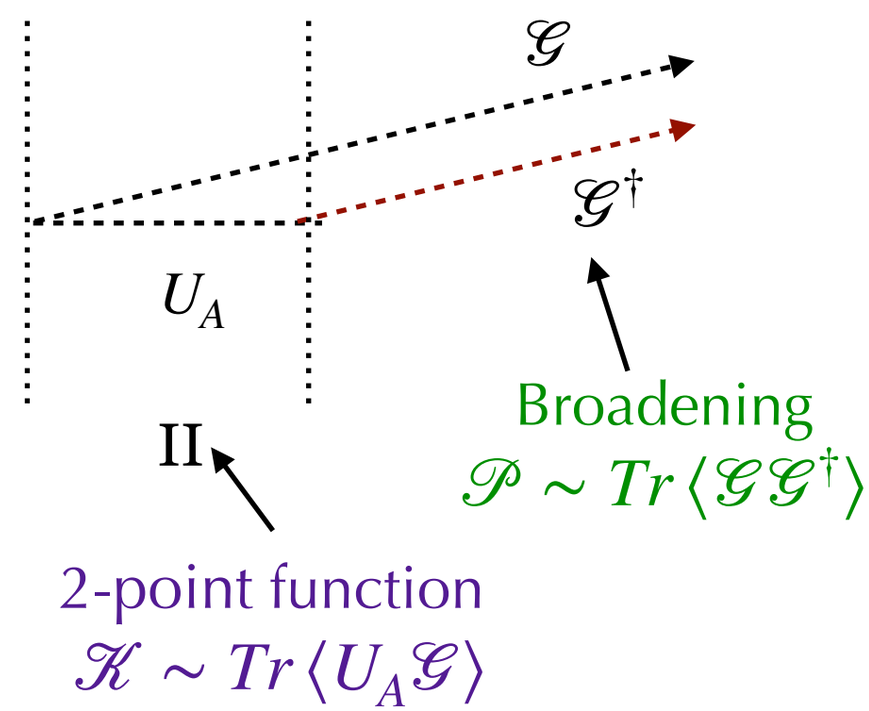
Medium-induced gluon spectrum

- For a soft emitted gluon ($\omega \ll E$)

Baier, Dokshitzer, Mueller, Peigné, Schiff (96)
Zaharov (97)
BDMPS-Z

$$\omega \frac{dI}{d\omega d^2\boldsymbol{k}} = \frac{2\alpha_s C_R}{(2\pi)^2\omega^2} \mathrm{Re} \int_0^\infty dt' \int_0^{t'} dt \int_{\boldsymbol{pq}} \boldsymbol{p}\cdot\boldsymbol{q}\, \widetilde{\mathcal{K}}(t',\boldsymbol{q};t,\boldsymbol{p})\mathcal{P}(\infty,\boldsymbol{k};t',\boldsymbol{q})$$





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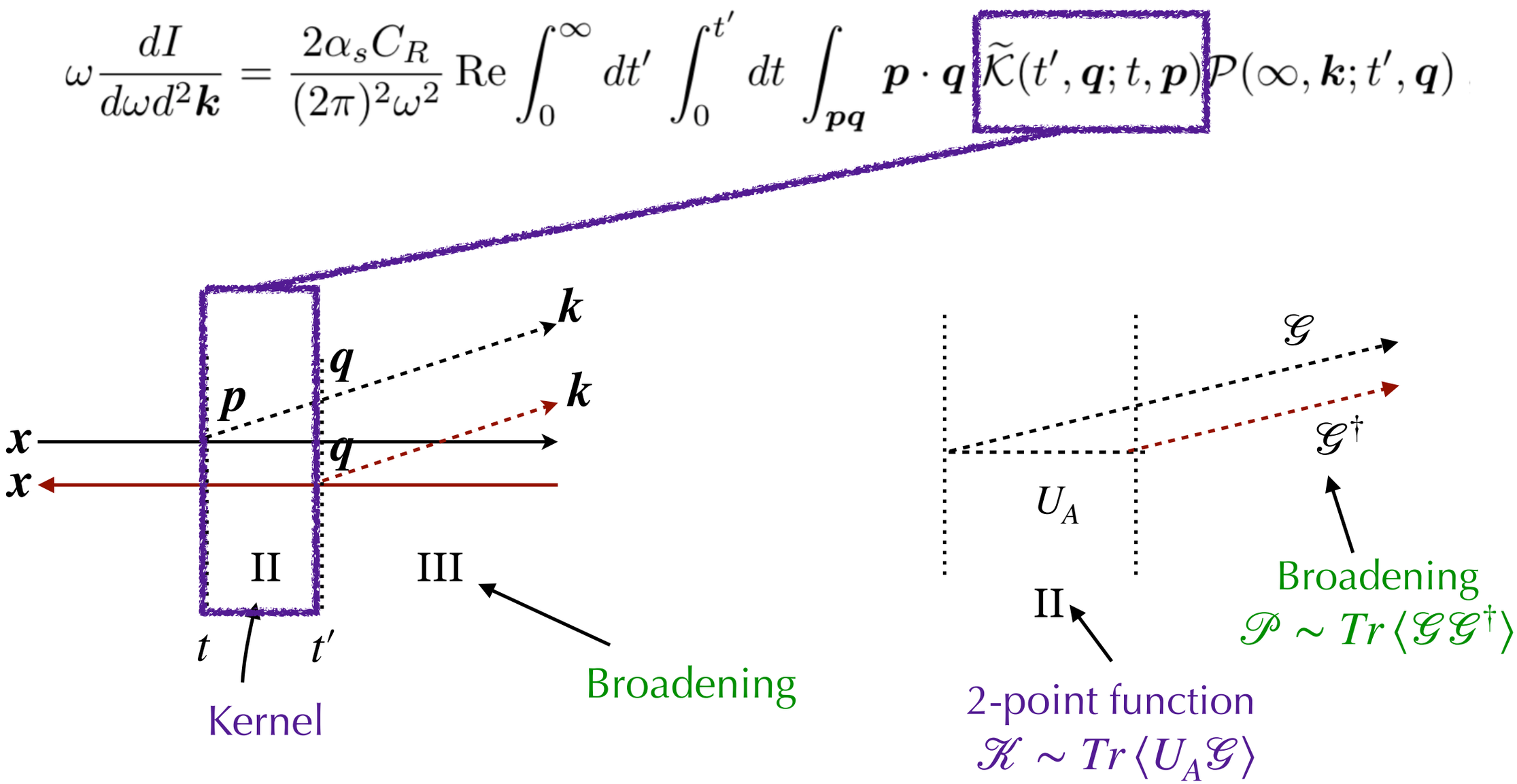

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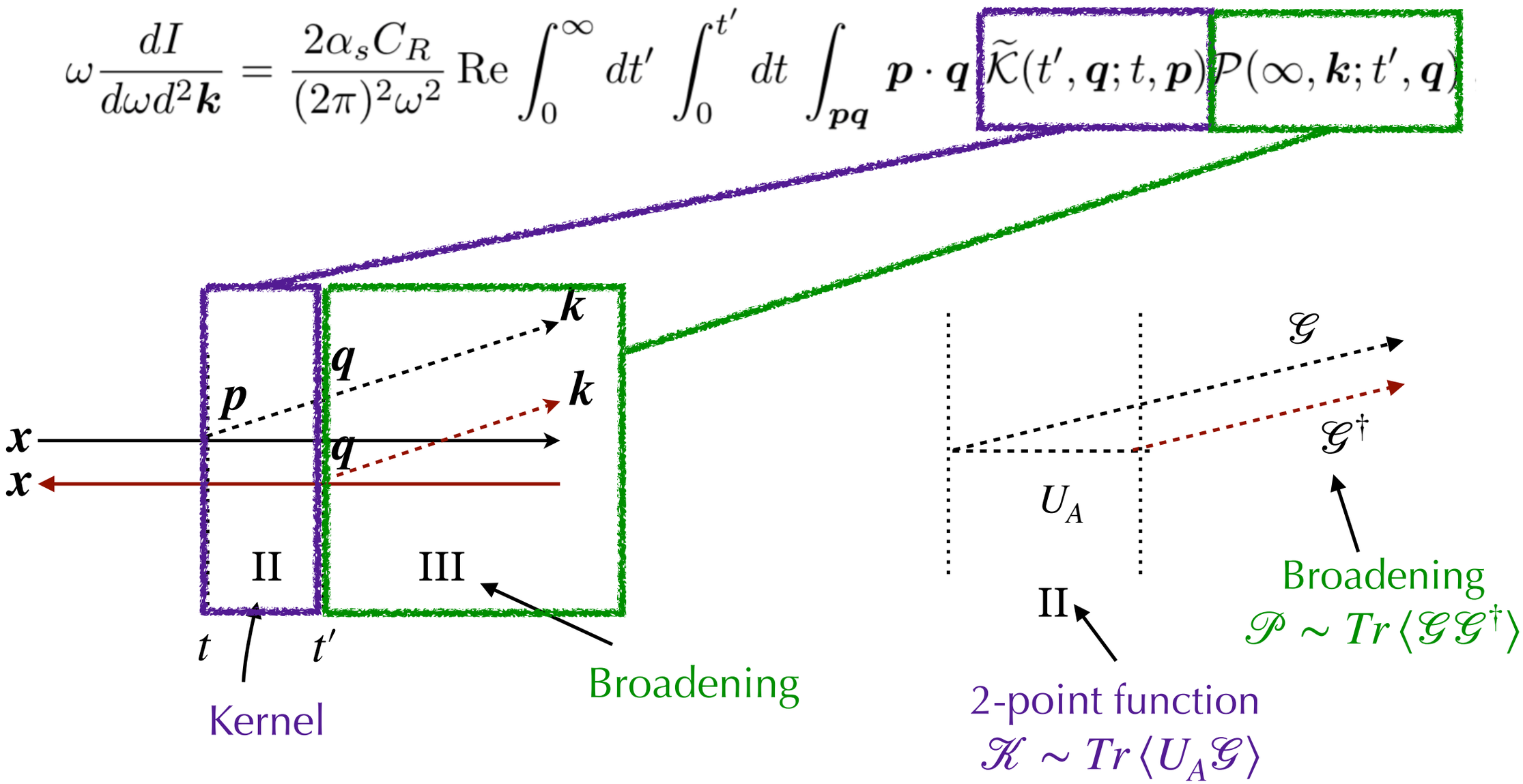
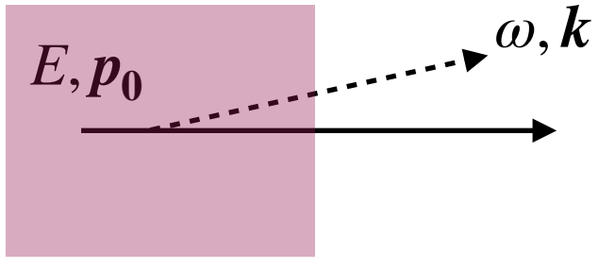


The master formula



- The in-medium spectrum is given by ($\omega \ll E$):

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BDMPS-Z

- Broadening

$$\mathcal{P}(t'',\boldsymbol{k};t',\boldsymbol{q}) \equiv \int d^2\boldsymbol{z}\, e^{-i(\boldsymbol{k}-\boldsymbol{q})\cdot\boldsymbol{z}} \exp\left\{-\frac{1}{2}\int_{t'}^{t''} ds\, n(s)\,\sigma(\boldsymbol{z})\right\}$$

- Emission Kernel

$$\mathcal{K}(t',\boldsymbol{z};t,\boldsymbol{y}) \equiv \int_{\boldsymbol{pq}} e^{i(\boldsymbol{q}\cdot\boldsymbol{z}-\boldsymbol{p}\cdot\boldsymbol{y})}\, \widetilde{\mathcal{K}}(t',\boldsymbol{q};t,\boldsymbol{p})$$
$$= \int_{\boldsymbol{r}(t)=\boldsymbol{y}}^{\boldsymbol{r}(t')=\boldsymbol{z}} \mathcal{D}\boldsymbol{r} \exp\left[\int_t^{t'} ds \left(\frac{i\omega}{2}\dot{\boldsymbol{r}}^2 - \frac{1}{2}n(s)\sigma(\boldsymbol{r})\right)\right]$$

Difficult to solve numerically for realistic $\sigma(\mathbf{r})$

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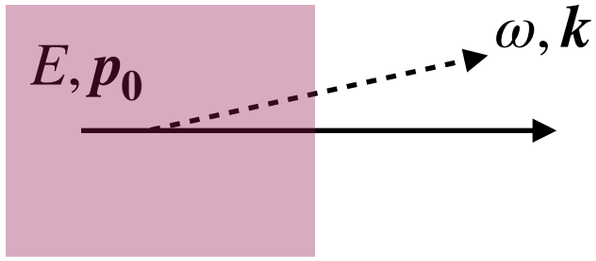

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Medium information

Difficult to solve numerically for realistic $\sigma(\mathbf{r})$

Analytic approximations

- Approximation 1: **Harmonic oscillator**

Perturbative tails neglected

$$n(s)\sigma(\boldsymbol{r}) \approx \frac{1}{2}\hat{q}(s)\boldsymbol{r}^2 + \mathcal{O}(\boldsymbol{r}^2 \ln \boldsymbol{r}^2)$$

The Kernel can be analytically computed (for a static medium)

- Approximation 2: **Opacity expansion** (GLV)

$\sigma(\mathbf{r})$ is taken as the full Yukawa cross-section $\sigma(\boldsymbol{r}) \equiv \int_{\boldsymbol{q}} V(\boldsymbol{q})\left(1 - e^{i\boldsymbol{q}\boldsymbol{r}}\right)$ $V(\boldsymbol{q}) = \frac{8\pi\mu^2}{\left(\boldsymbol{q}^2 + \mu^2\right)^2}$

The integrand in the Kernel is expanded in powers of $\left(n(s)\sigma(\boldsymbol{r})\right)^N$

$N = 1$: **First opacity** or **GLV** approximation

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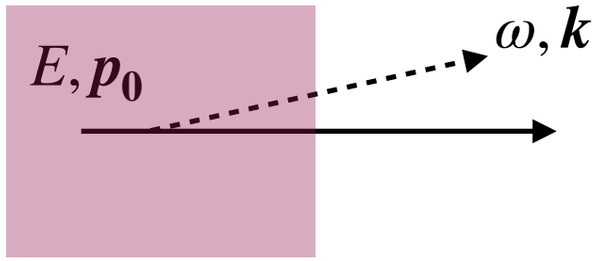
Full solution



- Solve the differential equation satisfied by the Kernel

$$\partial_t \widetilde{\mathcal{K}}(s,\boldsymbol{q};t,\boldsymbol{p}) = \frac{i\boldsymbol{p}^2}{2\omega}\widetilde{\mathcal{K}}(s,\boldsymbol{q};t,\boldsymbol{p}) + \frac{1}{2}n(t)\int_{\boldsymbol{k}'}\sigma(\boldsymbol{k}'-\boldsymbol{p})\widetilde{\mathcal{K}}(s,\boldsymbol{q};t,\boldsymbol{k}')$$

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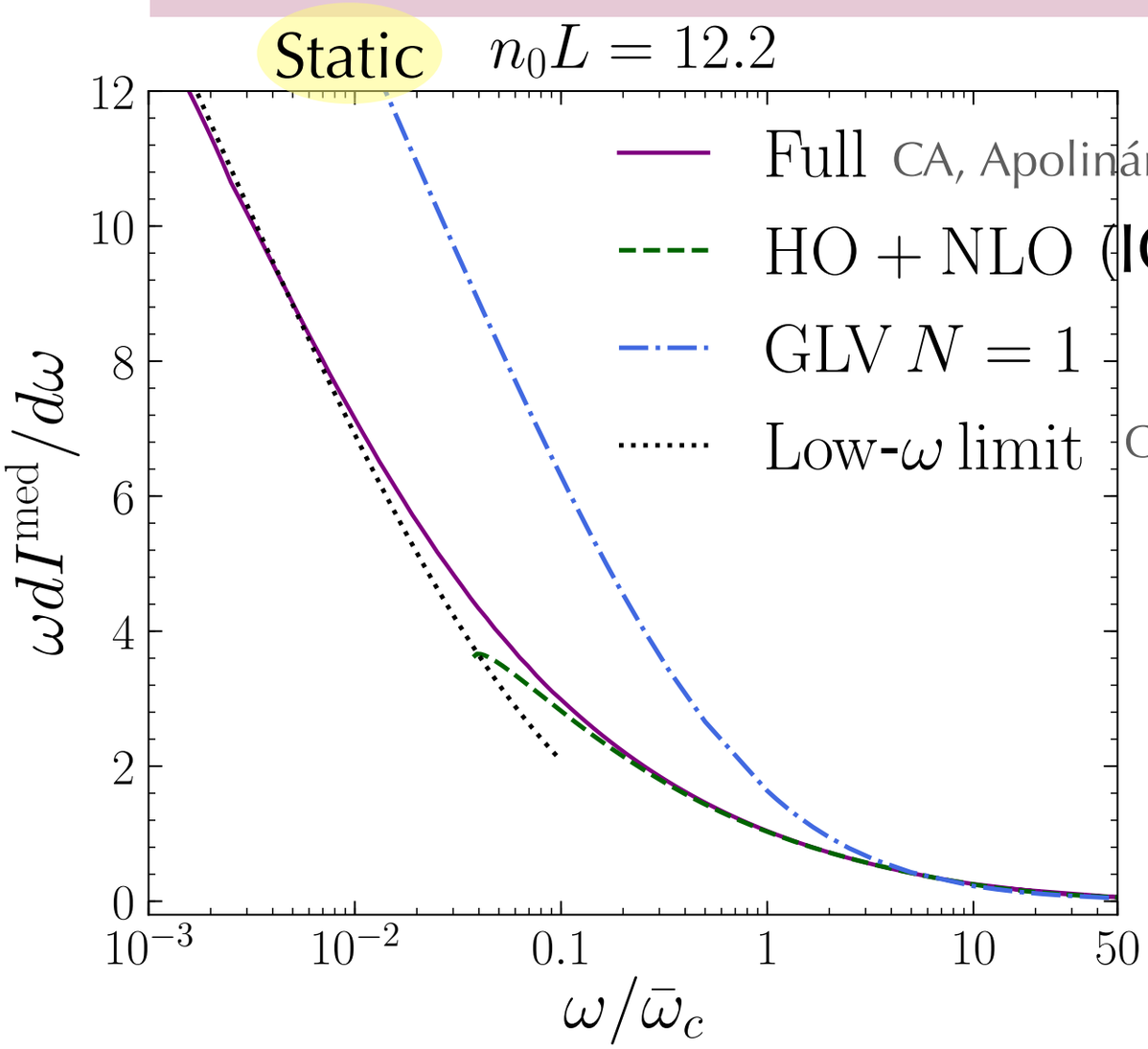

CA, Apolinário, Dominguez, 2002.01517

Mehtar-Tani, Barata, Soto-Ontoso, Tywoniuk

CA, M. G. Martinez, F. Dominguez, 2011.06522

$$\bar{\omega}_c = \frac{\mu^2 L}{2}$$

$$V(\boldsymbol{q}) = \frac{8\pi\mu^2}{(\boldsymbol{q}^2+\mu^2)^2}$$

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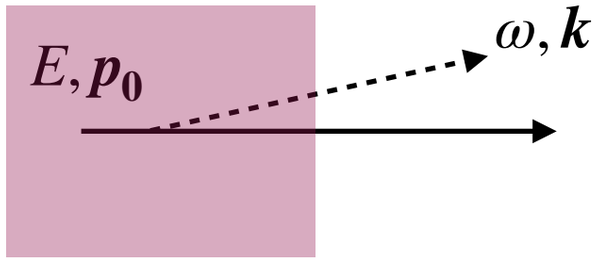

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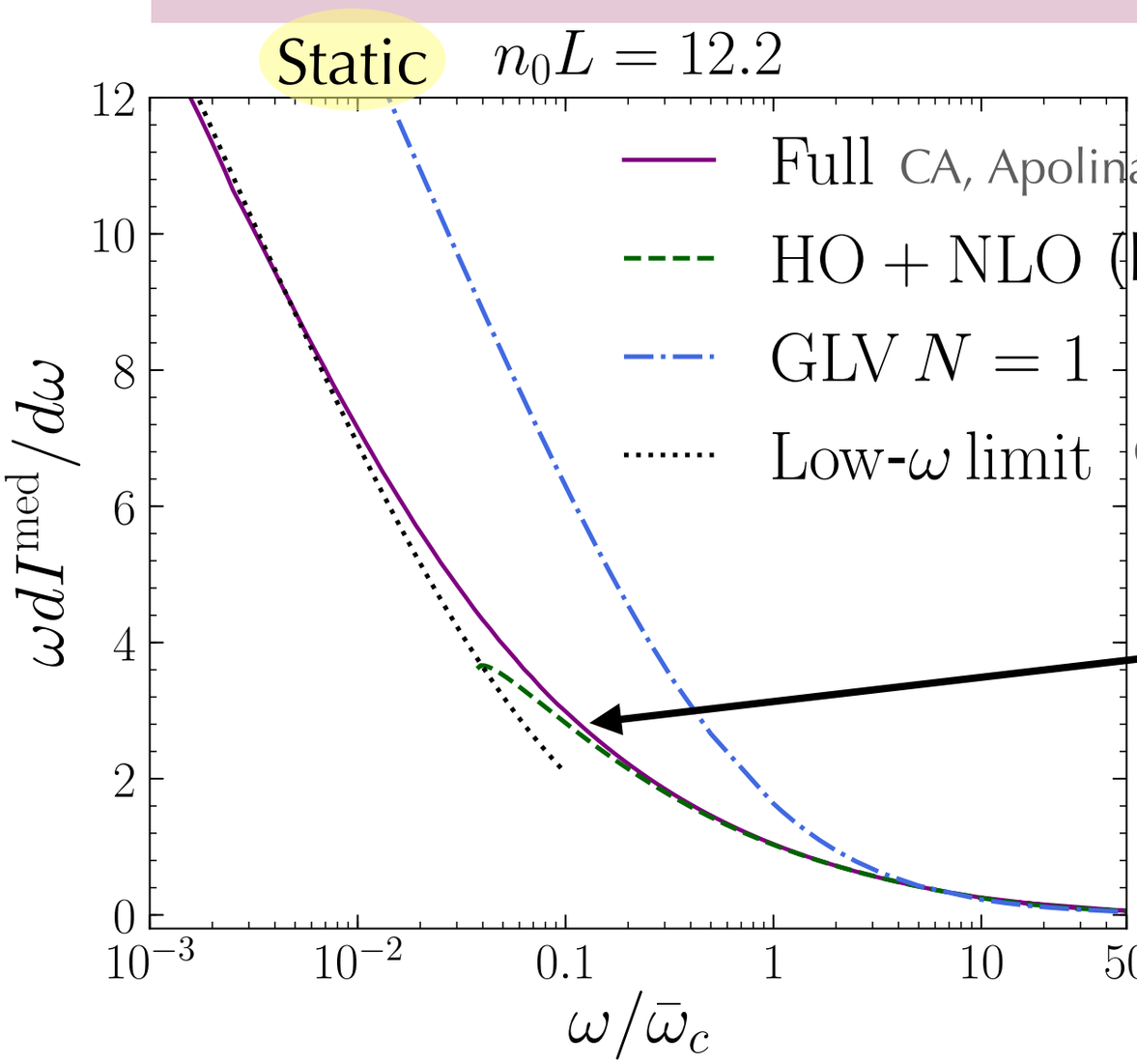
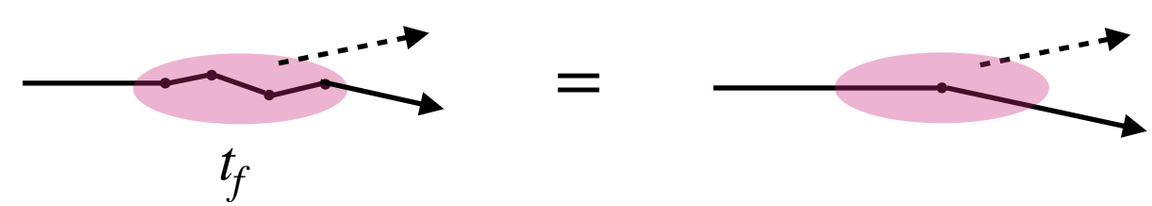


CA, Apolinário, Dominguez, 2002.01517

Mehtar-Tani, Barata, Soto-Ontoso, Tywoniuk

CA, M. G. Martinez, F. Dominguez, 2011.06522

LPM effect



$$\bar{\omega}_c = \frac{\mu^2 L}{2}$$

$$V(\boldsymbol{q}) = \frac{8\pi\mu^2}{(\boldsymbol{q}^2+\mu^2)^2}$$

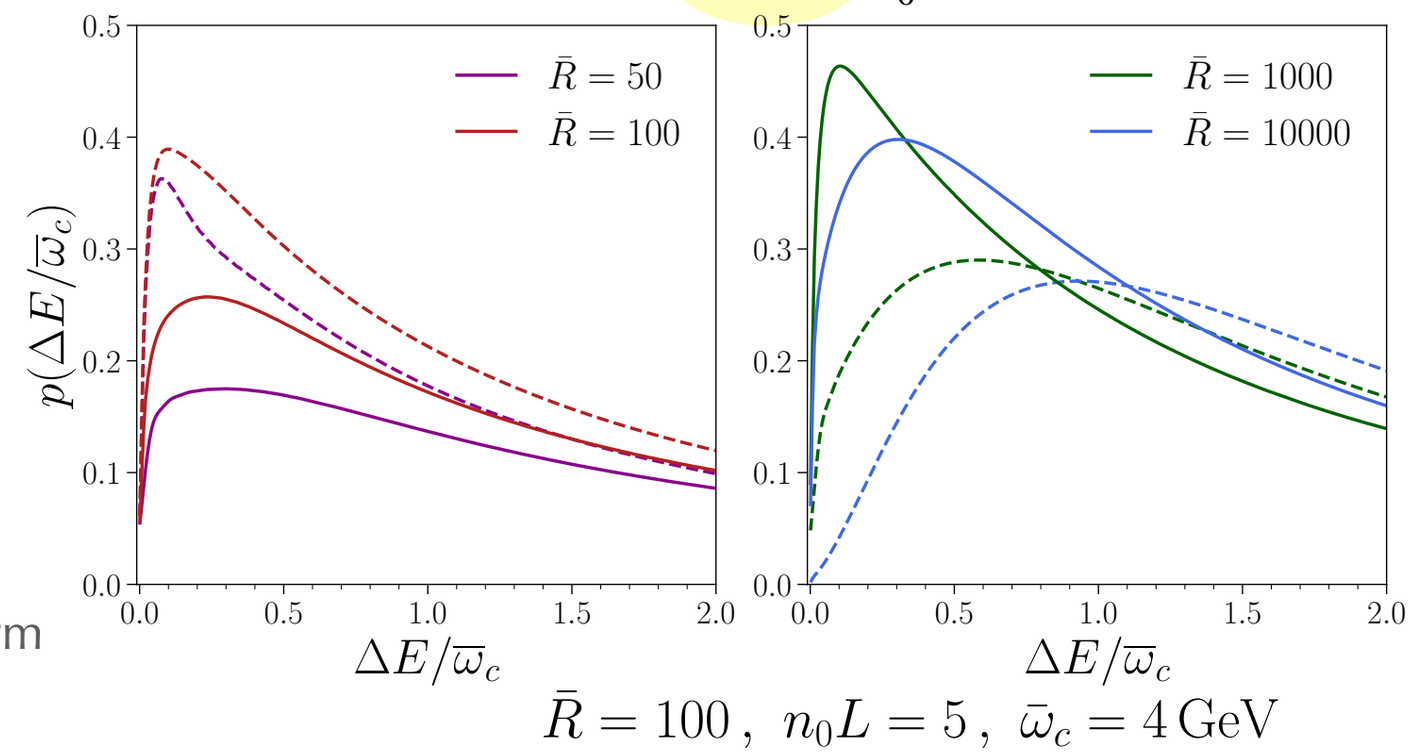
CA, M. G. Martinez, F. Dominguez, 2011.06522

Towards phenomenology?

Hadron suppression

GLV
Full

Static $n_0 L = 5$



Quenching weights ($P(\Delta E)$)

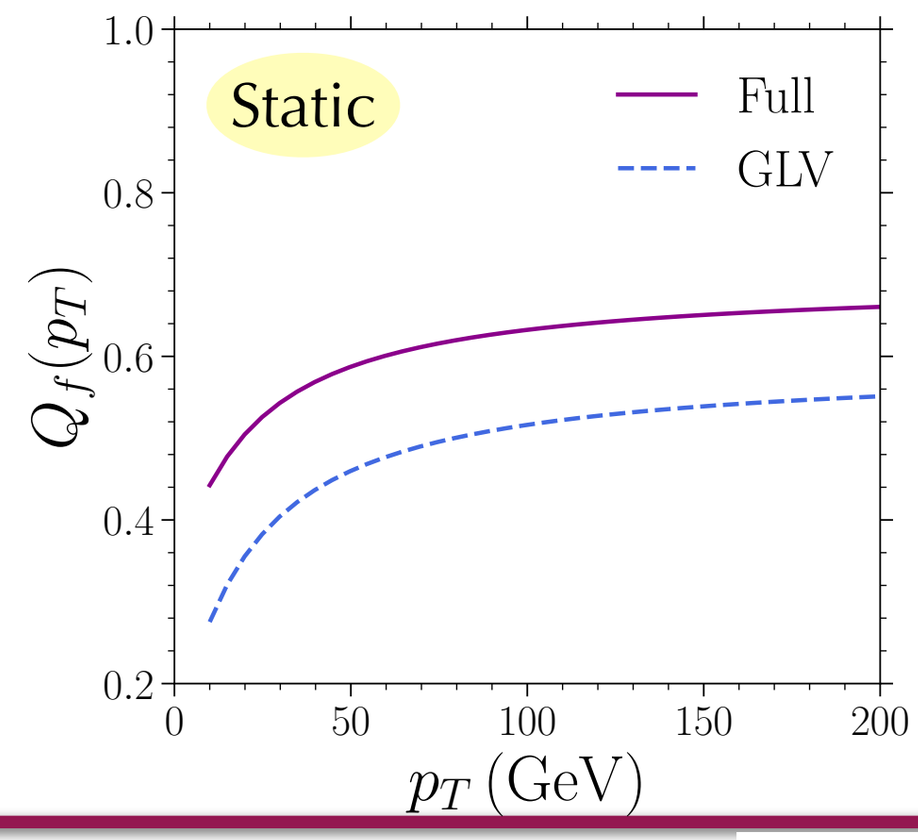
- *Probability* distribution of the hard parton of losing medium-induced energy ΔE

(assuming gluons emitted independently)

*Plotting just their continuous term

Quenching factor (Q_f)

$$Q_f(p_T) = \frac{d\sigma^{med}(p_T)/dp_T}{d\sigma^{vac}(p_T)/dp_T} \sim \int d\Delta E\, P(\Delta E) \left(\frac{p_t}{p_T + \Delta E} \right)^n$$

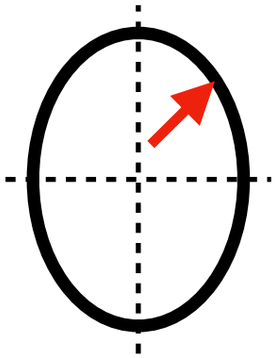


- The **suppression factors** of the GLV and fully resummed spectra seem **different**

How do we move to a realistic medium?

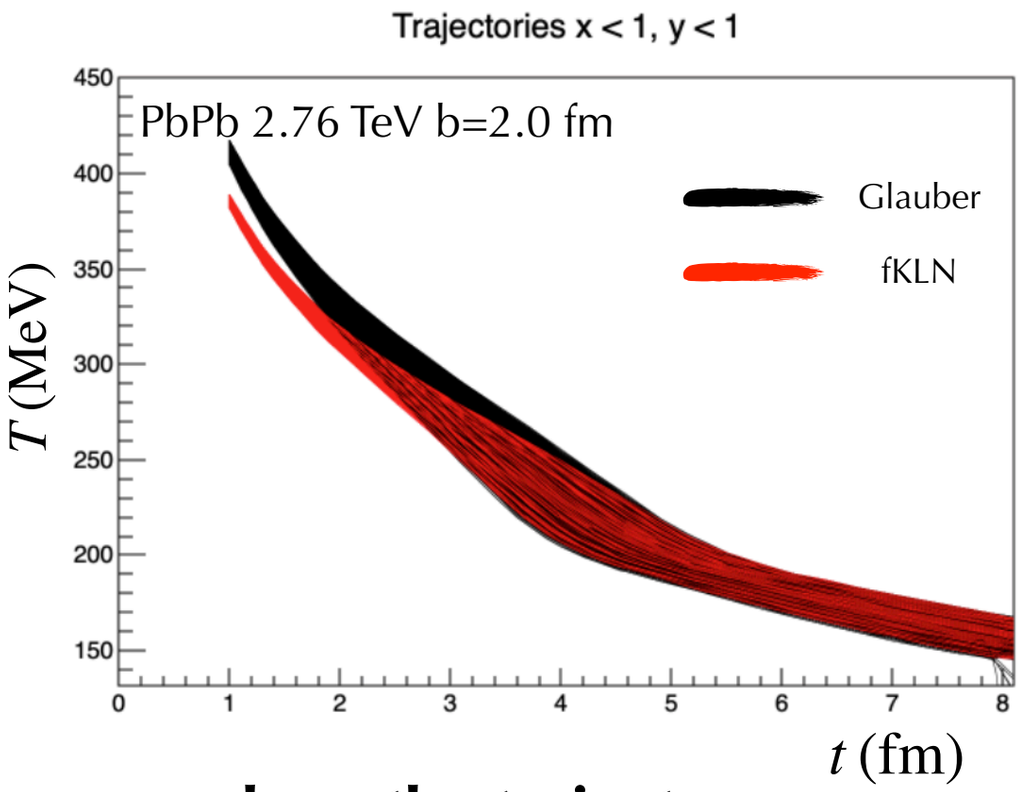
Beyond the brick

*with multiple scatterings



- The goal is to compute the energy lost by a hard parton along **its trajectory** within an **evolving** media

- One should read the medium properties (for instance, the temperature T) from the hydro at each point of the path



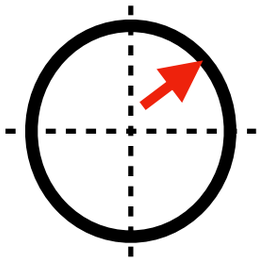
- Then, obtain the medium parameters entering the spectrum: $n(T(t))\,,\ \mu(T(t))\ldots$

- And feed them to the code and compute the spectrum **along the trajectory**

The spectrum depends on the full trajectory, there is no "per-point spectrum"

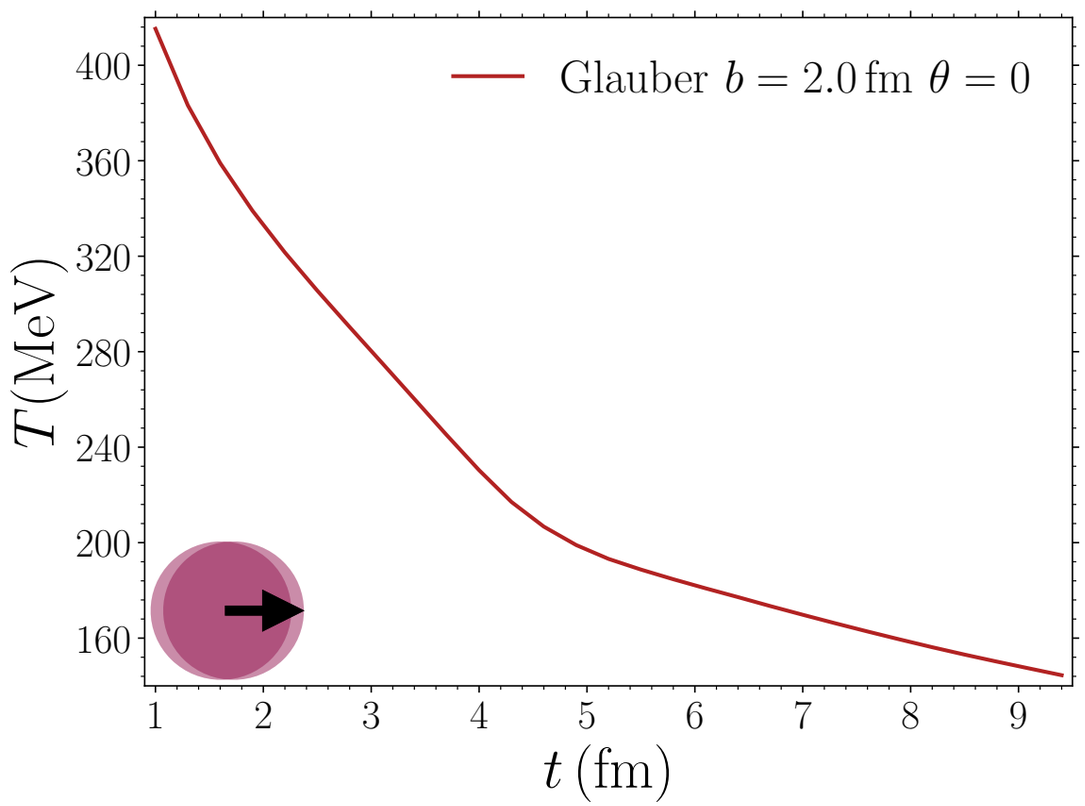
Beyond the brick II

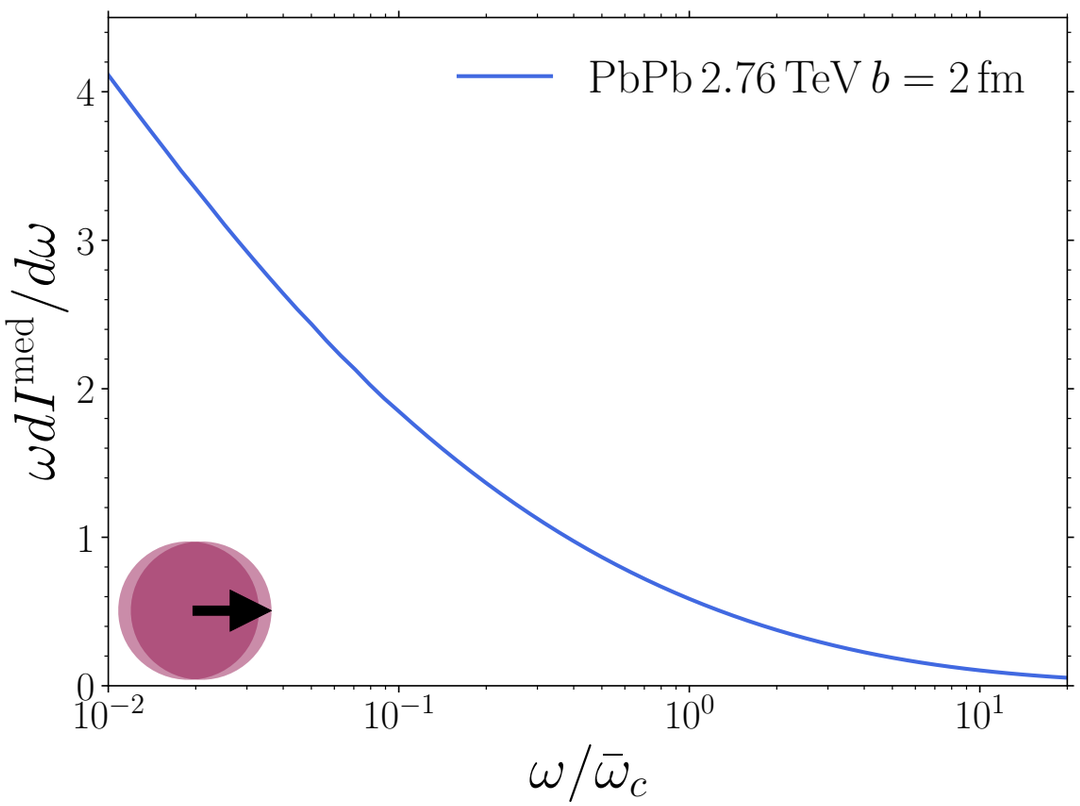
***with multiple scatterings**



- We can compute the spectrum with time-dependent variables along a path

$$n_{hydro}(t) = k_1 T(t), \quad \mu^2_{hydro}(t) = k_2 T^2(t)$$

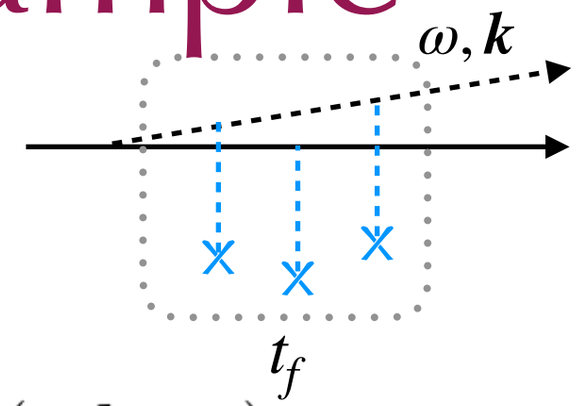




$$\bar{\omega}_c = \frac{\mu^2 L}{2}$$

- But this is computationally demanding. Currently, it seems too costly to do it for every trajectory *on the fly*
- Pre-tabulate it? **How do we know a priori how the medium parameters will behave along all possible paths**?

The full solution as an example



- Full one (soft) gluon emission in-medium calculation

$$\omega \frac{dI^{\rm med}}{d\omega} = \frac{2\alpha_s C_R}{\omega} \, {\rm Re} \int_0^L ds \, n(s) \int_0^s dt \int_{\boldsymbol{pql}} i \, \frac{\boldsymbol{p}\cdot\boldsymbol{q}}{\boldsymbol{q}^2} \sigma(\boldsymbol{q}-\boldsymbol{l}) \widetilde{\mathcal{K}}(s,\boldsymbol{l};t,\boldsymbol{p})$$

BDMPS-Z

(Vacuum subtracted)

- Emission Kernel

$$\partial_t \widetilde{\mathcal{K}}(s,\boldsymbol{q};t,\boldsymbol{p}) = \frac{i\boldsymbol{p}^2}{2\omega} \widetilde{\mathcal{K}}(s,\boldsymbol{q};t,\boldsymbol{p}) + \frac{1}{2} n(t) \int_{\boldsymbol{k}'} \sigma(\boldsymbol{k}'-\boldsymbol{p}) \widetilde{\mathcal{K}}(s,\boldsymbol{q};t,\boldsymbol{k}')$$

$$\sigma(\boldsymbol{q}) \equiv -V(\boldsymbol{q}) + (2\pi)^2 \delta^2(\boldsymbol{q}) \int_{\boldsymbol{l}} V(\boldsymbol{l}) \qquad V(\boldsymbol{q}) = \frac{8\pi\mu^2(t)}{\left(\boldsymbol{q}^2+\mu^2(t)\right)^2}$$

Need to solve this differential equation for each trajectory

CA, L. Apolinário, F. Dominguez, 2002.01517

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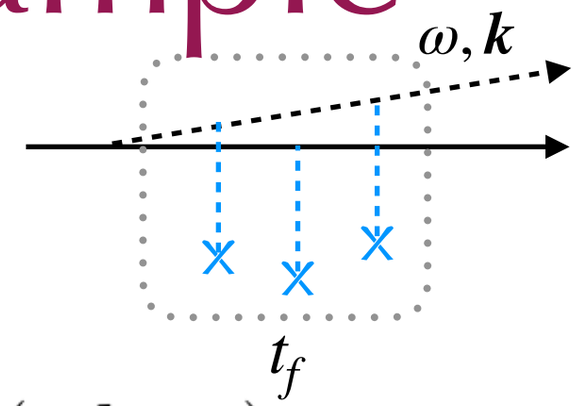

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Medium information

$$\sigma(\boldsymbol{q}) \equiv -V(\boldsymbol{q}) + (2\pi)^2 \delta^2(\boldsymbol{q}) \int_{\boldsymbol{l}} V(\boldsymbol{l})$$

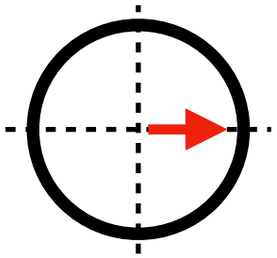
$$V(\boldsymbol{q}) = \frac{8\pi\mu^2(t)}{\left(\boldsymbol{q}^2 + \mu^2(t)\right)^2}$$

Screening mass

Need to solve this differential equation for each trajectory

CA, L. Apolinário, F. Dominguez, 2002.01517

How?



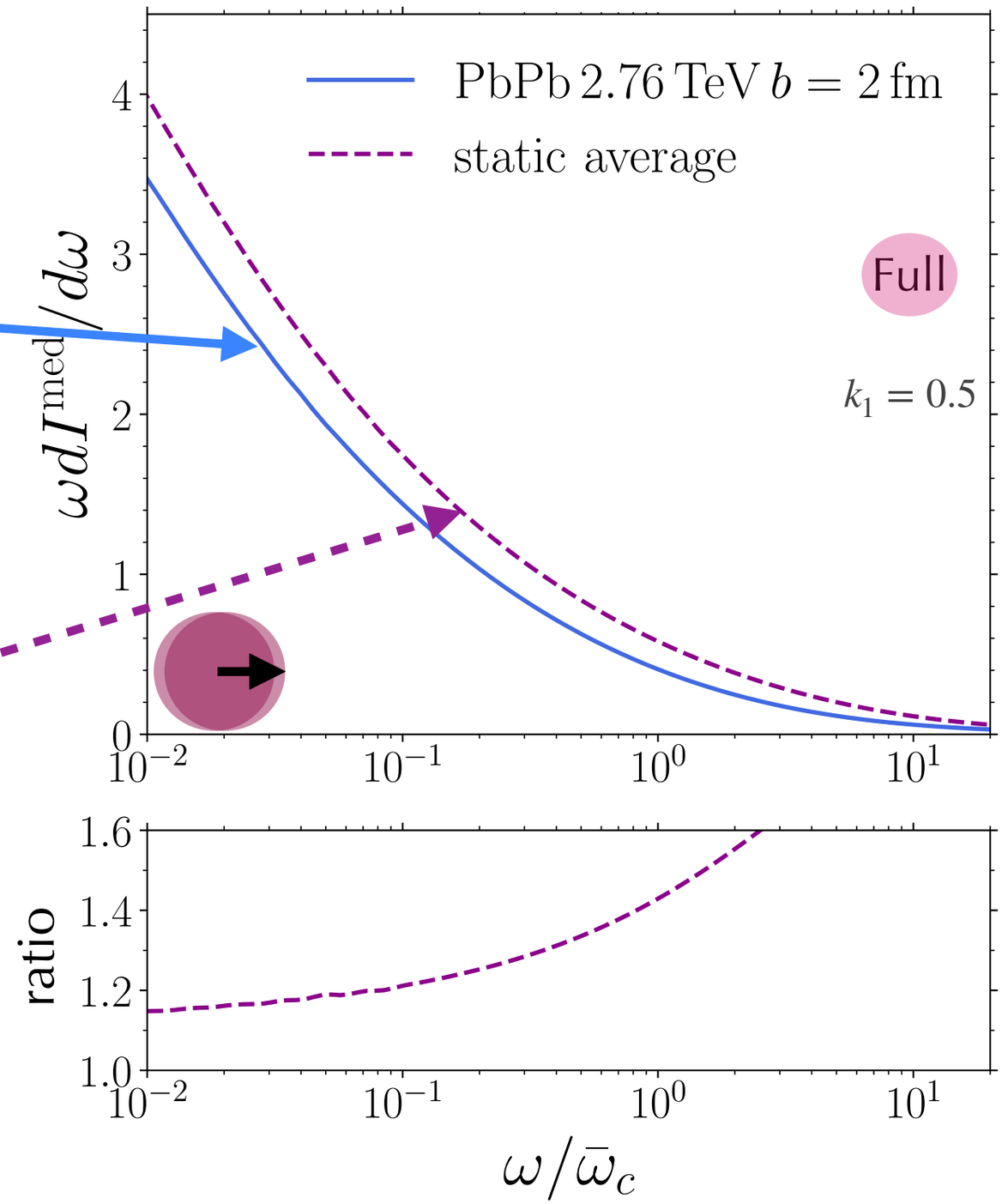
- Using average values?
- Compute the full solution along a path thorough a hydro

$$n_{hydro}(t) = k_1 T(t) \qquad \mu^2_{hydro}(t) = k_2 T^2(t)$$

- Compare to the static where the medium parameters are given by their average along the path:

$$n_0 L\Big|_{\text{static}} = k_1 \langle T \rangle L'$$

$$\bar{\omega}_c\Big|_{\text{static}} = \frac{\mu^2 L}{2}\Bigg|_{\text{static}} = \frac{1}{2} k_2 \langle T \rangle^2 L'$$

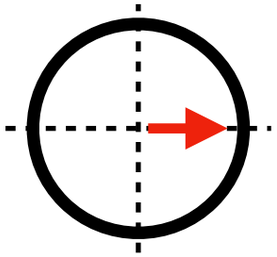


Using average values does not work well!

How?

Find an equivalent static scenario?

Scaling laws?



- The idea is to **find an equivalent static scenario**
  Find the values of the parameters that best approximate the dynamic spectrum along the path

  Salgado, Wiedemann, 0302184
  Priyam Adhya, Salgado, Spousta, Tywoniuk, 1911.12193

- Compute the full solution along a path thorough a hydro

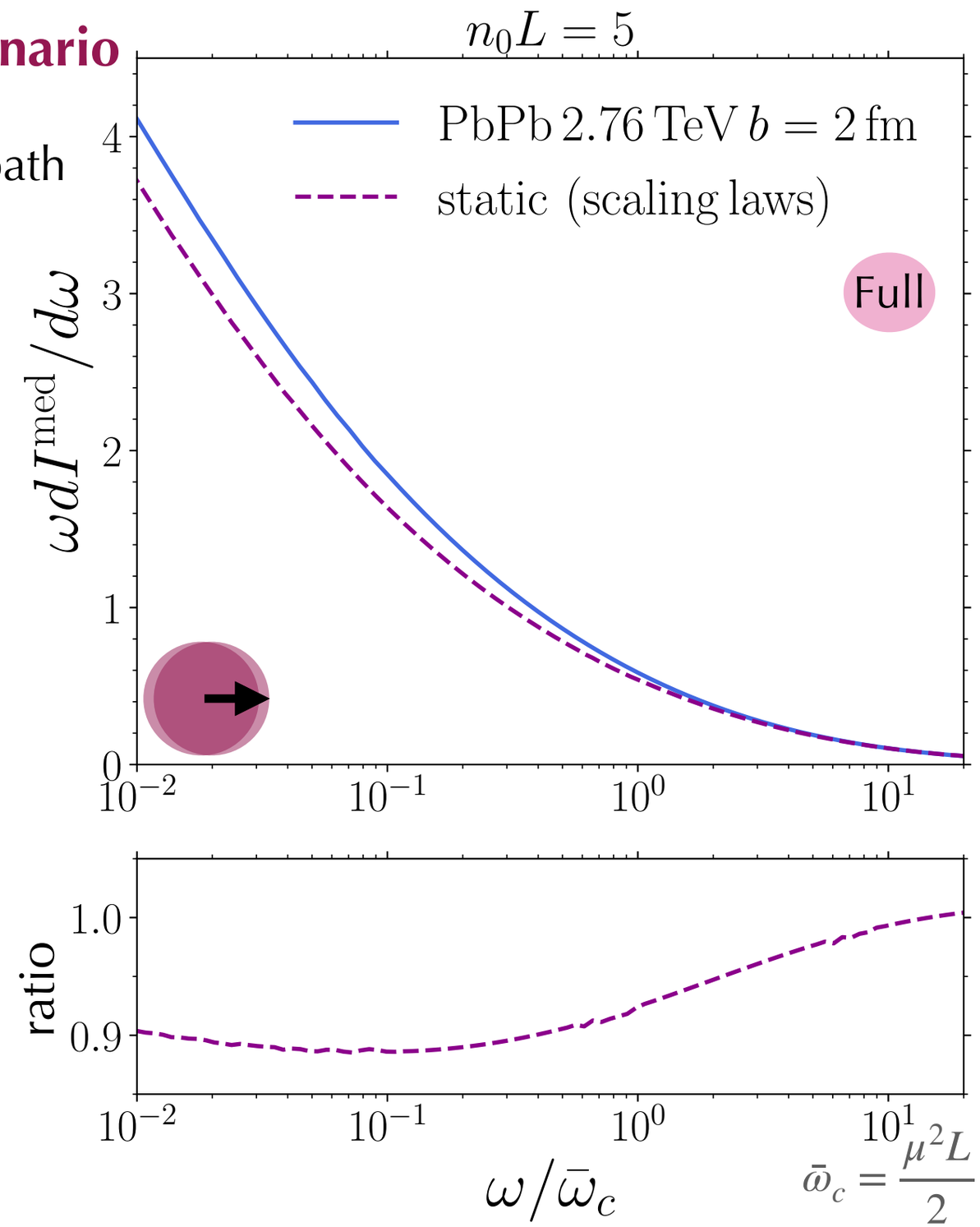
$$n_{hydro}(t) = k_1 T(t) \qquad \mu^2_{hydro}(t) = k_2 T^2(t)$$

- Find the values of the parameters of the static scenario we want to compare to:

$$n_0 L\Big|_{\text{static}} = \int_0^{L'} dt\, n_{hydro}(t)$$

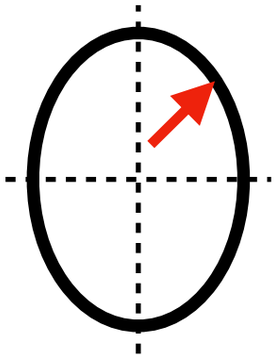
$$\frac{n_0 \mu^2 L^2}{2}\Bigg|_{\text{static}} = \int_0^{L'} dt\, t\, n_{hydro}(t)\, \mu^2_{hydro}(t)$$

- It works better than using average values, but the **errors go up to ~10-20%**



Hydro: Luzum and Romatschke, 0901.4588

Scaling laws w.r.t. a power-law case?



- Why the pre-tabulated spectrum needs to be the **static** one?

- The idea is to **find an equivalent scenario given by a power-law**

  Goal: find matching relations between the spectrum in the real world (along a path throughout a hydro) and a pre-tabulated power-law spectrum

- For instance:

  Compute the **spectrum along a path** throughout a hydro:

$$n_{hydro}(t) = k_1 T(t) \qquad \mu^2_{hydro}(t) = k_2 T^2(t)$$

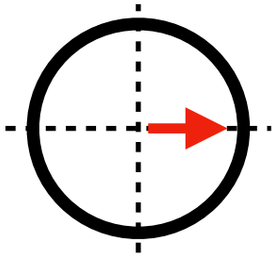
  Compare to a **power-law** spectrum for a profile given by:

$$n(t) = \frac{n'_0}{(t+t_0)^\alpha} \qquad \mu^2(t) = \frac{\mu'^2}{(t+t_0)^{2\alpha}}$$

  with a **scaling law** given by

$$\int_0^{L_1} dt\, n(t) = \int_0^{L_2} dt\, n_{hydro}(t) \qquad \int_0^{L_1} dt\, t\, n(t)\, \mu^2(t) = \int_0^{L_2} dt\, t\, n_{hydro}(t)\, \mu^2_{hydro}(t)$$

Scaling laws w.r.t. a power-law case



- Select a trajectory in central PbPb 2.76 TeV collisions

- Compute the full spectrum along this path

$$n_{hydro}(t) = k_1 T(t) \qquad \mu^2_{hydro}(t) = k_2 T^2(t)$$

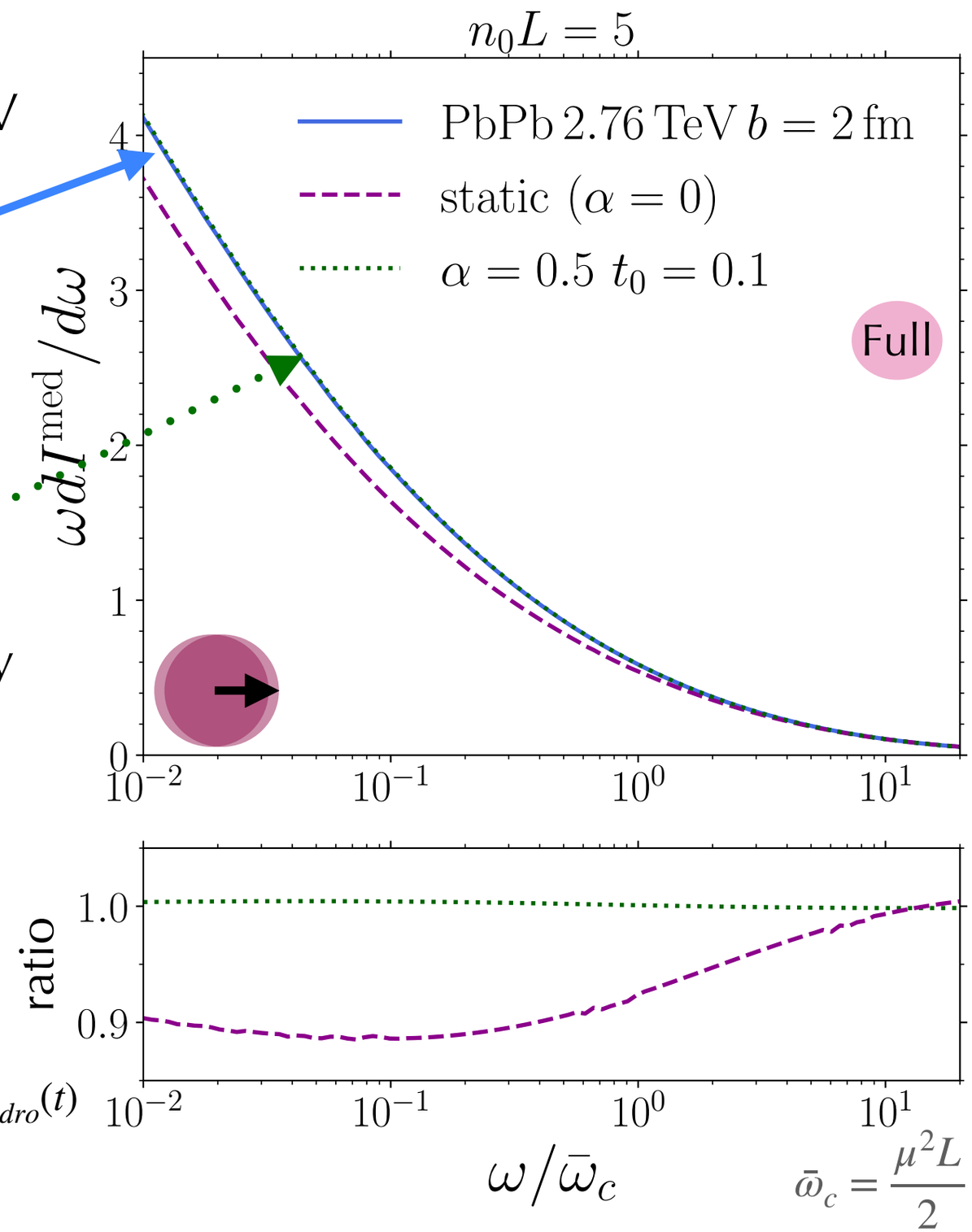
- Compare to a power-law spectrum given by

$$n(t) = \frac{n_0'}{(t+t_0)^\alpha} \qquad \mu^2(t) = \frac{\mu'^2}{(t+t_0)^{2\alpha}}$$

$$\alpha = 0.5$$

with a **scaling law** given by

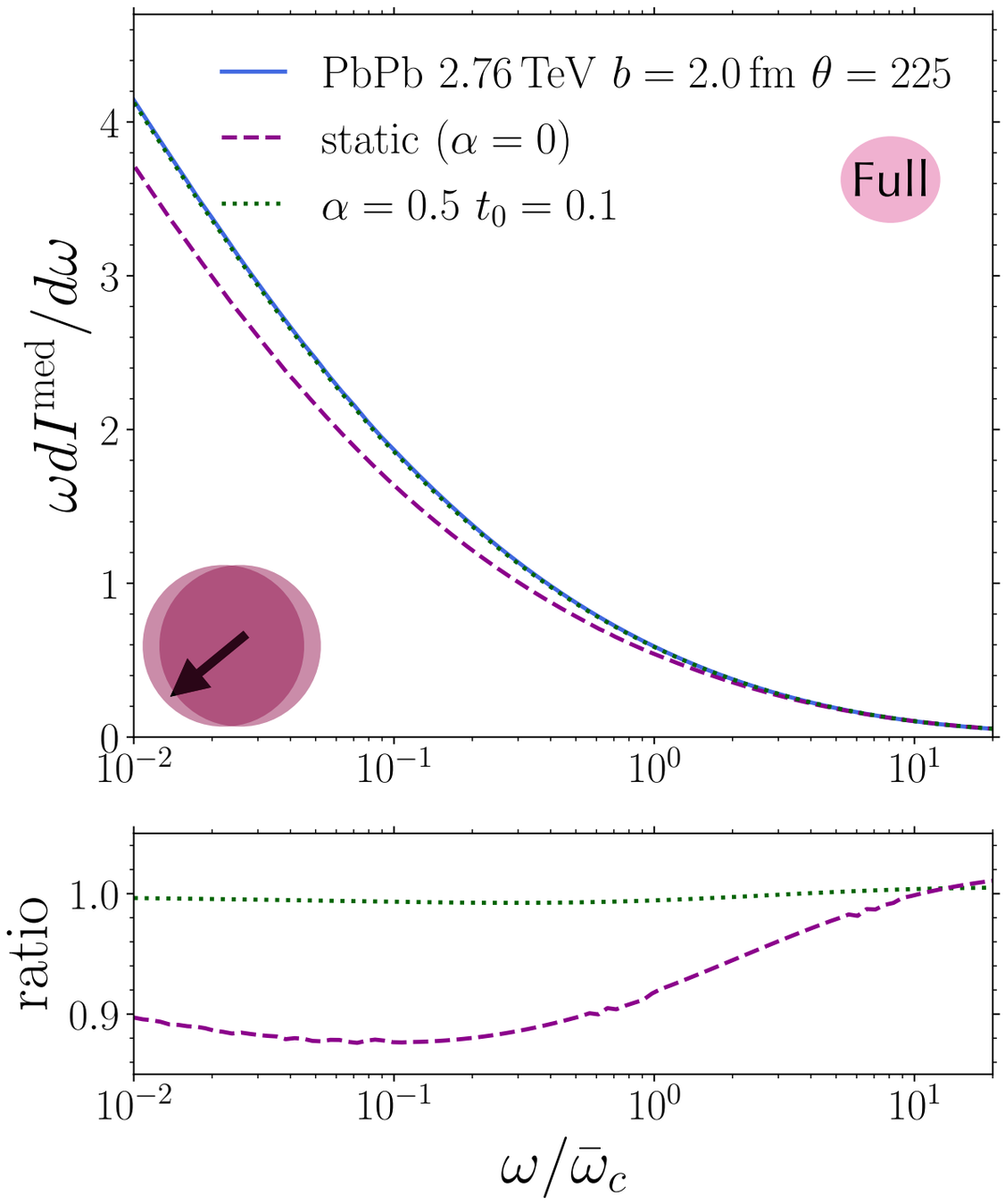
$$\int_0^{L_1} dt\, n(t) = \int_0^{L_2} dt\, n_{hydro}(t) \qquad \int_0^{L_1} dt\, t\, n(t)\, \mu^2(t) = \int_0^{L_2} dt\, t\, n_{hydro}(t)\, \mu^2_{hydro}(t)$$



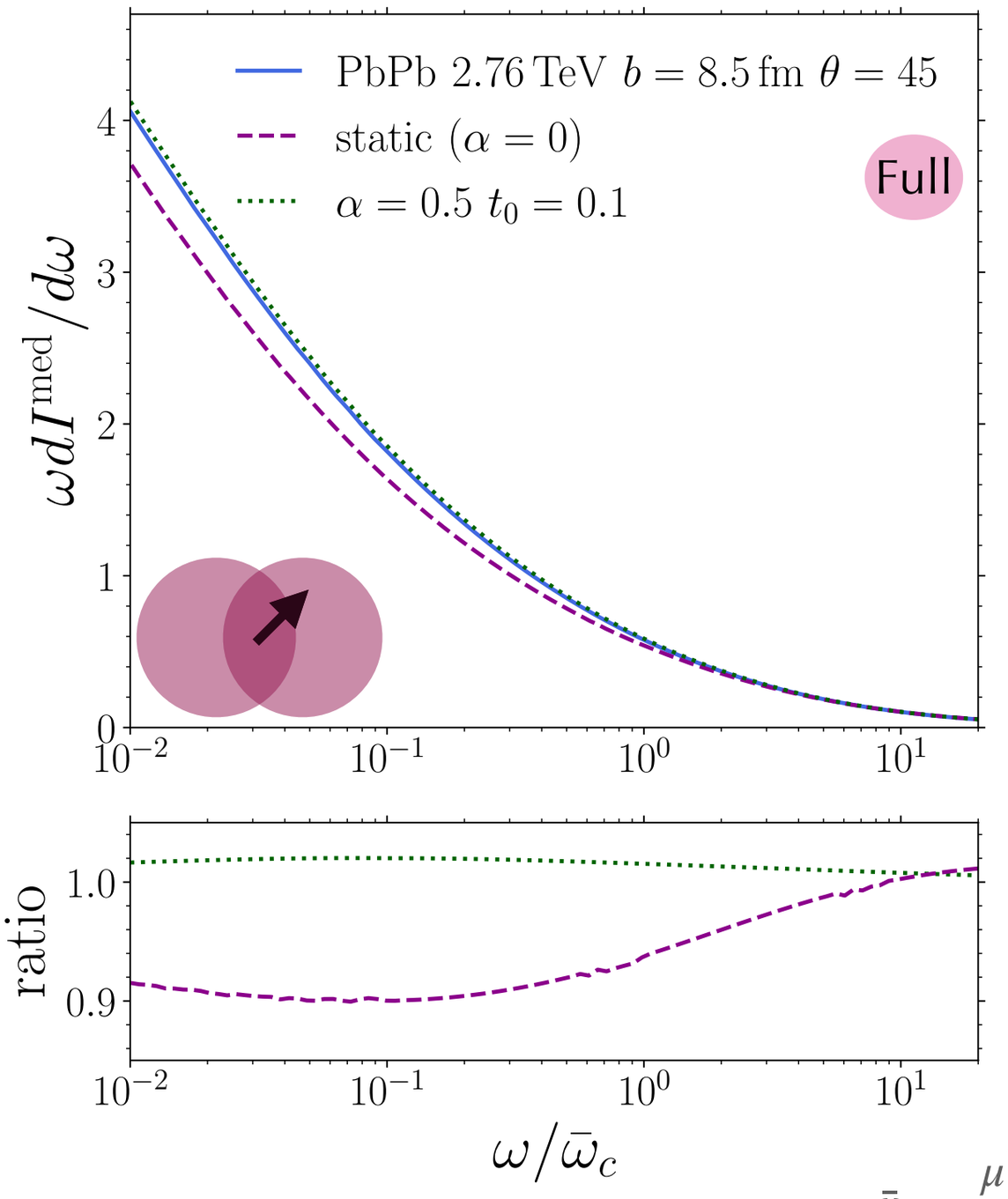
Hydro: Luzum and Romatschke, 0901.4588

Scaling laws w.r.t. a power-law case

- PbPb 2.76 TeV 0-5%



- PbPb 2.76 TeV 30-40%

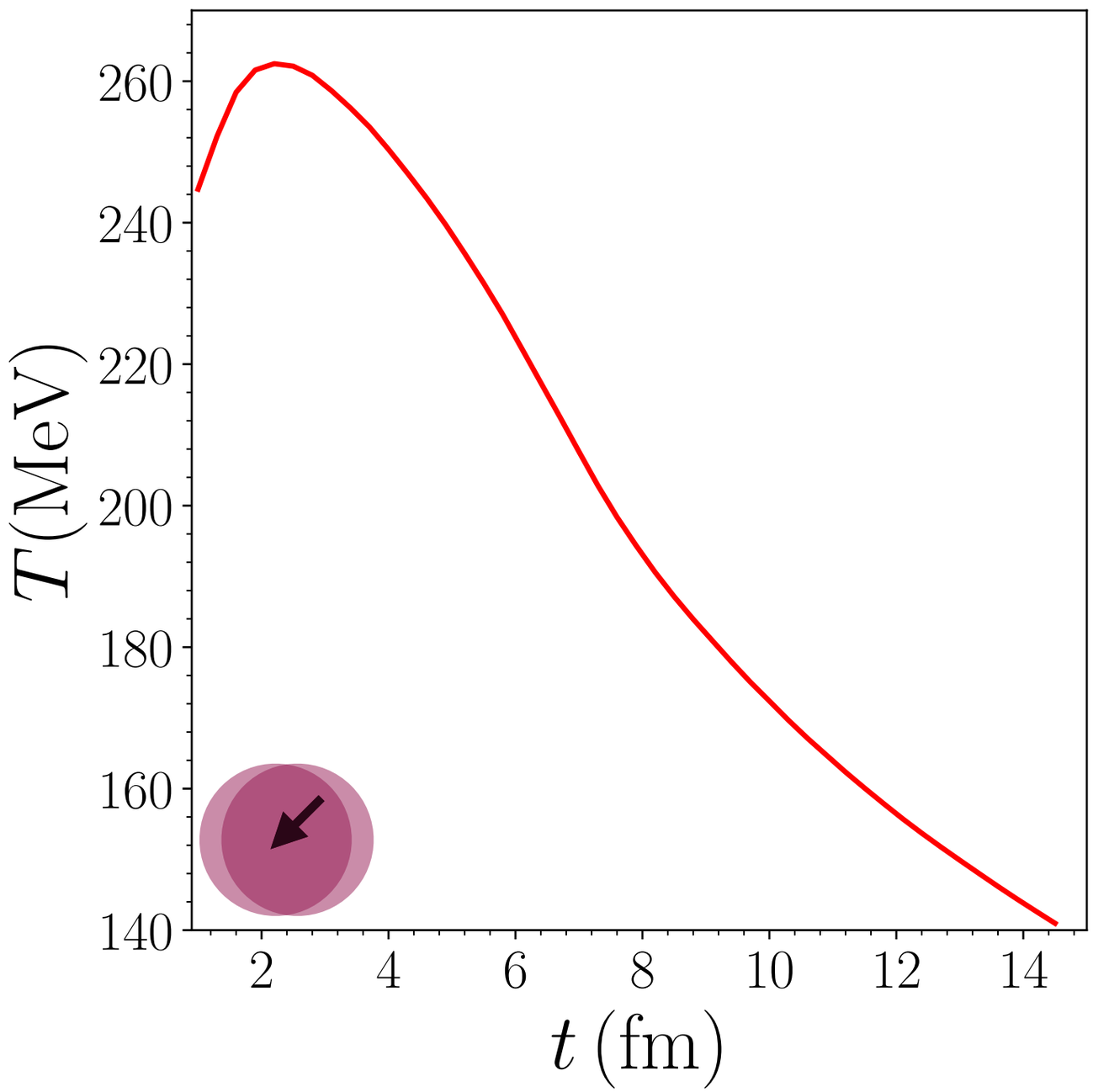


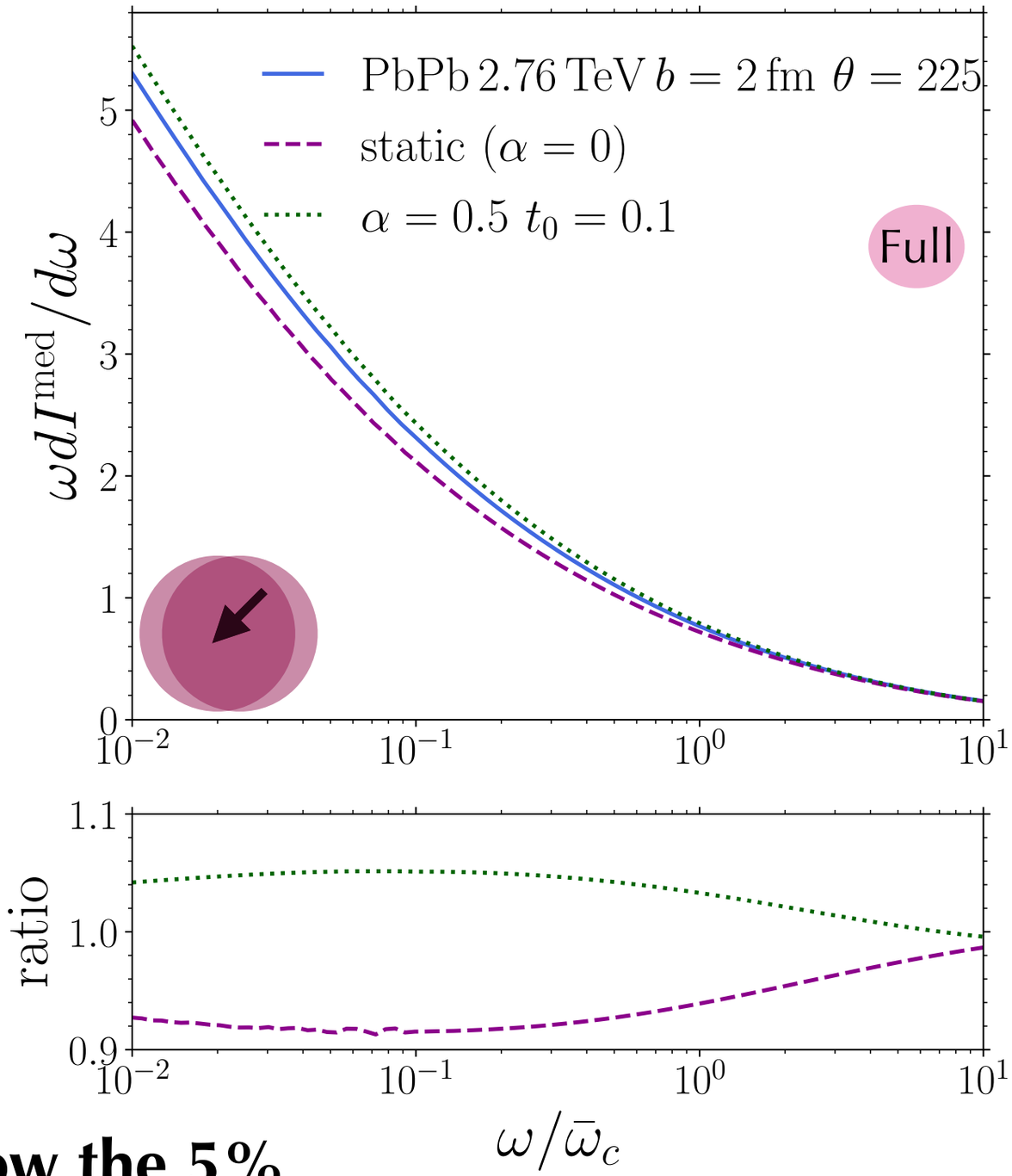
Hydro: Luzum and Romatschke, 0901.4588

$$\bar{\omega}_c = \frac{\mu^2 L}{2}$$

Rare cases

PbPb 2.76 TeV 0-5%





Errors below the 5%

$$\bar{\omega}_c = \frac{\mu^2 L}{2}$$

Hydro: Luzum and Romatschke, 0901.4588

Conclusions

- Many **new developments** in the computation of the **medium-induced radiation** spectrum (in **static media**)

- Numerical approaches **allow to compute the spectrum along a path in realistic media** (given by a hydrodynamics simulation)

- But it is computationally demanding

- We need to use **pre-compute the spectra** for a set of given profiles approximating realistic conditions (scaling laws)

- Using **power-law profiles reduces the errors substantially**

Merci!

Beyond the usual approximations

- Several **new approaches go beyond the usual approximations (in static media)**

Finite length rates:

Caron-Huot and Gale, 1006.2379

Fully resummed spectrum:

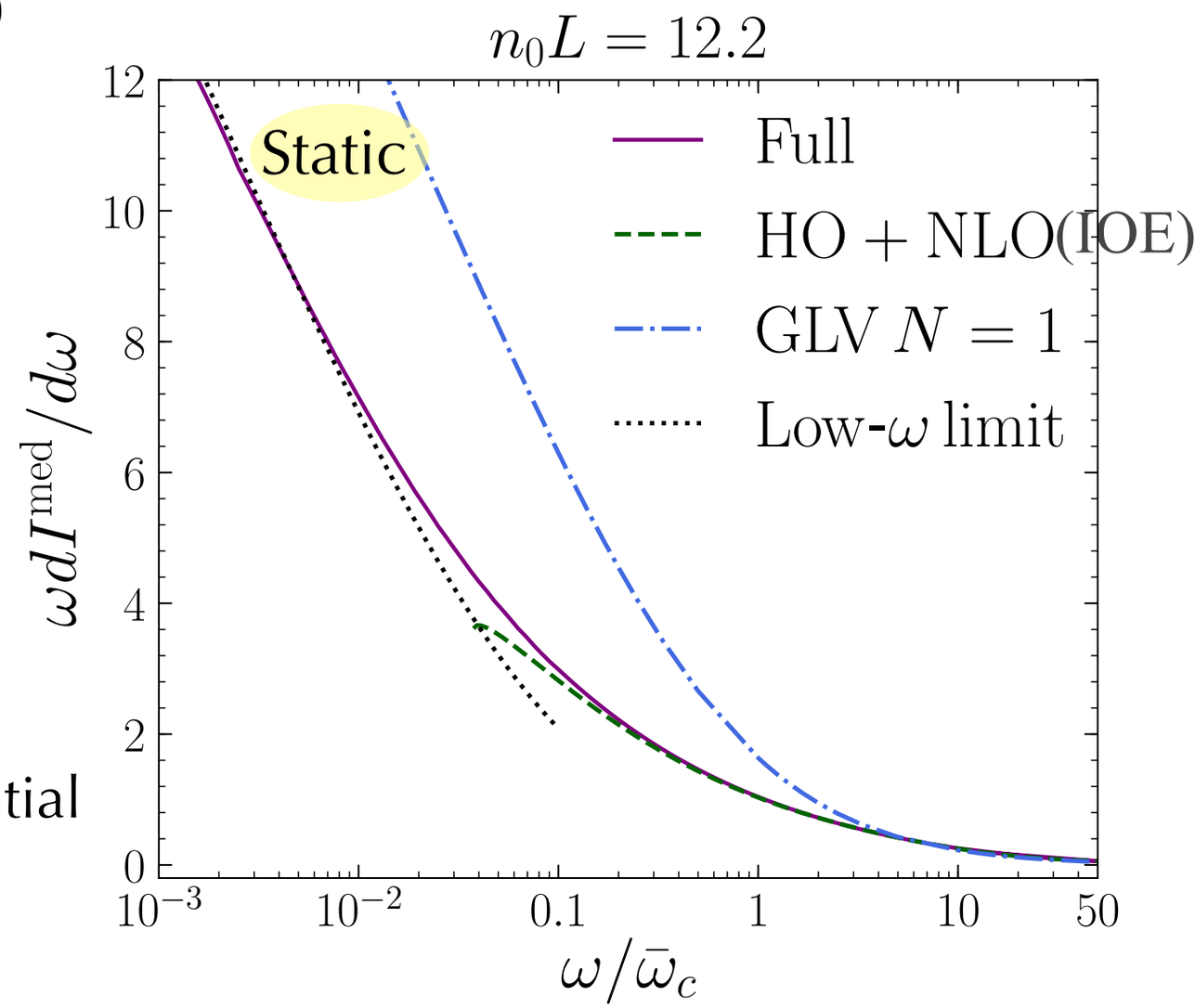
CA, Apolinario, Martinez, Dominguez 2002.01517, 2011.06522

IOE (expansion around the HO):

Mehtar-Tani, Barata, Soto-Ontoso, Tywoniuk, 1903.00506, 2106.07402

Finite length rates + non-perturbative potential

Schlichting, Soudi, 2111.13731



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CA, M. G. Martinez, F. Dominguez, 2011.06522

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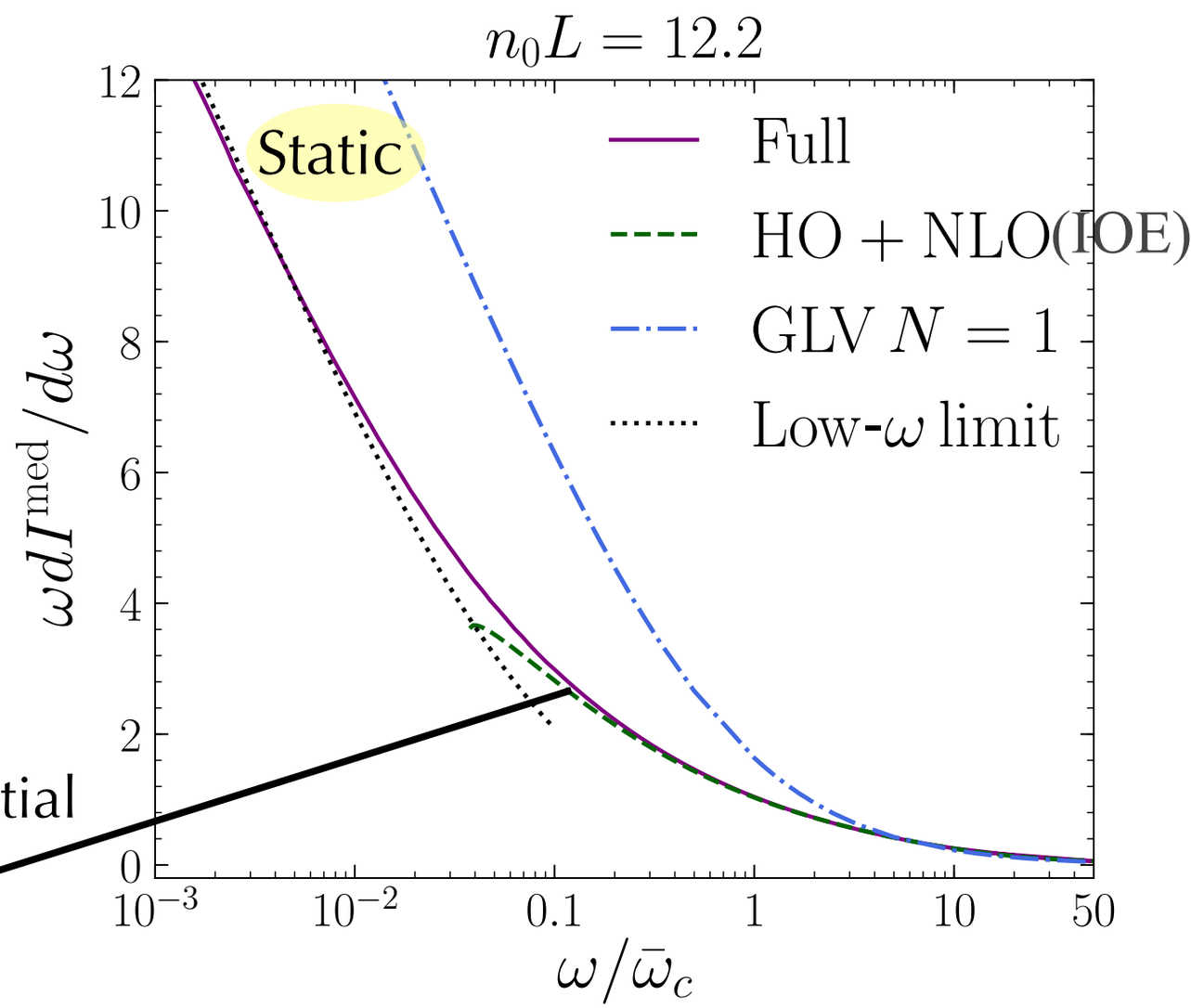

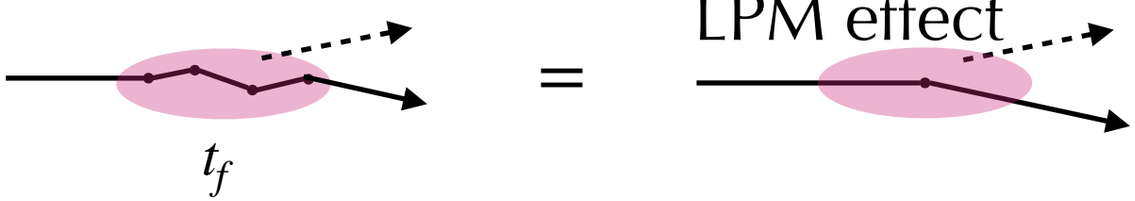

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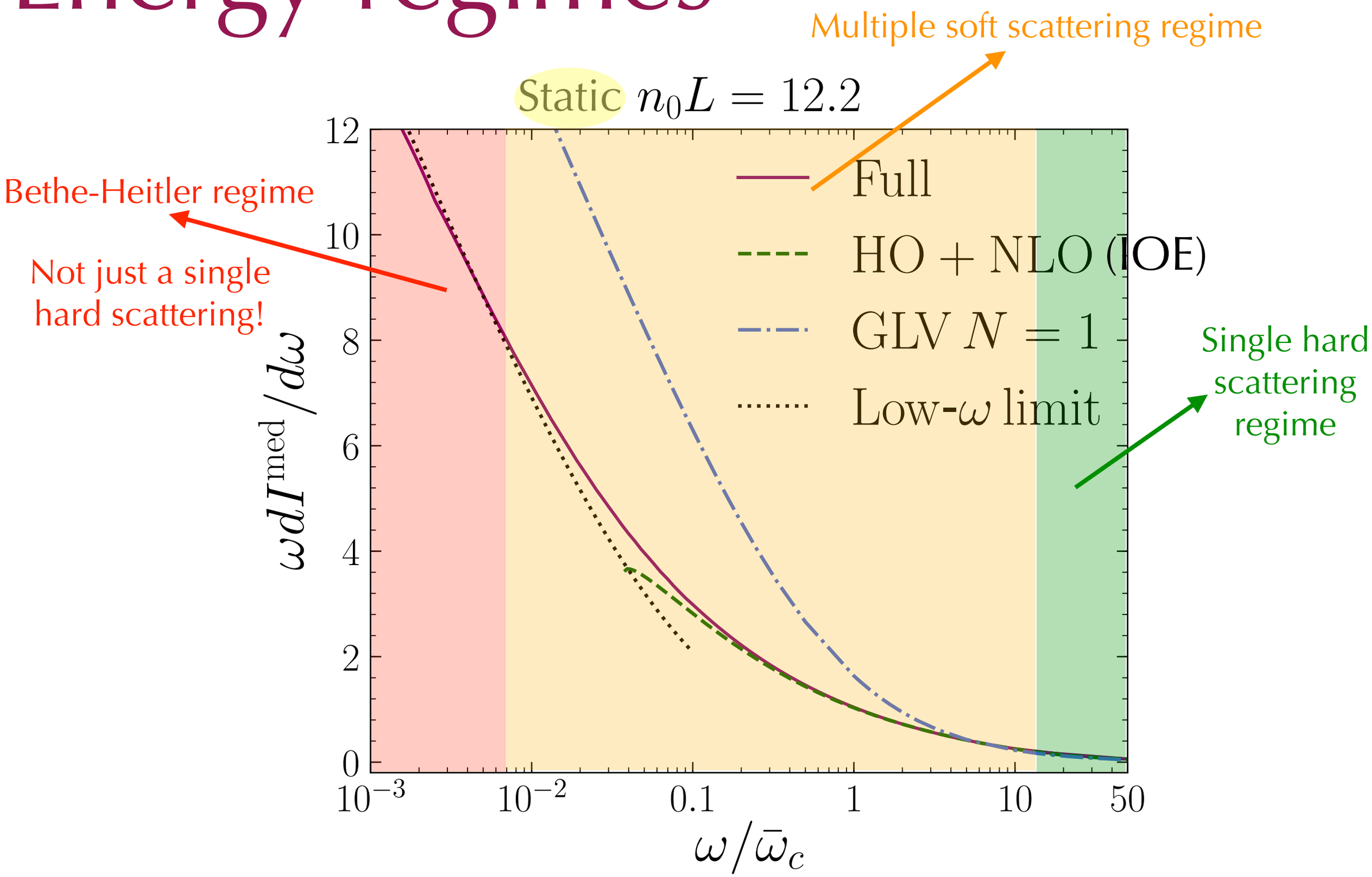
CA, M. G. Martinez, F. Dominguez, 2011.06522

Energy regimes



CA, M. G. Martinez, F. Dominguez, 2011.06522

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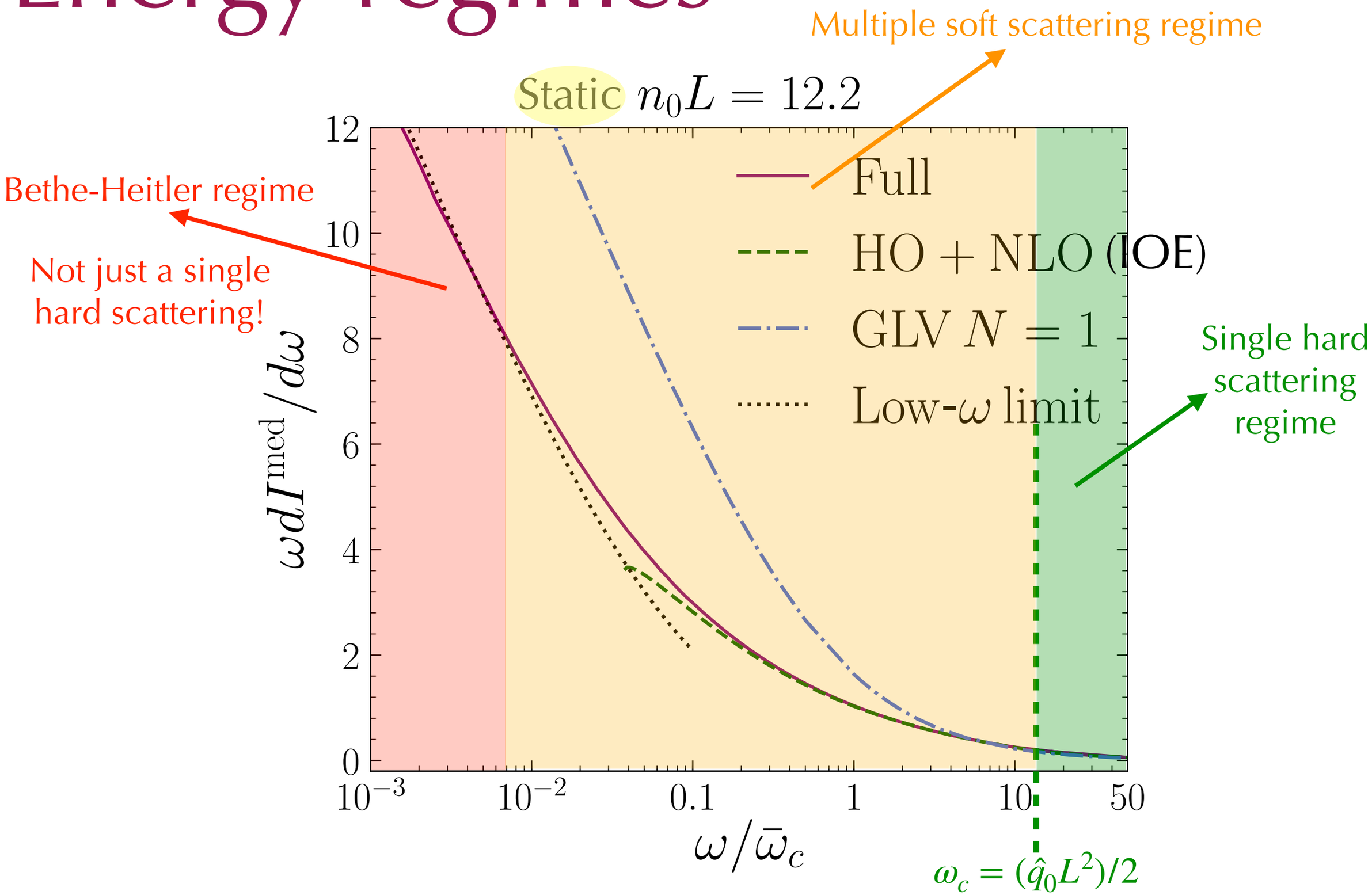

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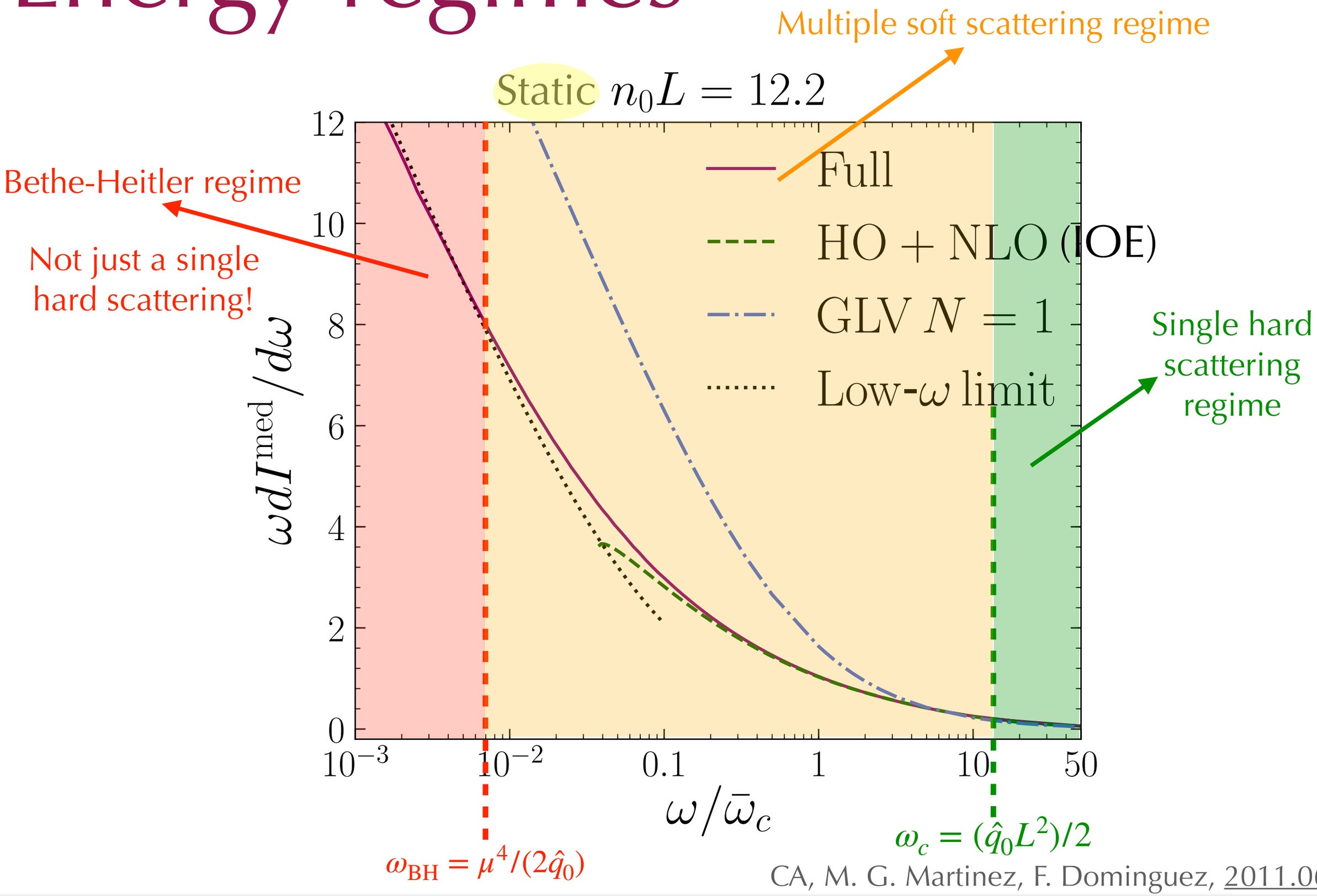

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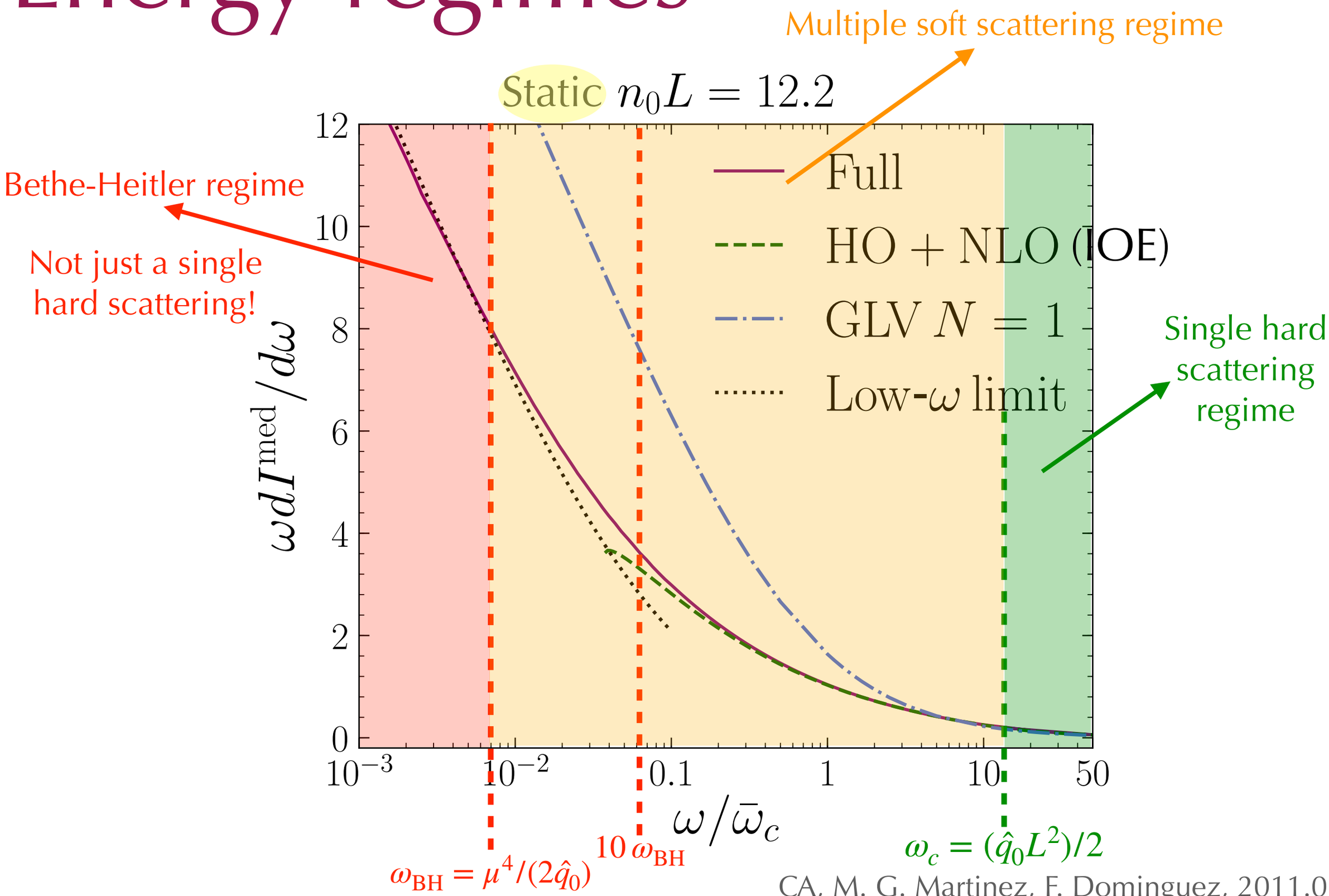

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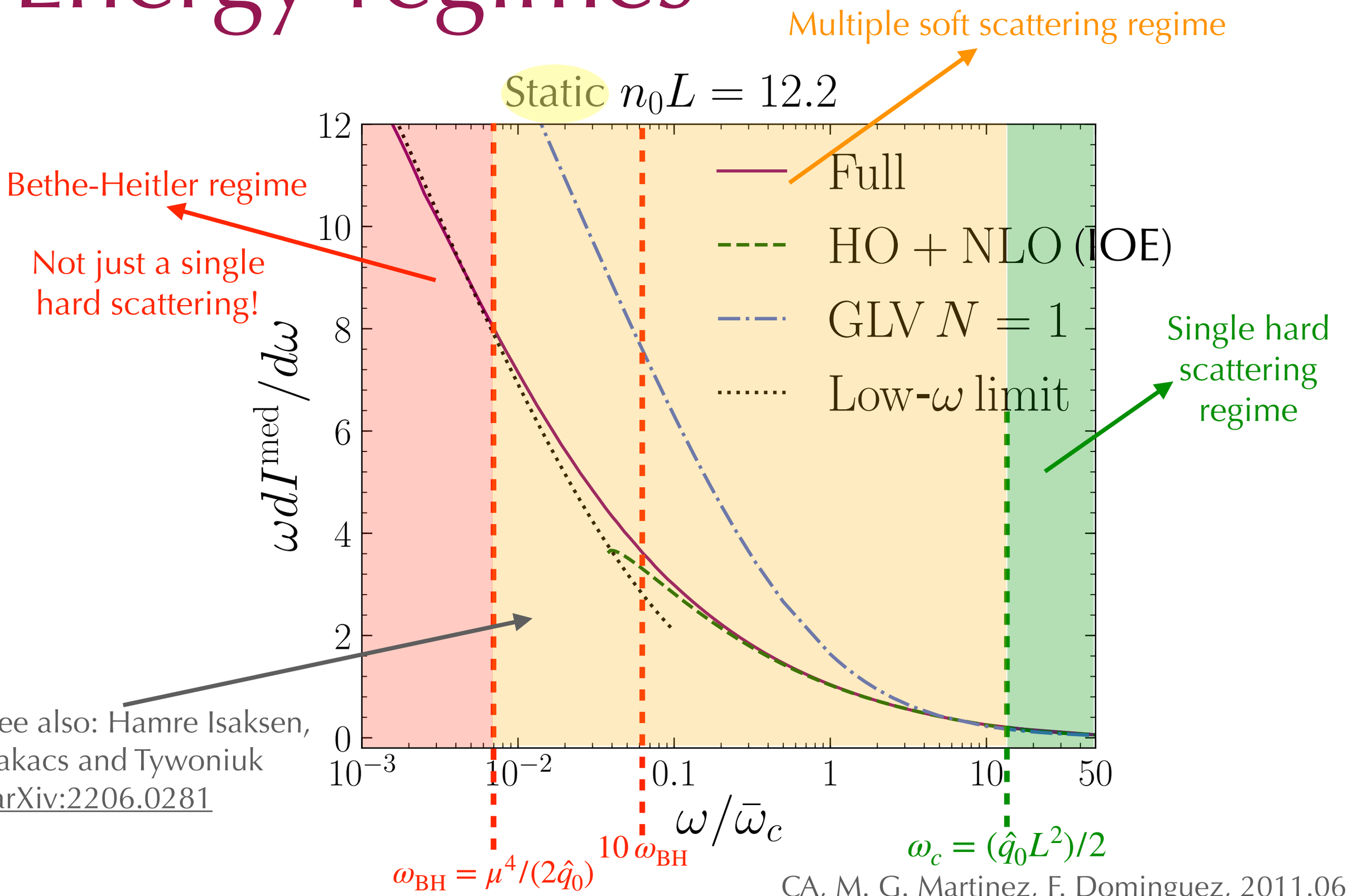


See also: Hamre Isaksen, Takacs and Tywoniuk arXiv:2206.0281

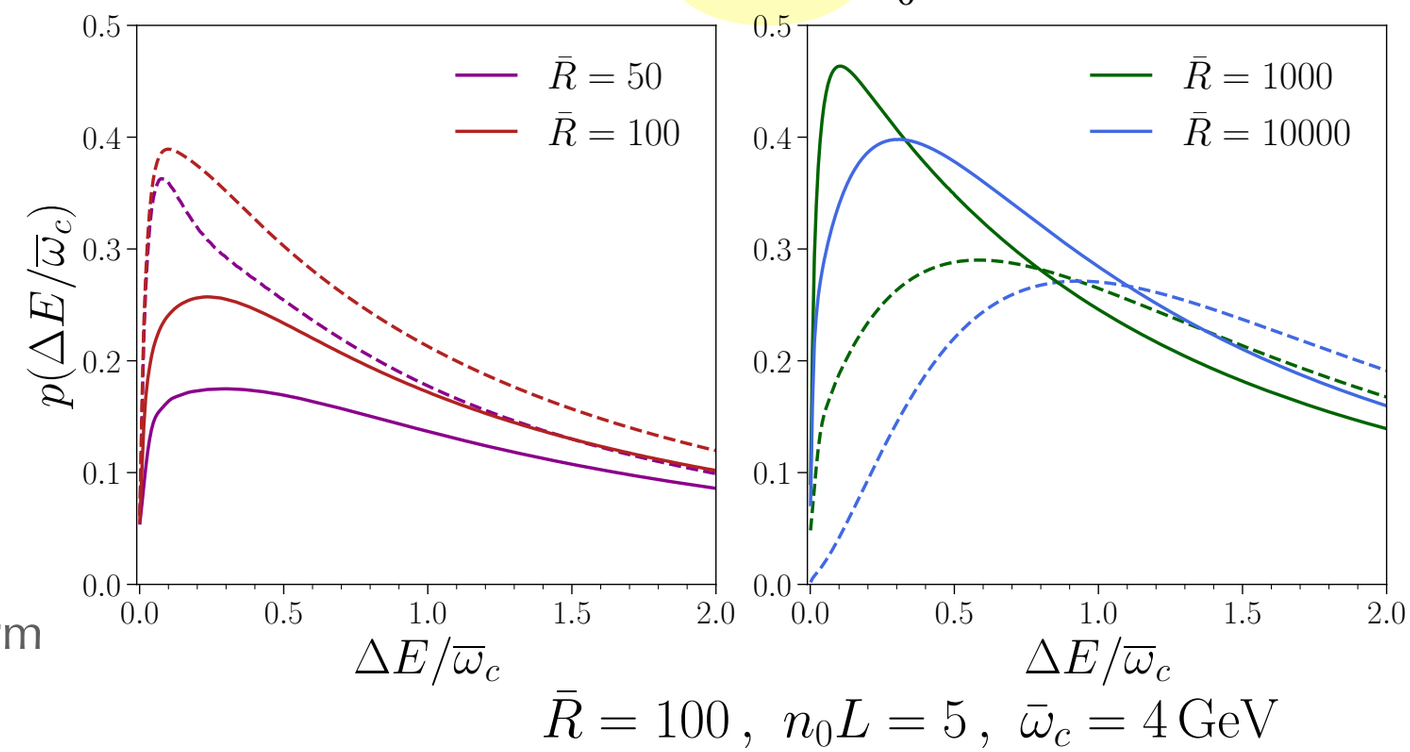
CA, M. G. Martinez, F. Dominguez, 2011.06522

Towards phenomenology

Hadron suppression

GLV
Full

Static $n_0 L = 5$



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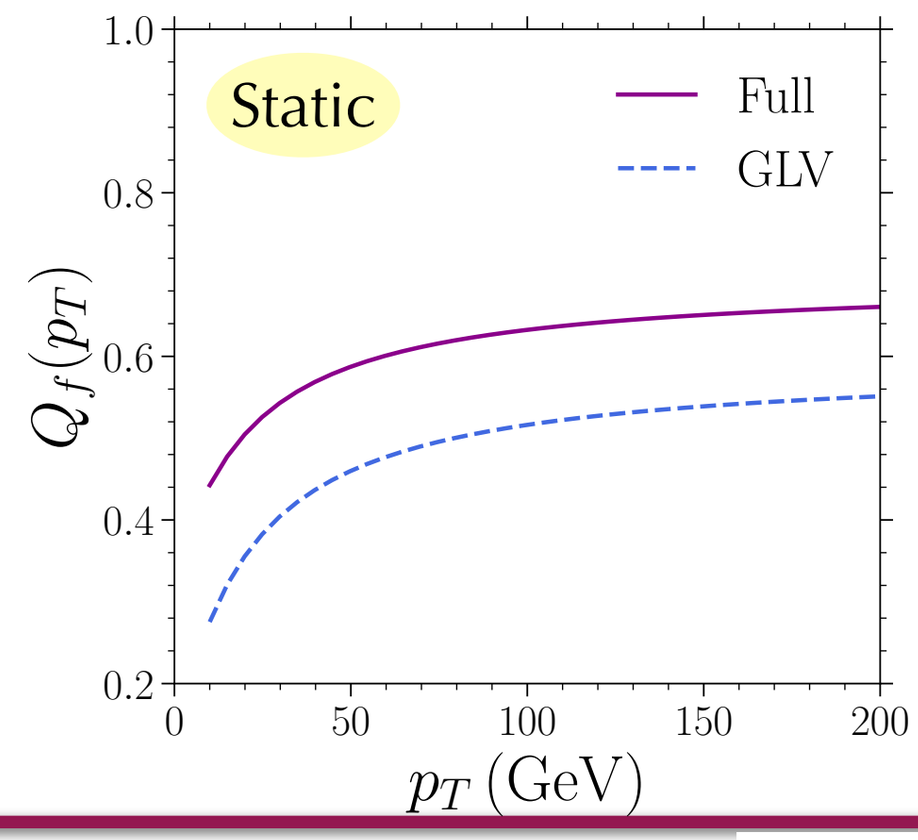

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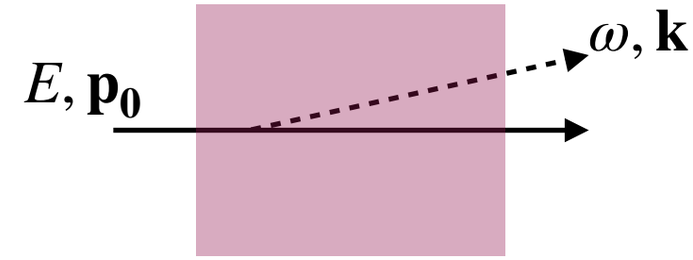
Energy distribution



$$\omega \frac{dI}{d\omega d^2\boldsymbol{k}} = \frac{2\alpha_s C_R}{(2\pi)^2\omega} \mathrm{Re} \int_0^\infty ds\, n(s) \int_0^s dt \int_{\boldsymbol{pql}} i\boldsymbol{p} \cdot \left(\frac{\boldsymbol{l}}{\boldsymbol{l}^2} - \frac{\boldsymbol{q}}{\boldsymbol{q}^2} \right) \sigma(\boldsymbol{l} - \boldsymbol{q}) \widetilde{\mathcal{K}}(s, \boldsymbol{q}; t, \boldsymbol{p}) \mathcal{P}(\infty, \boldsymbol{k}; s, \boldsymbol{l})$$

$k < \omega$

- Up to now: we have removed this kinematic constraint:

$$\omega \frac{dI}{d\omega} = \int_0^\infty \omega \frac{dI}{d\omega d^2\boldsymbol{k}^2} d\boldsymbol{k}^2$$

So we get rid of the broadening

$$\omega \frac{dI^{\mathrm{med}}}{d\omega} = \frac{2\alpha_s C_R}{\omega} \mathrm{Re} \int_0^\infty ds\, n(s) \int_0^s dt \int_{\boldsymbol{pql}} i \frac{\boldsymbol{p} \cdot \boldsymbol{q}}{\boldsymbol{q}^2} \sigma(\boldsymbol{q} - \boldsymbol{l}) \widetilde{\mathcal{K}}(s, \boldsymbol{l}; t, \boldsymbol{p})$$

- Integrate in transverse momentum with the constraint:

$$\omega \frac{dI}{d\omega} = \int_0^{\omega^2} \omega \frac{dI}{d\omega d^2\boldsymbol{k}^2} d\boldsymbol{k}^2$$

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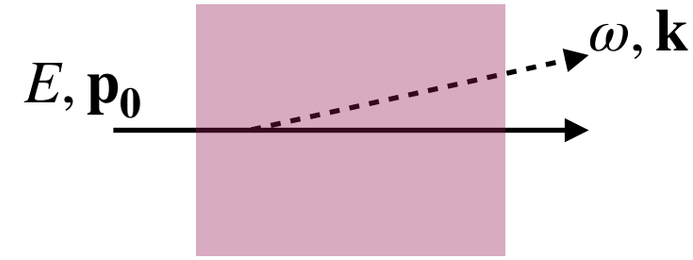

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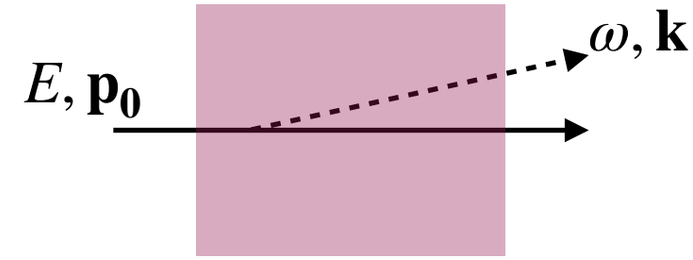
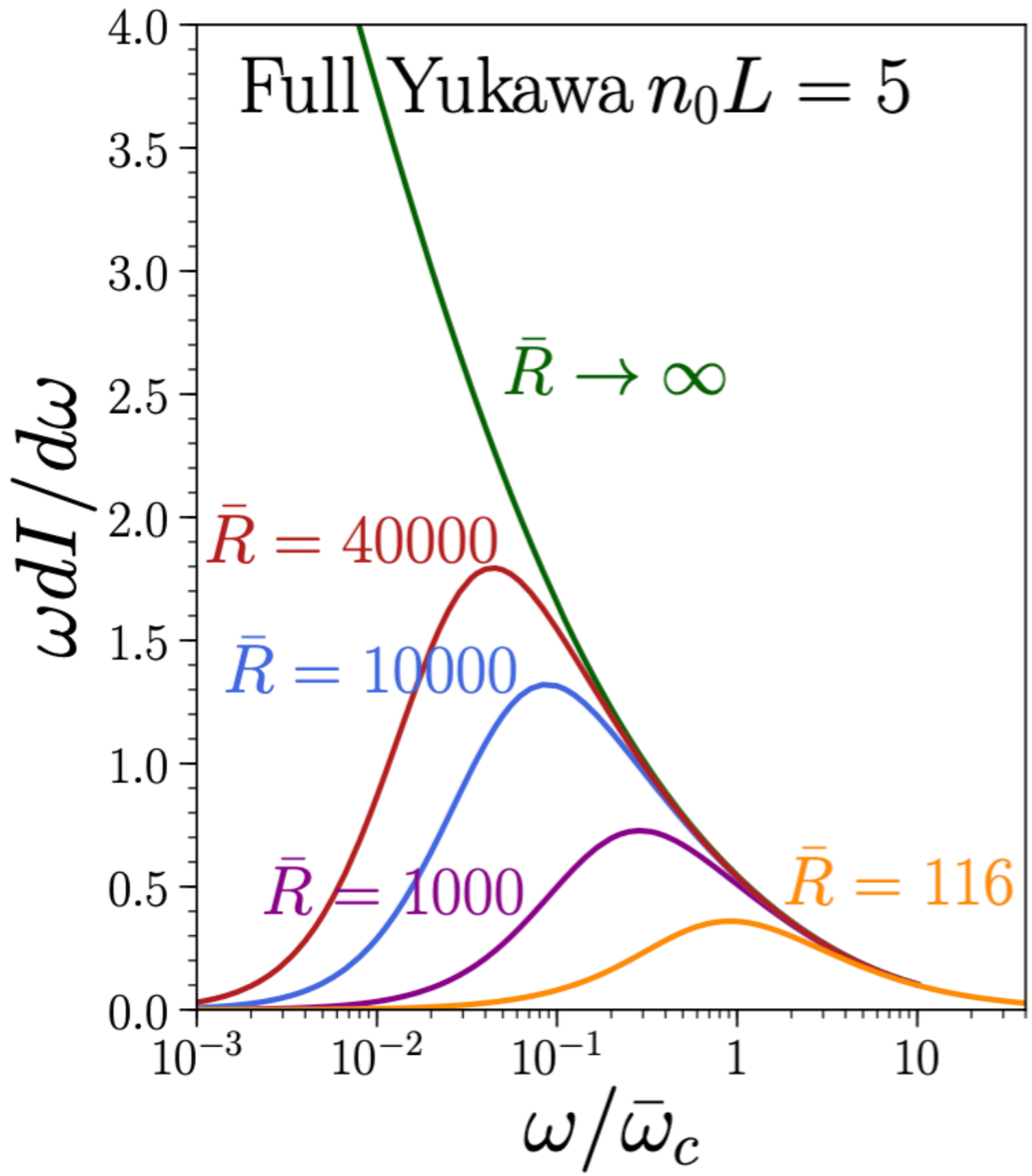
$$\bar{R} = \bar{\omega}_c L$$

Energy distribution



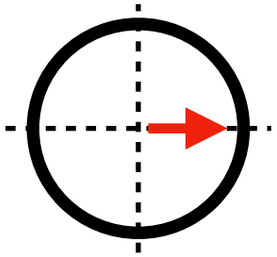


$$n_0 L$$

$$\bar{\omega}_c = \frac{\mu^2 L}{2}$$

$$\bar{R} = \bar{\omega}_c L$$

Scaling laws? Full



- Compute the full solution along a path thorough a hydro

$$n_{hydro}(t) = k_1 T(t) \qquad \mu^2_{hydro}(t) = k_2 T^2(t)$$

- Compare to a power-law spectrum given by

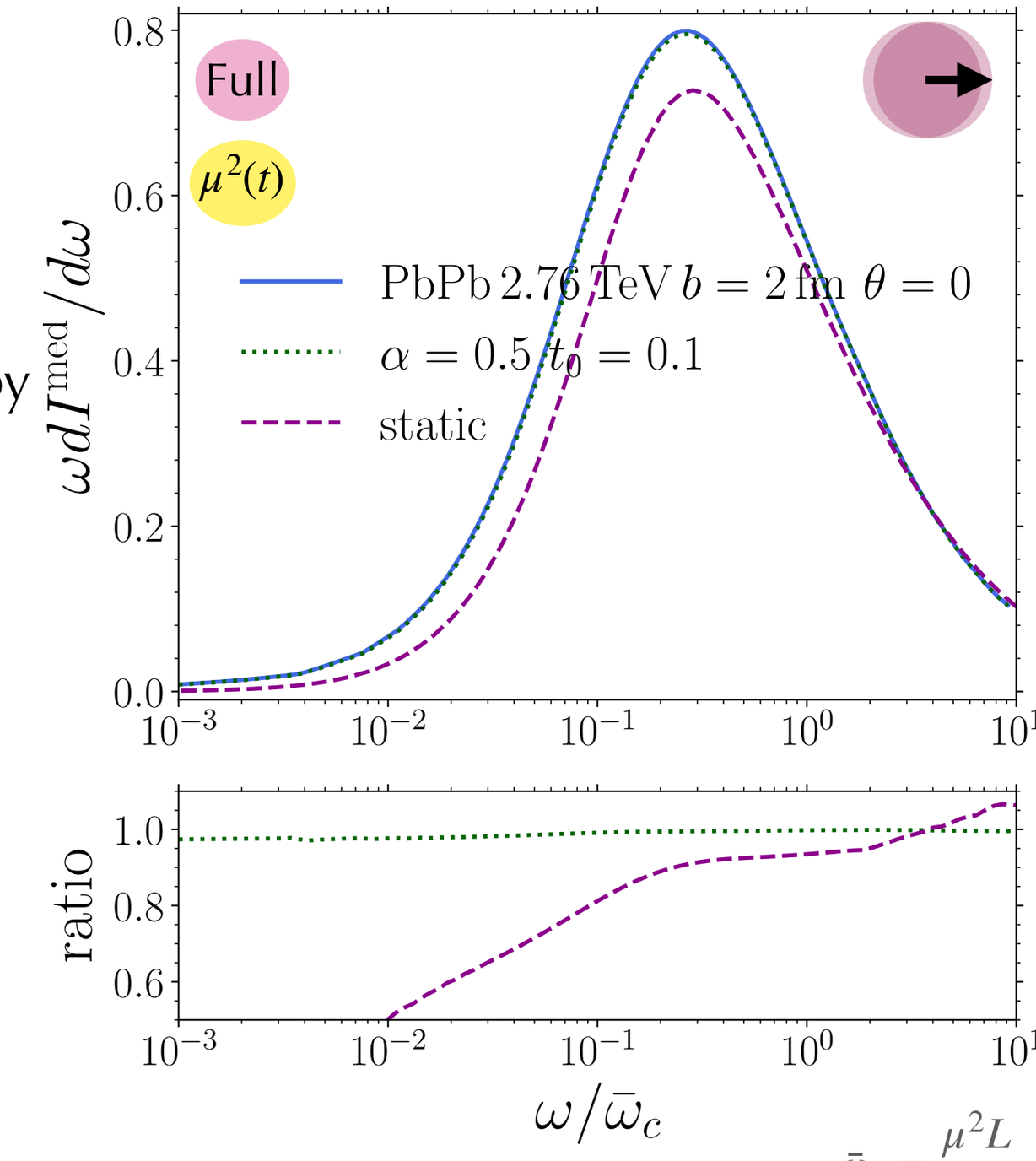
$$n(t) = \frac{n_0'}{(t+t_0)^\alpha} \qquad \mu^2(t) = \frac{\mu'^2}{(t+t_0)^{2\alpha}}$$

- Using the following scaling laws

$$\int_0^{L_1} dt\, n(t) = \int_0^{L_2} dt\, n_{hydro}(t)$$

$$\int_0^{L_1} dt\, t\, n(t)\, \mu^2(t) = \int_0^{L_2} dt\, t\, n_{hydro}(t)\, \mu^2_{hydro}(t)$$

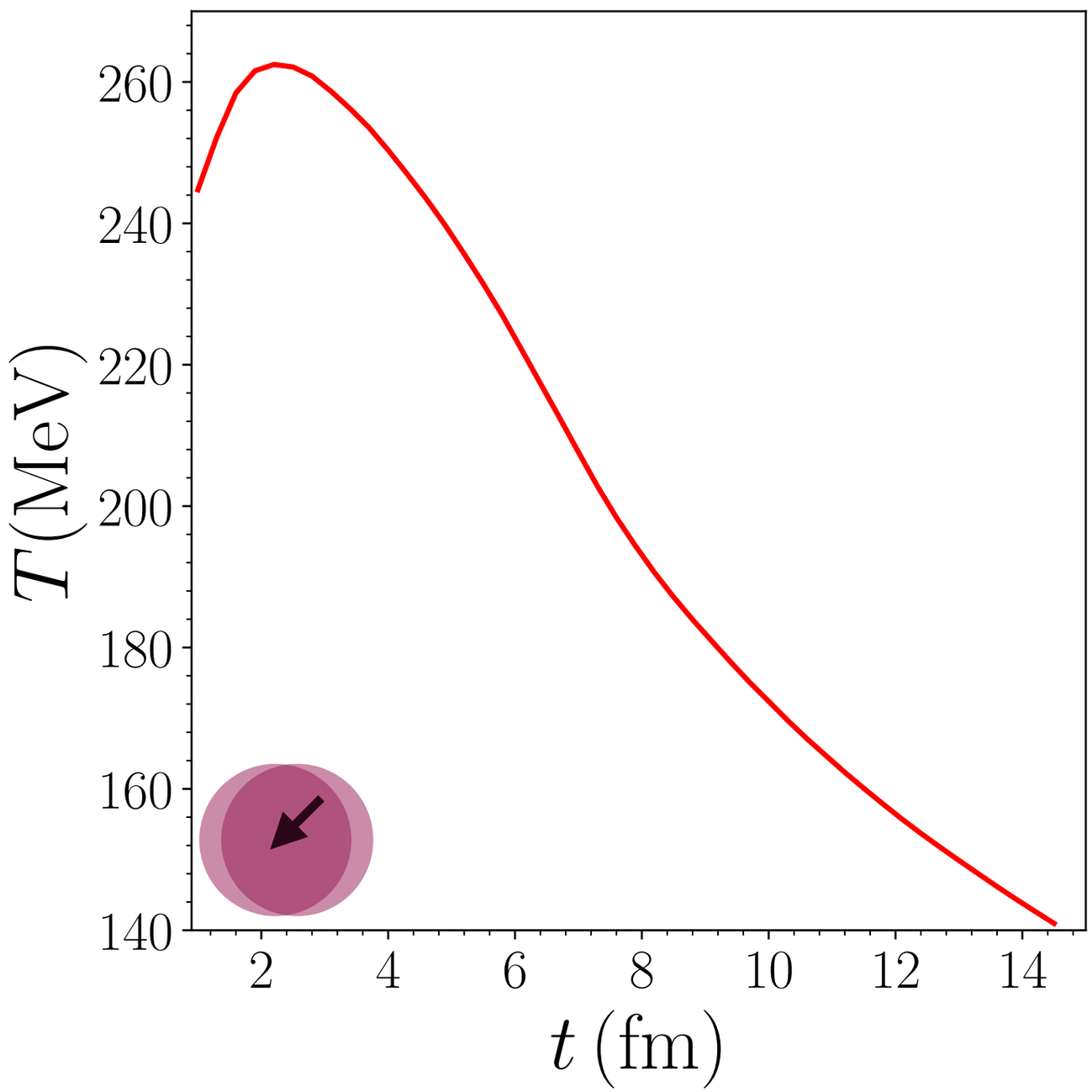
$$\int_0^{L_1} dt\, t^2\, n(t)\, \mu^2(t) = \int_0^{L_2} dt\, t^2\, n_{hydro}(t)\, \mu^2_{hydro}(t)$$

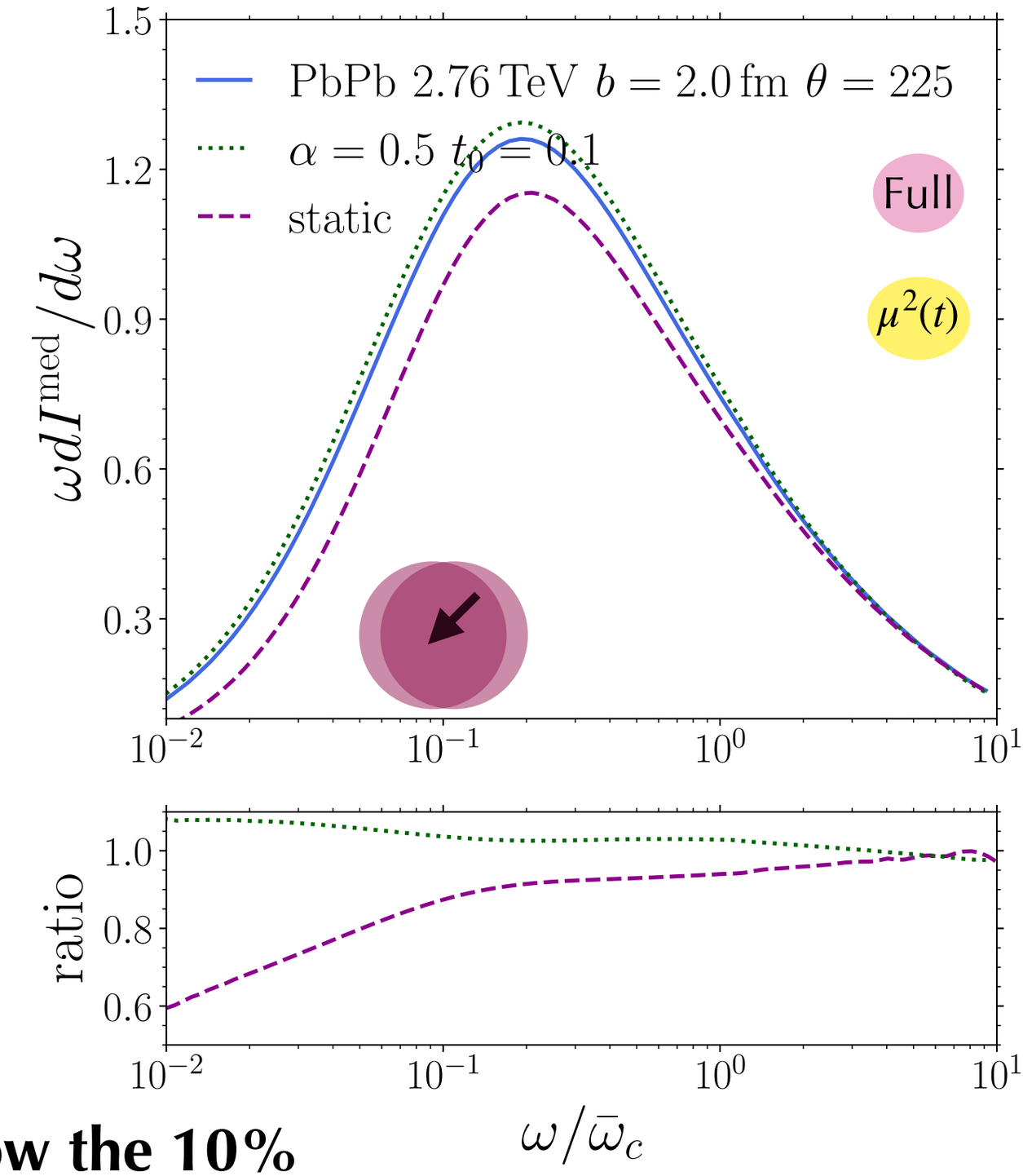


Hydro: Luzum and Romatschke, 0901.4588

Rare case

PbPb 2.76 TeV 0-5%





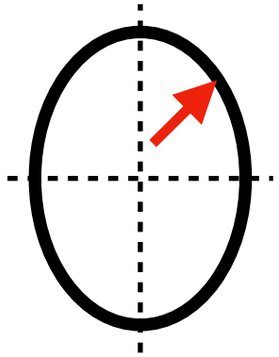
Errors below the 10%

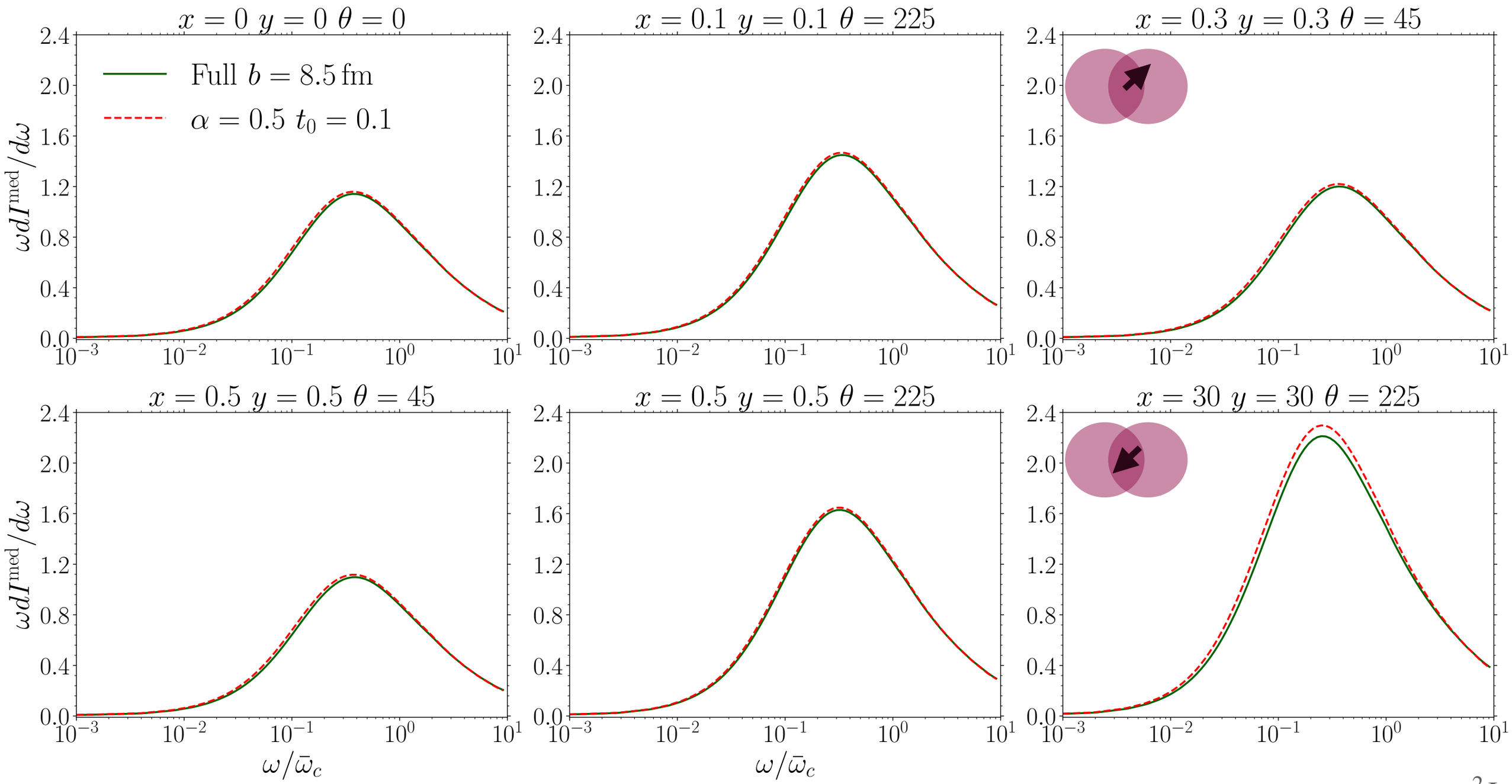
$$\bar{\omega}_c = \frac{\mu^2 L}{2}$$

Hydro: Luzum and Romatschke, 0901.4588

Summary

PbPb 2.76 TeV 30-40%





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Hydro: Luzum and Romatschke, 0901.4588