TMD distributions at the next-to-leading power.

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Outline

Transverse-momentum dependent distributions
What? Why? How?

Why going to next-to-leading power?

How? A few technical details

A complete example: quasi-TMD distribution in lattice QCD

What are TMD distributions?

Parton distributions that depends on the parton's
Longitudinal (light-cone) momentum and transverse momentum

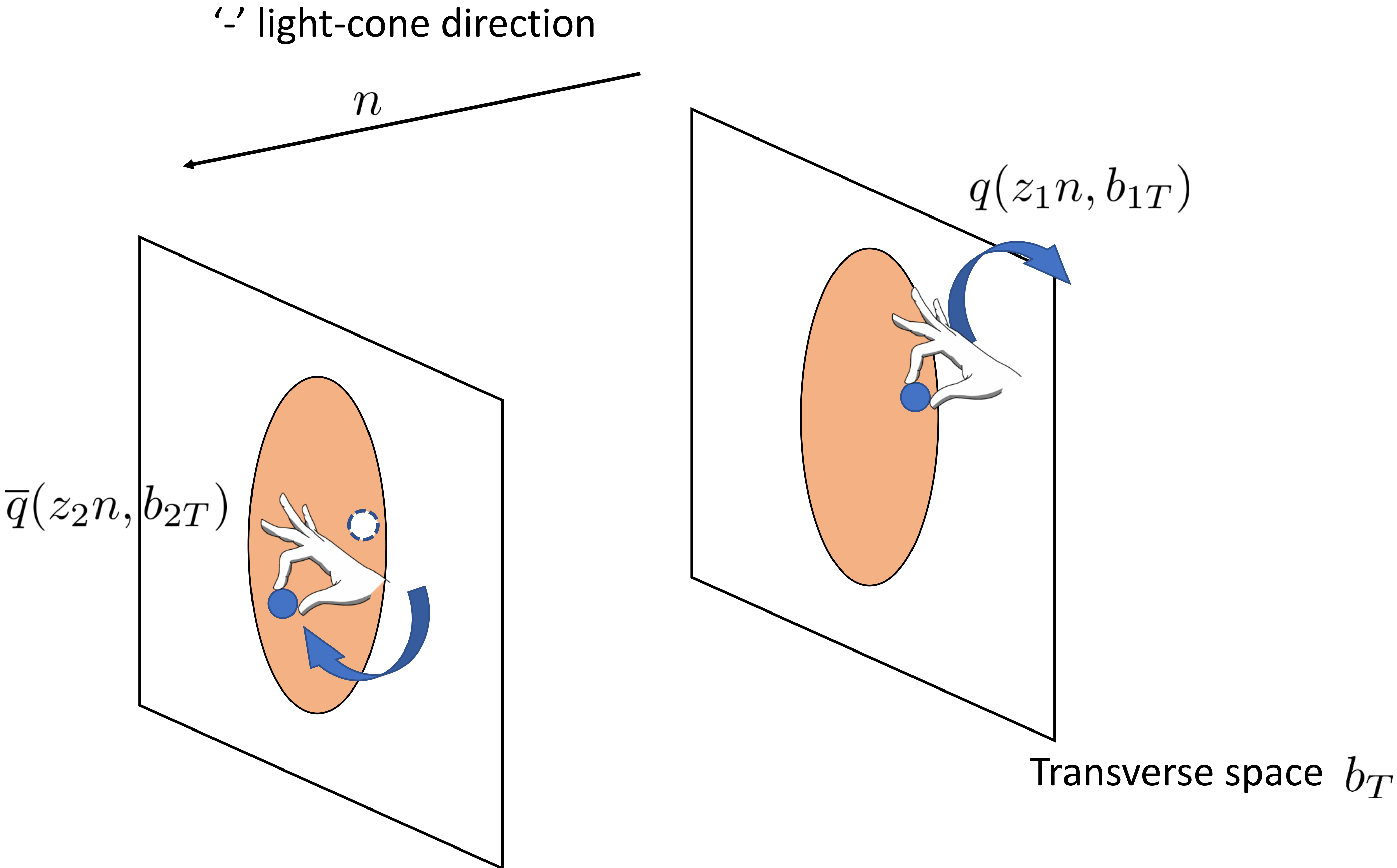
Often are considered only quark-quark and gluon-gluon parton distributions

$$\underbrace{\langle \text{proton}, p|}\bar{q}(\ldots)q(\ldots)|\text{proton}, p\rangle$$

Can be considered a medium filled with strongly-interacting QCD degrees of freedom

Fixed momentum in the '+' light-cone direction

What are TMD distributions?



A bit more formally

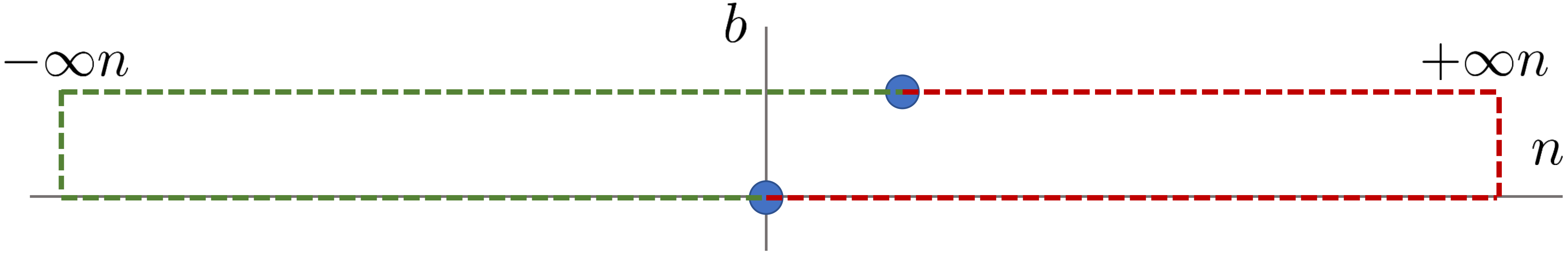
$$\Phi_{ij}(x,b) = \int \frac{dz}{2\pi} e^{-ixzp^+} \langle p,S|\bar{q}_j(zn+b)q_i(0)|p,S\rangle$$

How do we restore gauge-invariance?

⇔

What type of Wilson line path should we use?

Depends entirely on the process!



$$\Phi_{ij}(x,b) = \int \frac{dz}{2\pi} e^{-ixzp^+} \langle p,S|\bar{q}_j(zn+b)[zn+b,-\infty n+b][-\infty n,0]q_i(0)|p,S\rangle$$

A bit more formally

$$\mathrm{Tr}\left[\Phi(x,b)\Gamma\right]$$

$$\Gamma \in \{\gamma^+, \gamma^+\gamma_5, i\sigma^{\alpha+}\gamma_5\}$$

	U	H	T
U	f_1		$h_1^\perp$
L		g_1	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

Well-defined (they have close evolution equations)

For all other possible choices of Gamma this is not true anymore

Why we need TMD distributions?

Allow for more information on the internal structure of the proton

Spin-orbit correlations between proton spin and parton's transverse momentum

⇔

Sivers distribution

They enters the description of measured physical processes

Their scaling properties are related to the structure of QCD vacuum

Drell-Yan

proton, proton → lepton, lepton, X

SIDIS

proton, lepton → lepton, H, X

Can be accessed partially from lattice calculations

Why we need TMD distributions?

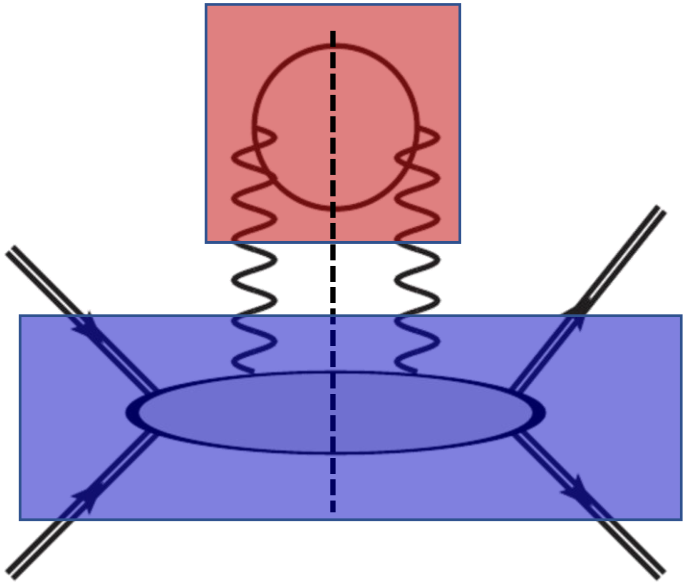
The language

Hadronic tensor

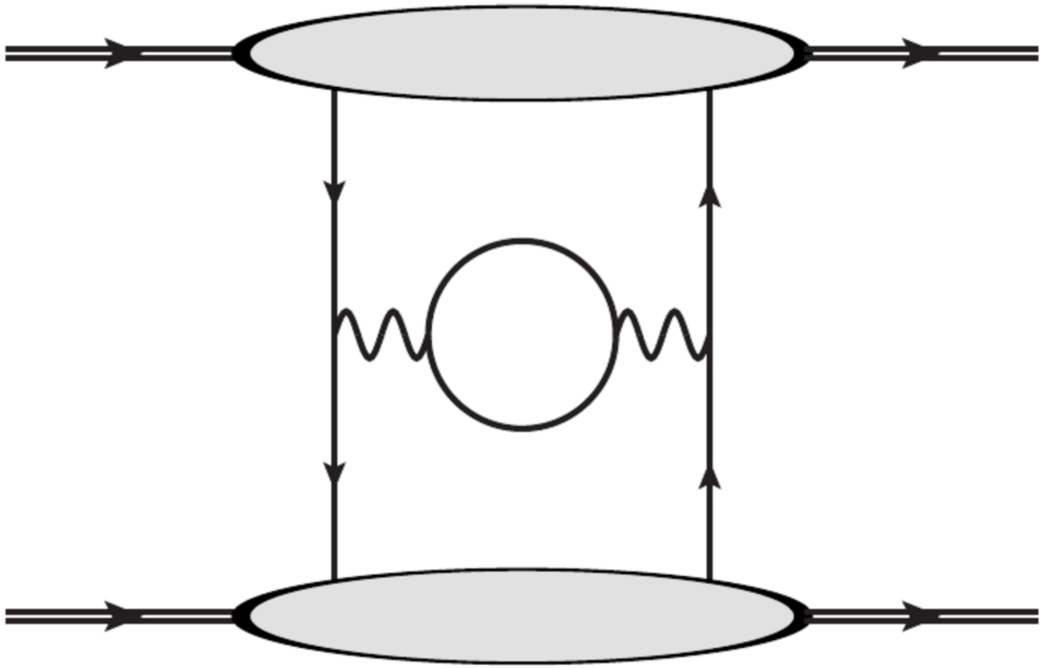
$$H^{\mu\nu} = \sum_X \langle p_1, p_2 | J^\mu(y) | X \rangle \langle X | J^\nu(0) | p_1, p_2 \rangle$$

TMD distributions

Leptonic tensor $L^{\mu\nu}$



Leading-power factorization



How do we extract them?

The measurement has to be differential in a ‘transverse’ variable

For DY the transverse component
of the virtual photon
respect to the proton-proton plane

For SIDIS the transverse component
of the produced hadron
respect to the proton-photon plane

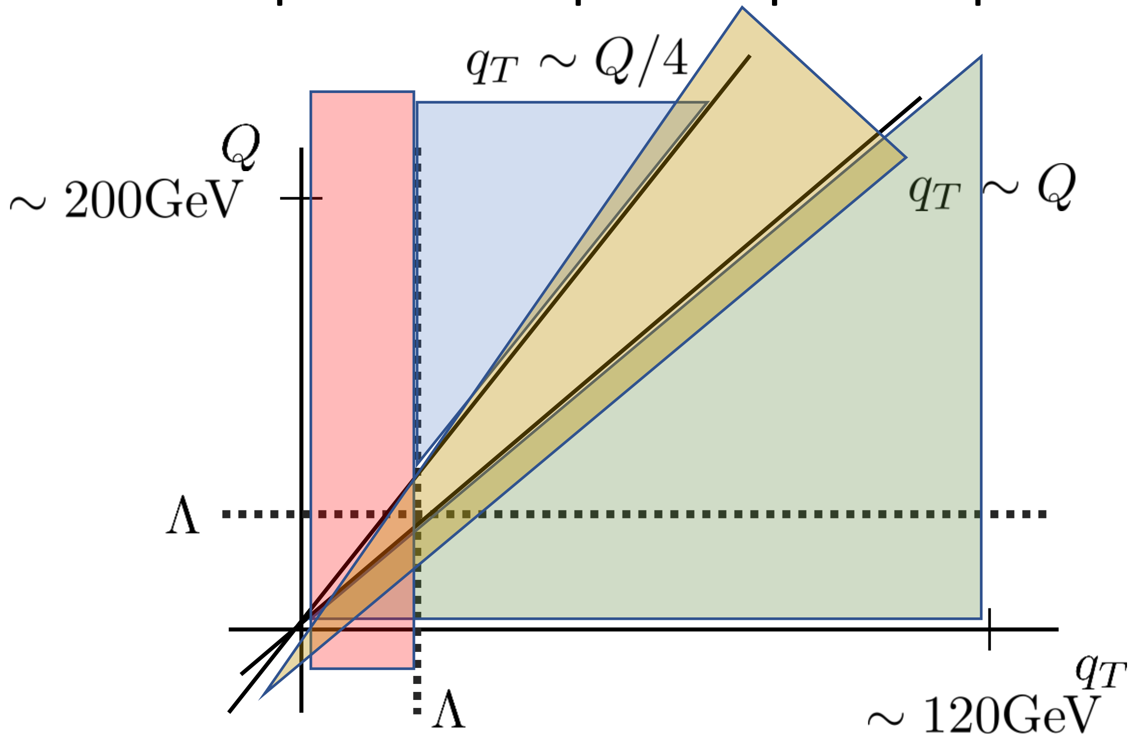
Need to select the
appropriate kinematic region

Fixed order

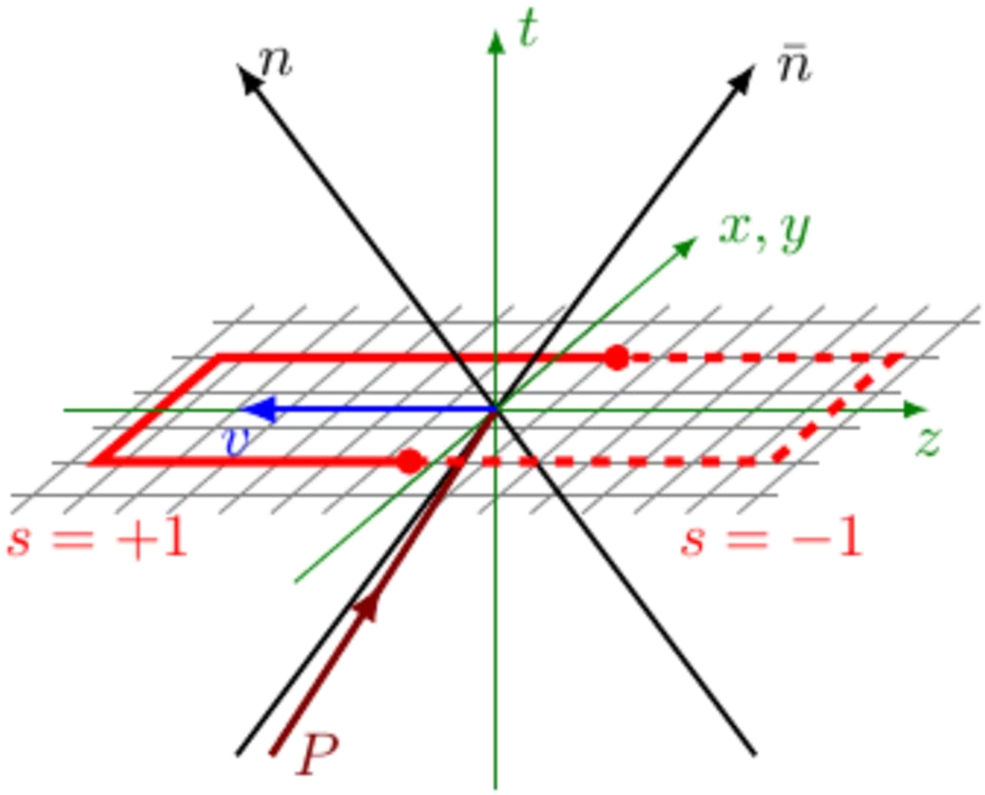
Non-perturbative TMD

Resummation

Next-to-leading power



Lattice observables



$$\boxed{W^{ij}(y_T; \ell, L; v, P, S)}$$
$$= \langle P, S | \bar{q}^j(y_T + \ell v)[y_T + \ell v; y_T + Lv][Lv; 0] q^i(0) | P, S \rangle$$

Conjugate of the current

Current, similar to DY/SIDIS

More similar to DY/SIDIS hadronic tensor than to TMD correlator

DY/SIDIS current

$$J^\mu(y) = \bar{q}(y)\gamma^\mu q(y)$$

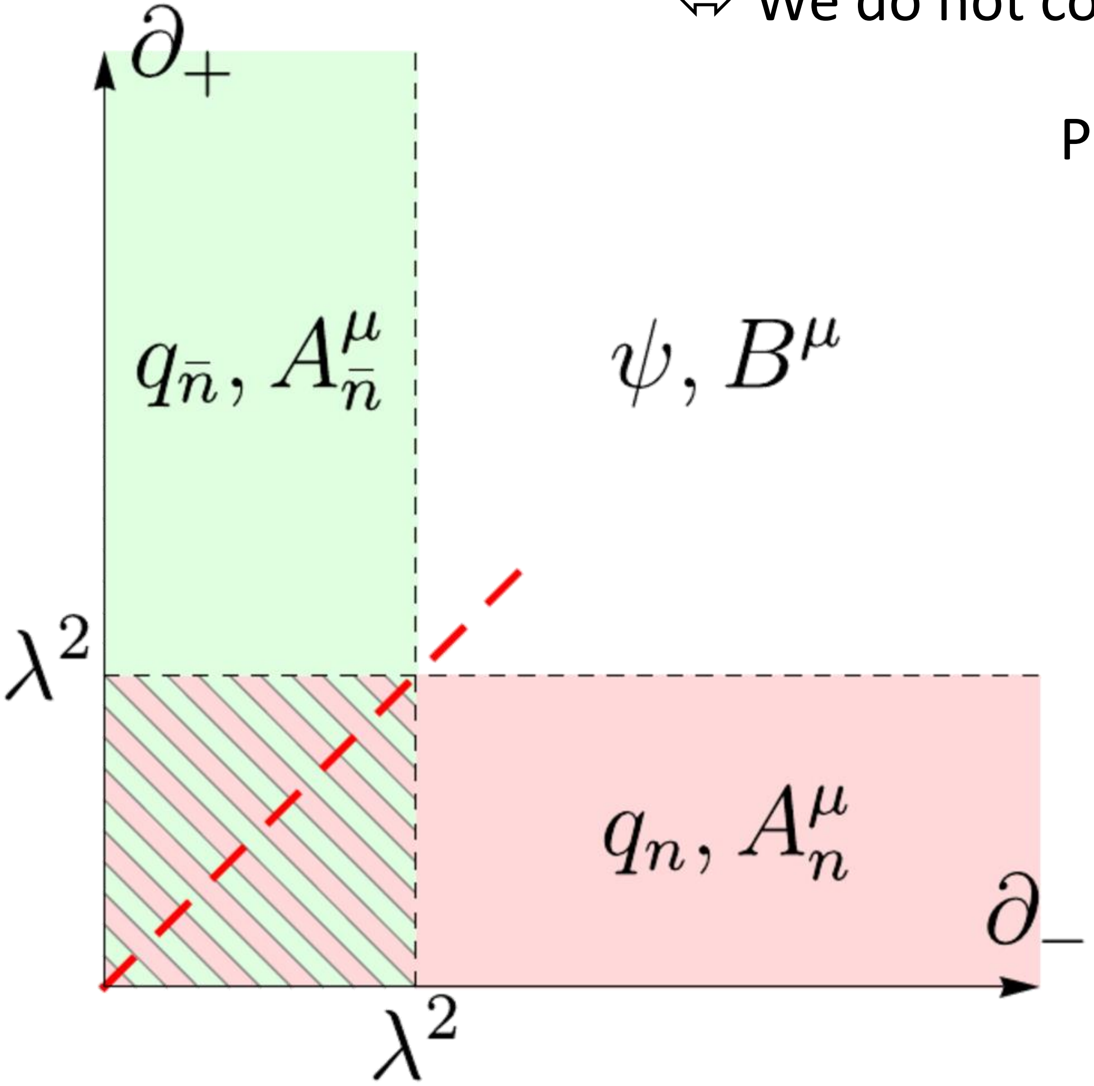
Quasi-TMD current

$$J^i_a(y, v) \equiv [Lv + y_T; y]_{ab} q^i_b(y)$$

We assume that $L \to \infty$

Background field approach

We assume that hadrons contains only collinear fields
⇔ We do not consider small-x effects (!)



Physical observables

$$\lambda = \frac{q_T^2}{Q^2 \pm q_T^2} \ll 1$$

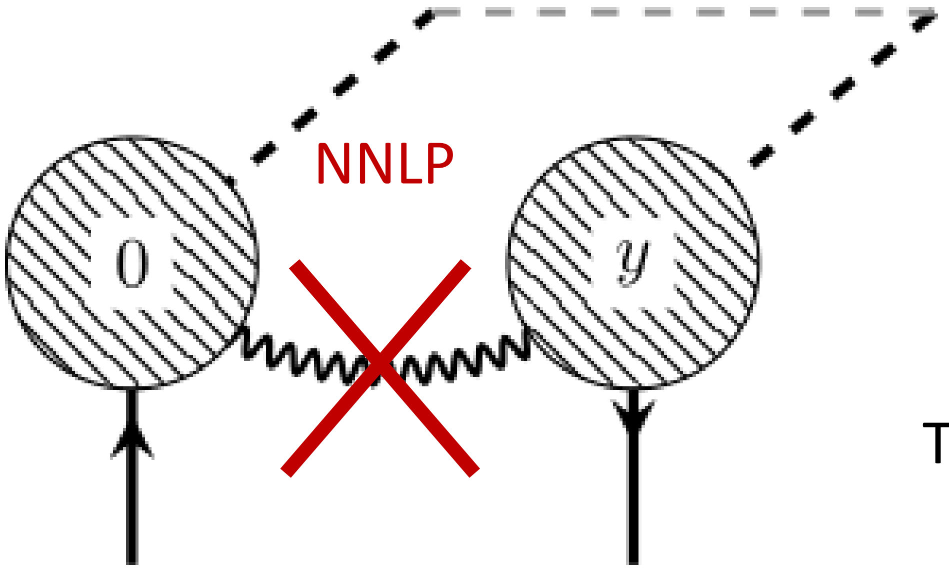
Lattice observables

$$\lambda = \frac{M}{P^+} \ll 1$$

$$\{\partial^+, \partial^-, \partial_T\}\xi_{\bar{n}} \lesssim P^+\{1, \lambda^2, \lambda\}\xi_{\bar{n}}$$

$$\xi_{\bar{n}} \sim \lambda, \quad \eta_{\bar{n}} \sim \lambda^2, \quad A^\mu_{\bar{n}} \sim \begin{cases} 1 & \text{if } \mu = + \\ \lambda^2 & \text{if } \mu = - \\ \lambda & \text{if } \mu = T \end{cases}$$

How one fixes the scaling of the position of the current?



Avoid the enhancement of collinear momentum
⇔ Keep small-x effects suppressed

Do not enhance anti-collinear momentum

To remain in the TMD region, ensure $b\partial_T \phi_{\bar{n}} \sim 1$

$$\{\ell, b\} \sim P_+^{-1}\{1, \lambda^{-1}\}$$

One handy advantage of BGF: different gauge choices for dynamical and background fields

Going to next-to-leading power

(And what it means?)

Phase-space factors
Suppressed contributions in the
Contraction of hadronic and leptonic tensors

Expansion of the kinematical parameters
in the ratio λ

Kinematical power corrections

Same operator as LP
that appears with suppression factor

Genuine power corrections

New operators

The next steps

Expand at the desired perturbative order and expand the fields in power of λ

Evaluate all the necessary loop integrals

Rewriting all the fields in terms of ‘good’ components only
using equation of motions

Recombination of divergences, renormalization

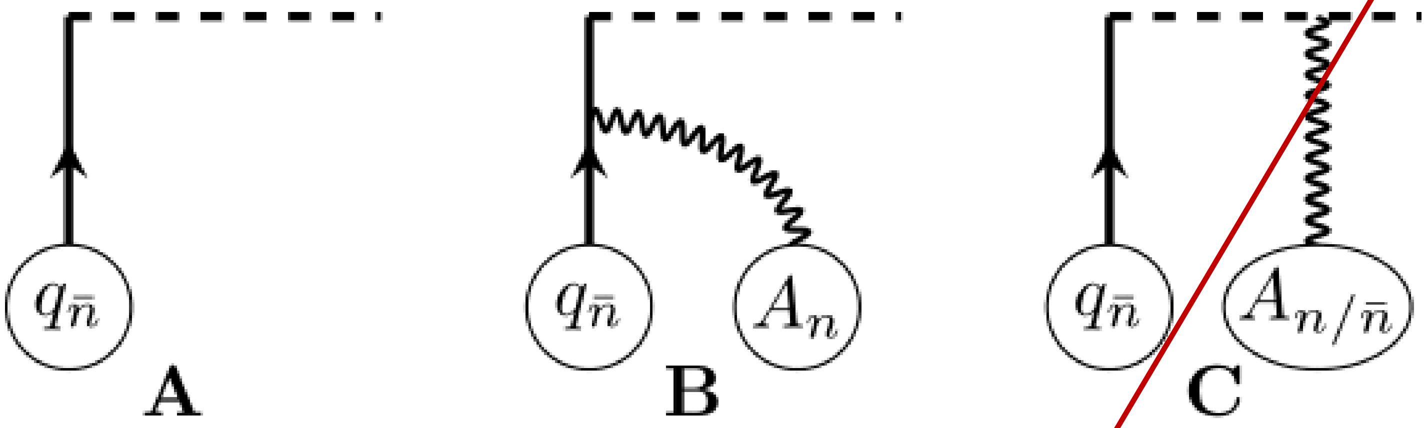
Assemble the final TMD operators

Few analytical details

$$H^\dagger(z) = P \exp\left[-ig\int_L^0 ds\, v^\mu A_{v,\mu}(sv+z)\right]$$

Leading-power $\mathcal{J}_{\mathrm{LP/LO}}(z) = H^\dagger(z)\xi_{\bar{n}}(z)$

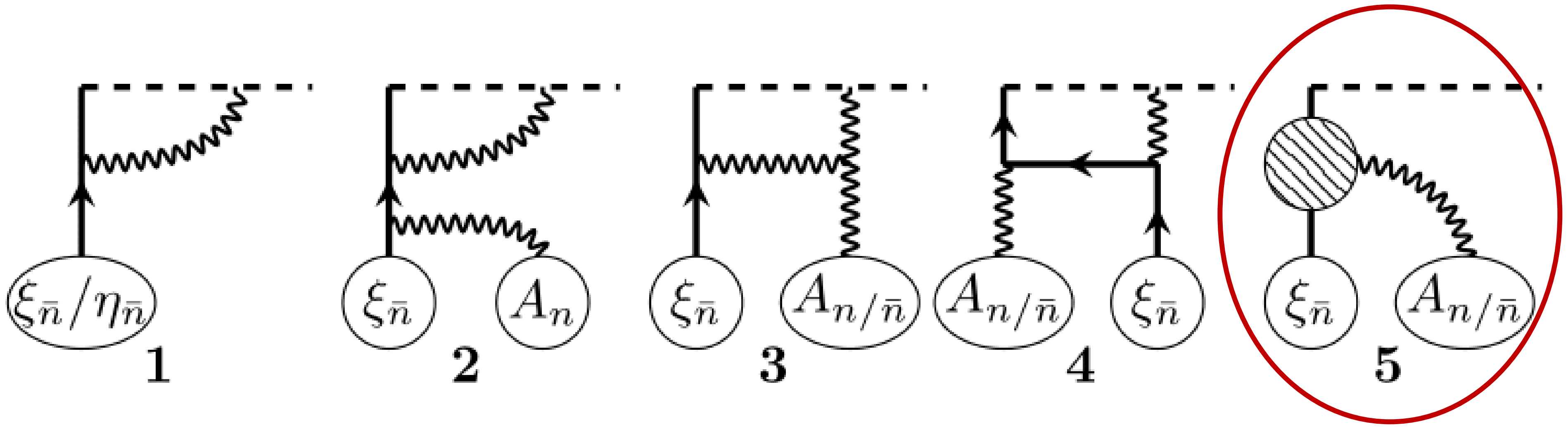
Vanishes with clever choice of gauge



Next-to-leading power $\mathcal{J}_{\mathrm{NLP/LO}}(z) = -\frac{1}{2}H^\dagger(z)\frac{\gamma^+}{\partial_+}\left(\not{\partial}_T - ig\not{A}_{\bar{n}T} - ig\not{A}_{vT}\right)\xi_{\bar{n}}(z)$

From bad component of quark field

Few analytical details



Linked to the absence (presence) of a hadron in the anti-collinear sector

Absent in quasi-TMD
Present in SIDIS/DY case(!)

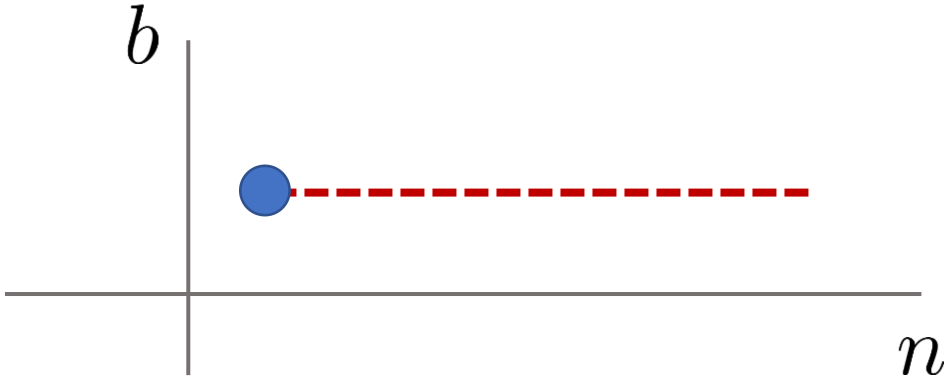
$$\mathcal{J}(z) = H^\dagger(z)\widehat{C}_1\xi_{\bar n}(z) - \frac{1}{2}\gamma^+\gamma^\mu H^\dagger(z)\widehat{C}_1\frac{\partial_\mu}{\partial_+}\xi_{\bar n}(z)$$
$$+\frac{ig}{2}\gamma^+\gamma^\mu H^\dagger(z)\frac{1}{\partial_+}\widehat{C}_2 A_{\bar n\mu}(z)\xi_{\bar n}(z) + \frac{ig}{2}\gamma^+\gamma^\mu H^\dagger(z)A_{v\mu}(z)\frac{1}{\partial_+}\widehat{C}_{2v}\xi_{\bar n}(z)$$

Restoring gauge invariance

After restoring gauge invariance with the appropriate Wilson lines
The relevant operators are semi-compact!

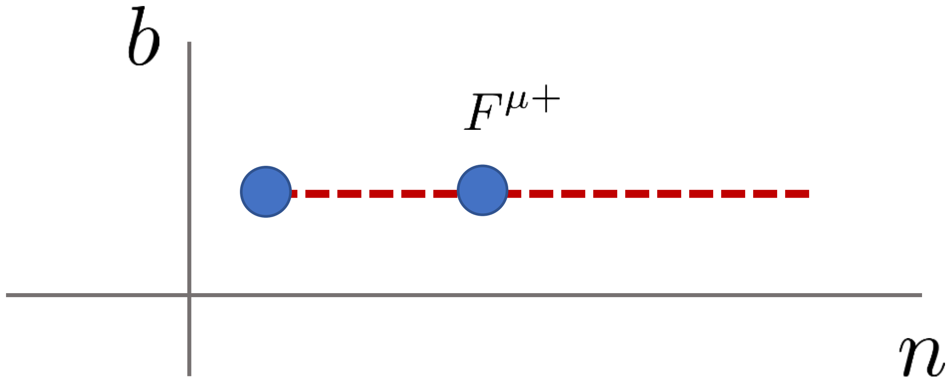
Leading power (twist-1)

$$U_1(z,b) = [\infty n + b, zn + b]\xi_{\bar{n}}(zn, b)$$



Next-to-leading power (twist-2)

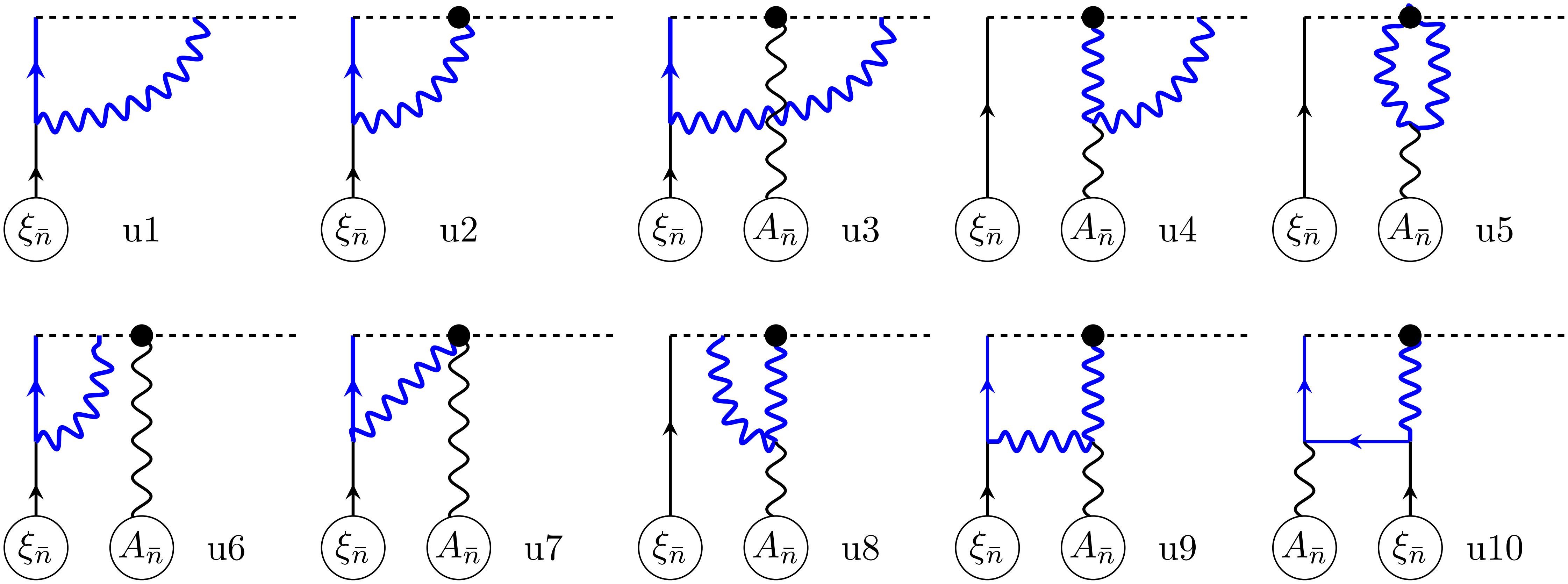
$$U_2(z_1 n, z_2 n) = [\infty n, z_1 n]F^{\mu+}(z_2 n + b)[z_1 n, z_2 n]\xi_{\bar{n}}(zn + b)$$



Semi-compact operators as building blocks

UV renormalization diagrams

$$U_N^{bare} = Z_N U_N(\mu^2)$$



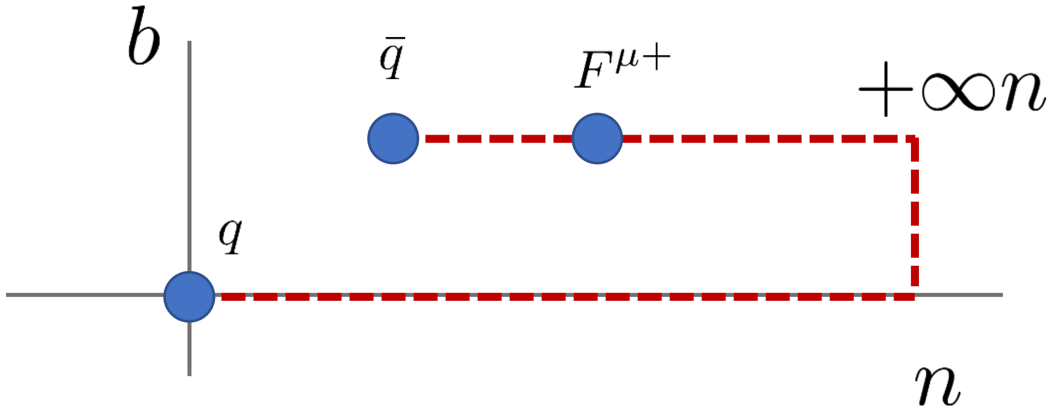
Leading power combination

$$\Phi_{ij}(x,b)=\int\frac{dz}{2\pi}e^{-ixzp^+}\langle p,S|\bar{q}_j(zn+b)[zn+b,-\infty n+b][-\infty n,0]q_i(0)|p,S\rangle$$

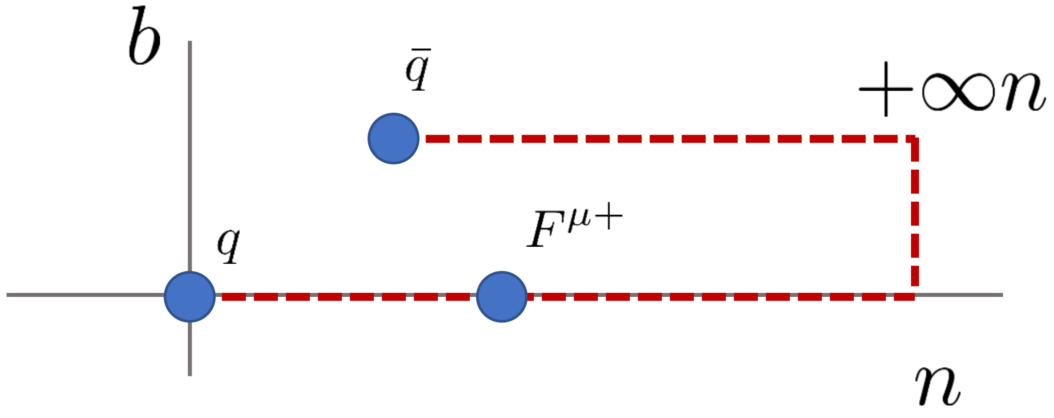
Now we interpret them as the combination of two twist-1 semi-compact operator

$$\Phi_{11,ij}(z,b)=\langle p,S|\bar{U}_{1,j}(zn+b)[\text{W.l. at T}\infty]U_{1,i}(0)|p,S\rangle$$

Next-to-leading power combinationS



or



$$\Phi_{12,ij}(z,b) = \langle p,S|\bar{U}_{1,j}(z_1 n + b)[...]U_{1,i}(0, z_2 n + b)|p,S\rangle$$

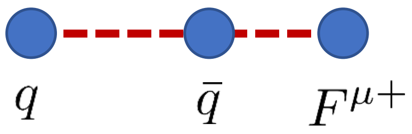
$$\Phi_{21,ij}(z,b) = \langle p,S|\bar{U}_{2,j}(z_1 n + b, z_2 n + b)[...]U_{1,i}(0)|p,S\rangle$$

Two distinct TMDs!

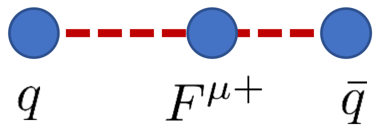
TMD twist is given by two numbers: at leading power just one case (1,1)

At next-to-leading power two cases (2,1) and (1,2)

Why does this not happen in the PDF case?



The same color structure as



Quasi-TMD peculiarity

In the quasi-TMD case we also have to introduce a TMD-like object
As vacuum matrix element of specific Wilson loops

Plays the role as the leading-twist distribution

$$\Psi(b) = \langle 0|\frac{\mathrm{Tr}}{N_c}[-\bar{n}\infty + b, b]H(b)H^\dagger(0)[0, -\bar{n}\infty]|0\rangle$$

$$\Psi_{\mu,12}(z,b) = \langle 0|\frac{\mathrm{Tr}}{N_c}[-\bar{n}\infty + b, b]H(b)H^\dagger(0)[0, zn]F_{\mu-}[zn, -\bar{n}\infty]|0\rangle$$

$$\Psi_{\mu,21}(z,b) = \langle 0|\frac{\mathrm{Tr}}{N_c}[-\bar{n}\infty + b, zn + b]F_{\mu-}[zn + b, b]H(b)H^\dagger(0)[0, -\bar{n}\infty]|0\rangle$$

$$\zeta\frac{d}{d\zeta}\Psi(b;\mu,\zeta) = -\mathscr{D}(b,\mu)\Psi(b;\mu,\zeta)$$

Divergences and scale dependence

$$\Phi_{NM,ij} = \langle p, S|\bar{U}_{N,j}(\{z_l n\}, b)[...]U_{M,i}(\{z_k n\}, 0)|p, S\rangle$$

$$\Phi^{bare}_{NM} = R(b^2) Z_N(\{z_l n\}) \otimes Z_M(\{z_k n\}) \otimes \Phi_{NM}(\mu^2, \zeta)$$

The UV divergences for $\bar{U}_N$

The UV divergences for U_M

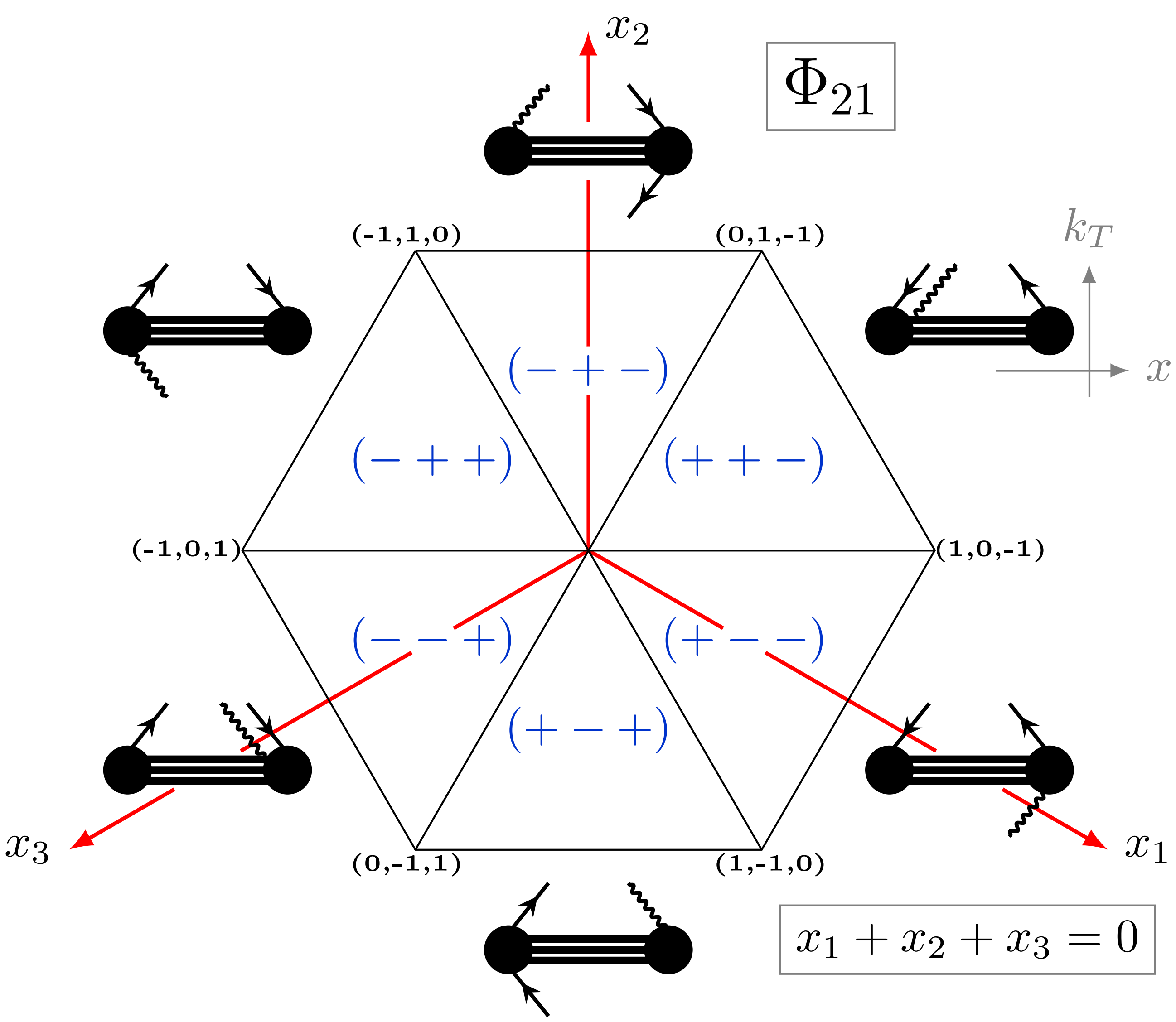
Rapidity divergences

Three independent divergences
Three renormalization constants
Three anomalous dimensions

$$\mu^2 \frac{d}{d\mu^2} \widetilde{\Phi}_{NM}(\{z_i n\}, b; \mu, \zeta) = (\widetilde{\gamma}_N(\{z_l n\}, \mu, \zeta) + \widetilde{\gamma}_M(\{z_k n\}, \mu, \zeta)) \otimes \widetilde{\Phi}_{NM}(\{z_i n\}, b; \mu, \zeta)$$

$$\zeta \frac{d}{d\zeta} \widetilde{\Phi}_{NM}(\{z_i n\}, b; \mu, \zeta) = -\mathcal{D}(b^2, \mu^2) \widetilde{\Phi}_{NM}(\{z_i n\}, b; \mu, \zeta)$$

Momentum-fraction interpretation



Different contributions in different slices

$$\mu^2 \frac{d}{d\mu^2}\Phi^{[\Gamma]}_{\mu,21}$$
$$\left(\frac{\Gamma_{\text{cusp}}}{2}\ln\left(\frac{\mu^2}{\zeta}\right) + \Upsilon_{x_1x_2x_3} + 2\pi i\, s\,\Theta_{x_1x_2x_3}\right)\Phi^{[\Gamma]}_{\mu,21}$$
$$+\mathbb{P}^A_{x_2x_1}\otimes\Phi^{[\gamma^\nu\gamma^\mu\Gamma]}_{\nu,21} + \mathbb{P}^B_{x_2x_1}\otimes \Phi^{[\gamma^\mu\gamma^\nu\Gamma]}_{\nu,21}$$

Process dependent evolution equations!

Imaginary evolution equations?!

Complex evolution for complex functions!

$$[\Phi^{[\Gamma]}_{\mu,12}(x_1,x_2,x_3,b)]^* = \Phi^{[\gamma^0\Gamma^\dagger\gamma^0]}_{\mu,21}(-x_3,-x_2,-x_1,-b)$$
$$[\Phi^{[\Gamma]}_{\mu,21}(x_1,x_2,x_3,b)]^* = \Phi^{[\gamma^0\Gamma^\dagger\gamma^0]}_{\mu,12}(-x_3,-x_2,-x_1,-b)$$

No definite complexity

No definite time-reversal parity

$$\mathscr{P}\mathscr{T}\Phi^{[\Gamma]}_{\mu,12}(x_1,x_2,x_3,b;s,L)(\mathscr{P}\mathscr{T})^{-1} = -\Phi^{[\gamma^0 T\Gamma^* T^{-1}\gamma^0]}_{\mu,21}(-x_3,-x_2,-x_1,-b;-s,-L)$$
$$\mathscr{P}\mathscr{T}\Phi^{[\Gamma]}_{\mu,21}(x_1,x_2,x_3,b;s,L)(\mathscr{P}\mathscr{T})^{-1} = -\Phi^{[\gamma^0 T\Gamma^* T^{-1}\gamma^0]}_{\mu,12}(-x_3,-x_2,-x_1,-b;-s,-L)$$

Definite T-parity combinations

To parametrize the twist-(2,1) –(1,2)

$$\Phi^{[\Gamma]}_{\mu,\oplus}(x_1,x_2,x_3,b) = \frac{\Phi^{[\Gamma]}_{\mu,21}(x_1,x_2,x_3,b) + \Phi^{[\Gamma]}_{\mu,12}(-x_3,-x_2,-x_1,b)}{2}$$

$$\Phi^{[\Gamma]}_{\mu,\ominus}(x_1,x_2,x_3,b) = i\frac{\Phi^{[\Gamma]}_{\mu,21}(x_1,x_2,x_3,b) - \Phi^{[\Gamma]}_{\mu,12}(-x_3,-x_2,-x_1,b)}{2}$$

Only non-vanishing traces with $\Gamma \in \{\gamma^+, \gamma^+\gamma_5, i\sigma^{\alpha+}\gamma_5\}$

32 distributions in total
16 T-even, 16 T-odd
J=0,1,2 tensor contributions
⇔
More possible spin combinations with three fields

	U	L	$\mathrm{T}_{J=0}$	$\mathrm{T}_{J=1}$	$\mathrm{T}_{J=2}$
U	$f_\bullet^\perp$	$g_\bullet^\perp$		$h_\bullet$	$h_\bullet^\perp$
L	$f_{\bullet L}^\perp$	$g_{\bullet L}^\perp$	$h_{\bullet L}$		$h_{\bullet L}^\perp$
T	$f_{\bullet T}$, $f_{\bullet T}^\perp$	$g_{\bullet T}$, $g_{\bullet T}^\perp$	$h_{\bullet T}^{D\perp}$	$h_{\bullet T}^{A\perp}$	$h_{\bullet T}^{S\perp}$, $h_{\bullet T}^{T\perp}$

Definite T-parity combinations

In terms of definite T-parity distributions the evolution equations are non-diagonal, real, and mixes the two type of distributions

The mixing is proportional to the direction of the Wilson line
i.e. mixing of T-even and T-odd distributions with a coefficient that change sign under time reversal

TMD distributions of twist-three are generalized functions
No definite value at $x_2 = 0$, but definite integrals

Special rapidity divergences

What about generic twist-3 TMD distributions?

$$\mathrm{Tr}\left[\Phi(x,b)\Gamma\right] \qquad \Gamma \in \{1, \gamma_5, \gamma^\mu, \gamma^\mu\gamma_5, i\sigma^{\alpha\beta}\gamma_5, i\sigma^{+-}\gamma_5\}$$

Isolate one ‘good’ and one ‘bad’ quark component,
we should use quark EOM to express everything in terms of good components,
i.e. operators of definite twist

Generic twist-3 $\quad x\, f_T = \quad \mathbf{f}^{(0)}_{\ominus,T} - \mathbf{g}^{(0)}_{\oplus,T} - f^{\perp}_{1T} - b^2 \frac{\partial f^{\perp}_{1T}}{\partial b^2}$ Derivative of twist-2

Genuine twist-1+2,2+1 Twist-2

BUT evolution equations are NOT closed

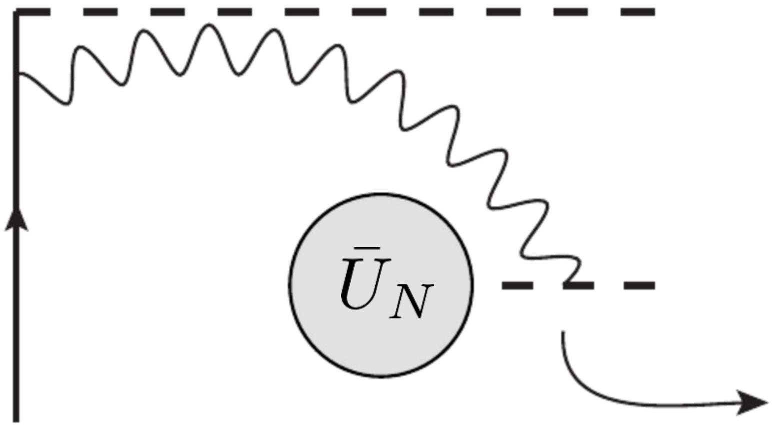
$$\mu^2 \frac{d}{d\mu^2}\begin{pmatrix} F_+ \\ F_- \end{pmatrix} = \left(\frac{\Gamma_{\text{cusp}}}{2}\ln\left(\frac{\mu^2}{\zeta}\right) + \gamma_1\right)\begin{pmatrix} F_+ \\ F_- \end{pmatrix} - \left(\gamma_1 + \frac{\gamma_V}{2}\right)\frac{1}{x}\begin{pmatrix} f_+ \\ f_- \end{pmatrix} + \frac{1}{x}\int \frac{[dx]}{x_2}\begin{pmatrix} 2\mathbb{P} & 2\pi s\Theta \\ -2\pi s\Theta & 2\mathbb{P} \end{pmatrix}\begin{pmatrix} \Phi_+ \\ \Phi_- \end{pmatrix}$$

$$\{F_+, F_-\} = \{f, g\} \text{ or } \{h, e\}\{f_+, f_-\} = \text{twsit-2 of}\{F_+, F_-\}$$

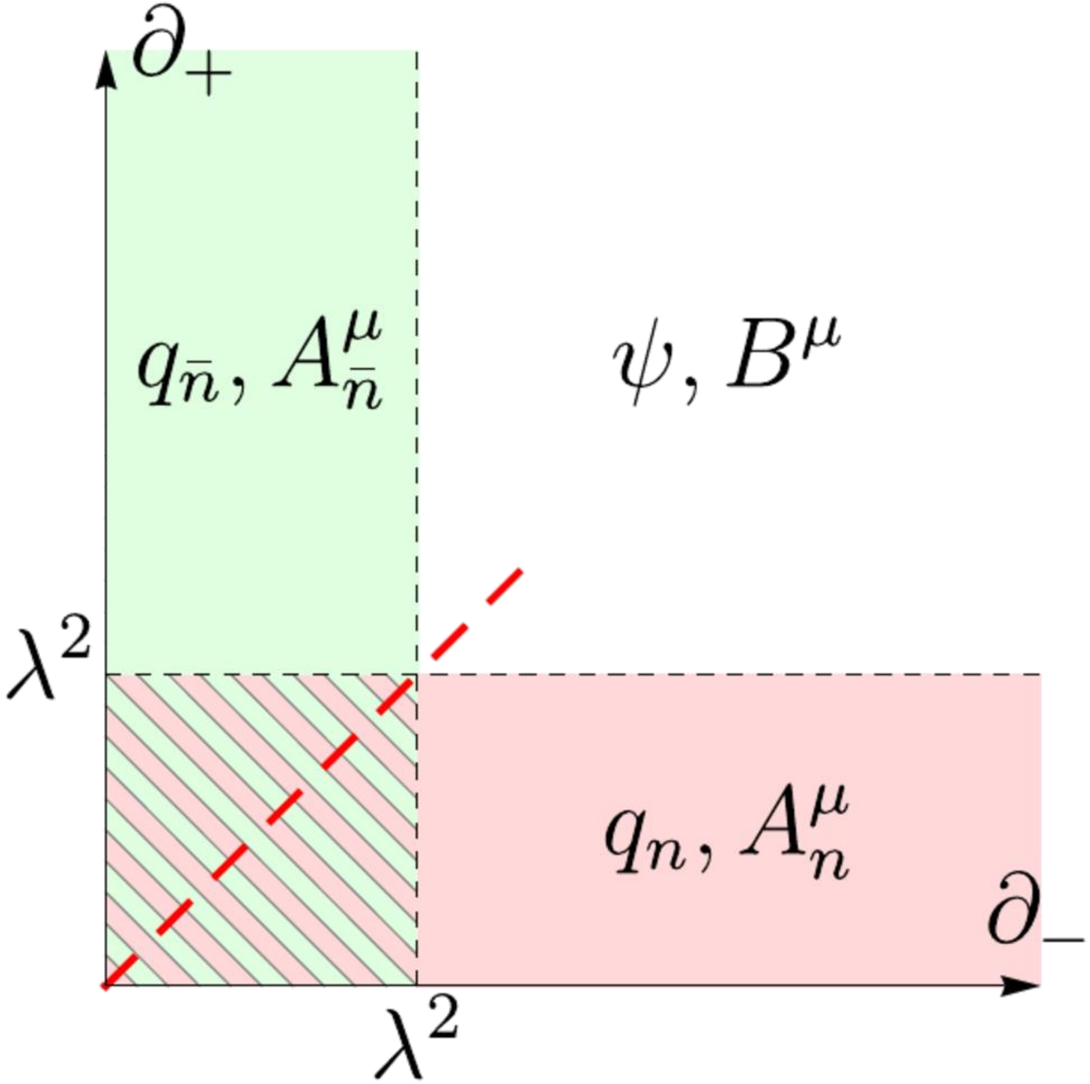
(Standard) Rapidity divergences

Rapidity divergences are soft divergences
and are removed by the cancellation
of the double-counting of the soft region

Diagrammatically they emerges from

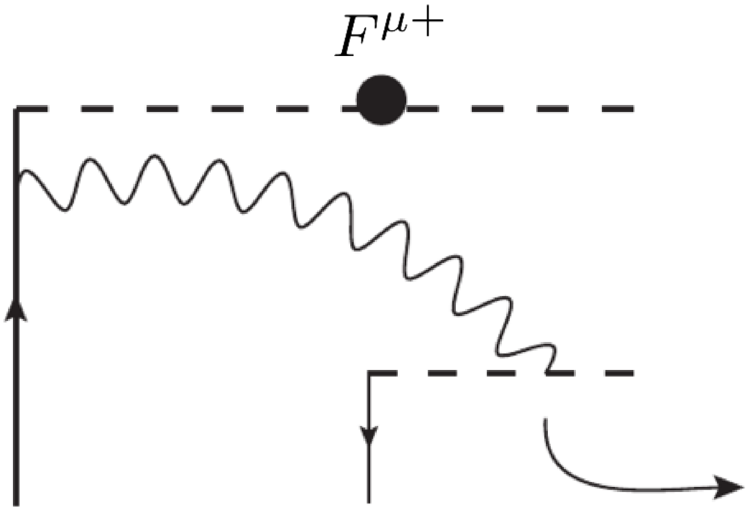


And are given by the standard R factor

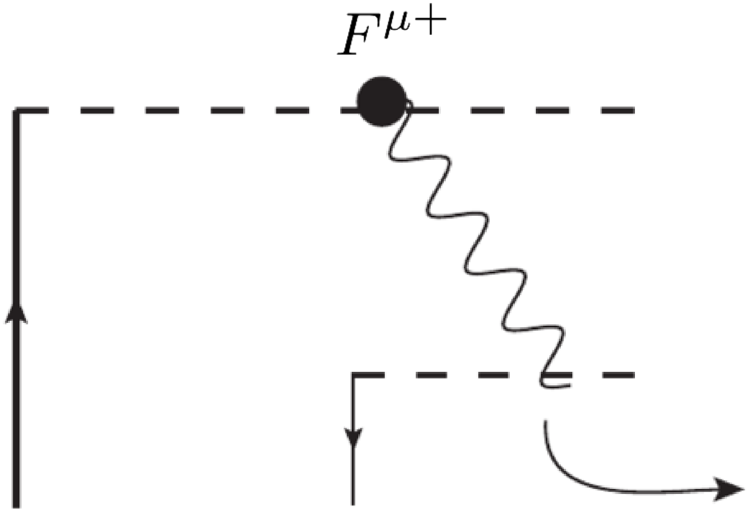


(Special) Rapidity divergences

For twist-2 semi-compact operators we also have another kind of rapidity divergences



Give the standard R factor as in LP



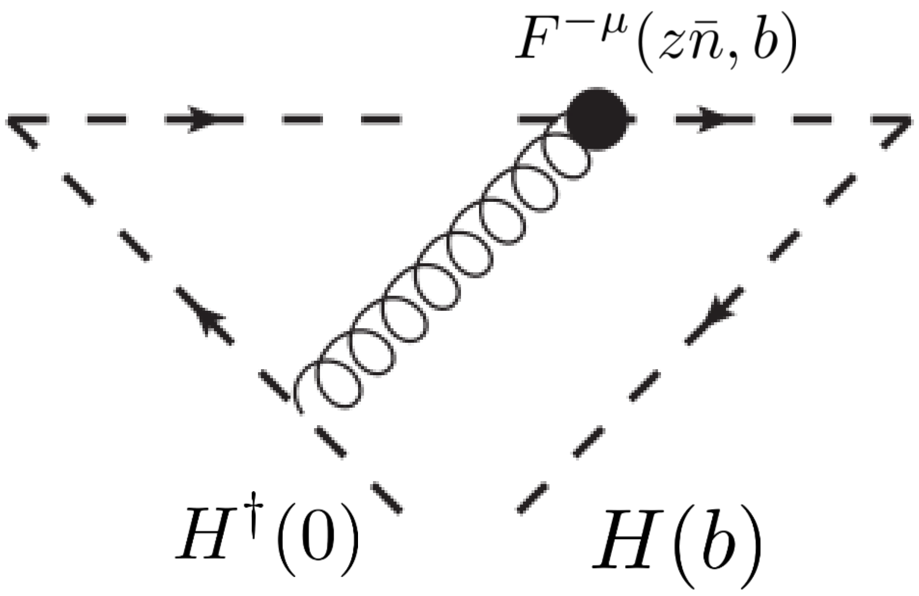
For both (2,1) and (1,2) case

$$= -\ln\left(\frac{\delta^+}{q^+}\right)\partial_\mu \mathcal{D}(b)\Phi_{11}^{[\Gamma]}(x,b)$$

These are cancelled in-between different terms of the cross-section
Not by the soft-factor

Can also be seen as divergences for vanishing gluon momentum fraction

What about the quasi-TMD case?



$$= i \ln\left(\frac{\delta^-}{q^-}\right) \partial_\mu \mathscr{D}(b)\Psi(b) + \ldots$$

$$\begin{aligned}
\widetilde{\Omega}^{[\Gamma]}_{\text{bare}}(y) \quad = \quad & \Psi(b)\widehat{C}_1^\dagger \widehat{C}_1 \left[\widetilde{\Phi}_{11}^{[\Gamma]}(\ell, b) - \frac{1}{2}\frac{\partial_\mu}{\partial_+}\widetilde{\Phi}_{11}^{[\gamma^\mu\gamma^+\Gamma + \Gamma\gamma^+\gamma^\mu]}(\ell, b)\right] \\
& + \frac{i}{2}\Psi(b)\int_{s\infty}^0 d\sigma \frac{1}{\partial_+}\left[\widehat{C}_2^\dagger \widehat{C}_1 \widetilde{\Phi}_{\mu,21}^{[\gamma^\mu\gamma^+\Gamma]}(\ell, \ell+\sigma, 0, b) + \widehat{C}_1^\dagger \widehat{C}_2 \widetilde{\Phi}_{\mu,12}^{[\Gamma\gamma^+\gamma^\mu]}(\ell, \sigma, 0, b)\right] \\
& + \frac{i}{2}\int_{-\infty}^0 d\sigma \frac{1}{\partial_+}\left[\Psi_{\mu,21}(\sigma, b)\widehat{C}_{2v}^\dagger \widehat{C}_1 \widetilde{\Phi}_{11}^{[\gamma^\mu\gamma^+\Gamma]}(\ell, b) + \Psi_{\mu,12}(\sigma, b)\widehat{C}_1^\dagger \widehat{C}_{2v}\widetilde{\Phi}_{11}^{[\Gamma\gamma^+\gamma^\mu]}(\ell, b)\right]
\end{aligned}$$

What now?

Insert the renormalization of TMD operators

Remove standard rapidity divergences with the soft factor

Subtract the special rapidity divergences to obtain an expression finite term by term

$$\Psi(b) i\partial_\mu \Phi_{11}(x,b) + i\Psi(b)\int \frac{[dx]}{x_2 - is0}\delta(x-x_3)\Phi_{\mu,21}(x_{1,2,3},b) + \Psi^{(0)}_{\mu,21}(b)\Phi_{11}(x,b)$$

$$i\Psi(b)\left(\partial_\mu - \frac{1}{2}[\partial_\mu \mathscr{D}(b)]\ln\left(\frac{\bar\zeta}{\zeta}\right)\right)\Phi_{11}(x,b) + i\Psi(b)\int \frac{[dx]}{x_2 - is0}\delta(x-x_3)\mathbf{\Phi}_{\mu,21}(x_{1,2,3},b) + \mathbf{\Psi}^{(0)}_{\mu,21}(b)\Phi_{11}(x,b)$$

Restored boost invariance! $\zeta \to \dfrac{\zeta}{\alpha} \qquad \bar\zeta \to \alpha\bar\zeta$

Final steps

Write everything in terms of definite T-parity TMD

Make explicit the complex structure (also present in the coefficient functions)

$$
\begin{aligned}
\Omega^{[\Gamma]}(x,b,\mu) \quad = \quad & \Psi(b;\mu,\bar{\zeta})\mathbb{C}_{11}\Phi_{11}^{[\Gamma]}(x,b;\mu,\zeta) \\
& + \frac{i}{2xP_+}\mathbb{C}_{11}\Psi(b)\left(\partial_\mu - \frac{1}{2}[\partial_\mu \mathcal{D}(b,\mu)]\ln\left(\frac{\bar{\zeta}}{\zeta}\right)\right)\Phi_{11}^{[\gamma^\mu\gamma^+\Gamma+\Gamma\gamma^+\gamma^\mu]}(x,b;\mu,\zeta) \\
& + \frac{1}{2xP_+}\mathbb{C}_{11v}\boldsymbol{\Psi}_{\mu,21}^{(0)}(b;\mu,\bar{\zeta})\Phi_{11}^{[\gamma^\mu\gamma^+\Gamma+\Gamma\gamma^+\gamma^\mu]}(x,b;\mu,\zeta) \\
& + \frac{i}{2xP_+}\Psi(b;\mu,\bar{\zeta})\int_{-1}^{1} dx_2 \Big[\mathbb{C}_R(x,x_2)\boldsymbol{\Phi}_{\mu,\oplus}^{[\gamma^\mu\gamma^+\Gamma-\Gamma\gamma^+\gamma^\mu]}(\tilde{x},b;\mu,\zeta) \\
& \qquad + s\pi\mathbb{C}_I(x,x_2)\boldsymbol{\Phi}_{\mu,\ominus}^{[\gamma^\mu\gamma^+\Gamma-\Gamma\gamma^+\gamma^\mu]}(\tilde{x},b;\mu,\zeta) \\
& \qquad - i\mathbb{C}_R(x,x_2)\boldsymbol{\Phi}_{\mu,\ominus}^{[\gamma^\mu\gamma^+\Gamma+\Gamma\gamma^+\gamma^\mu]}(\tilde{x},b;\mu,\zeta) \\
& \qquad + is\pi\mathbb{C}_I(x,x_2)\boldsymbol{\Phi}_{\mu,\oplus}^{[\gamma^\mu\gamma^+\Gamma+\Gamma\gamma^+\gamma^\mu]}(\tilde{x},b;\mu,\zeta)\Big]
\end{aligned}
$$

Using the factorization theorem

Extract Collins-Soper kernel

Determine physical TMD distributions

At LP the ratio in momentum space is systematically improvable

At NLP no ratios are systematically improvable

$$\frac{\widetilde{F}(\ell=0,b;\mu;P_1)}{\widetilde{F}(\ell=0,b;\mu;P_2)} = \left(\frac{(vP_2)}{(vP_1)}\right)^{2\mathcal{D}(b,\mu)} \mathbf{r}^{(0)}_{\text{NLP}}(b,\mu)$$

$$\mathbf{r}^{(0)}_{\text{NLP}}(b,\mu) \quad = \quad 1 + 4a_s(\mu)C_F \ln\left(\frac{(vP_1)}{(vP_2)}\right)\left\{\ln\left(\frac{\mu^2}{4(vP_1)(vP_2)}\right) - 2\mathbf{M}^{(0)F}_{\text{NLP}}(b,\mu)\right\}$$

@ LP one assume M=const

@ NLP M=const works only for few selected cases

Only a limited number of quasi-TMD at NLP (6 out of 16) satisfy the constraints

Conclusions

Physical processes, TMD distributions

Quasi-TMD distributions on the lattice

Factorization theorem, power counting and semi-compact operators

Rapidity divergence: standard and special

Special rapidity divergences cancellation and boost-invariance restoration

How to use the quasi-TMD factorization theorem