# Integrability in and beyond AdS/CFT

João Caetano
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Institut de Mathématiques de Bourgogne
28 March 2022

INSTITUT DE MATHÉMATIQUES DE BOURGOGNE

# Integrability **in** First Part
# and
# Second Part **beyond** AdS/CFT

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# Gauge Theory with (large) N colors

['t Hooft' 1974]

Interplay between 4D and 2D

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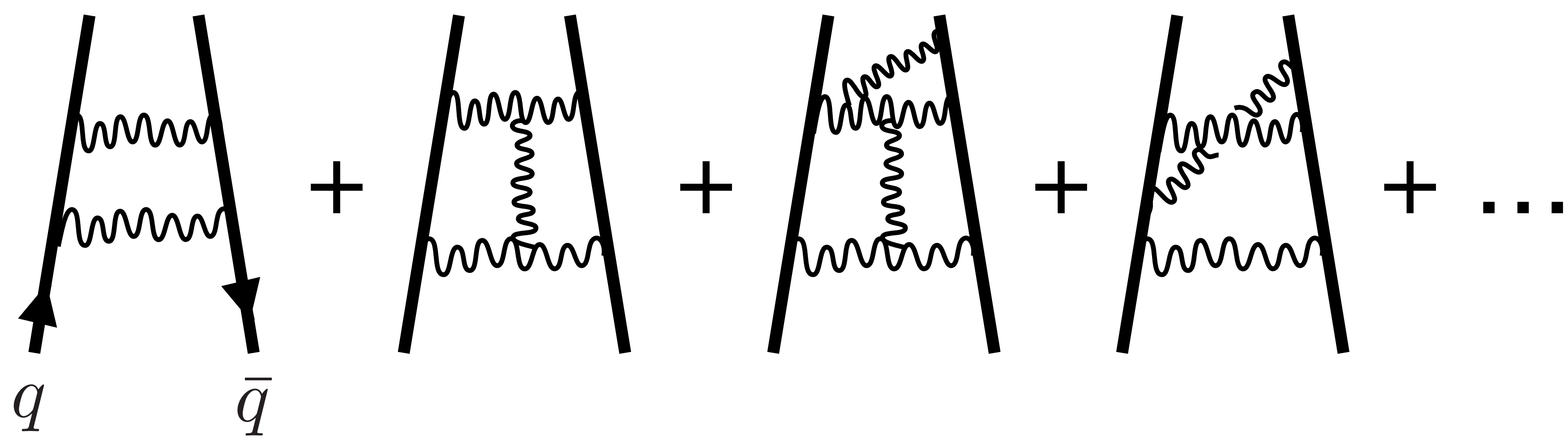

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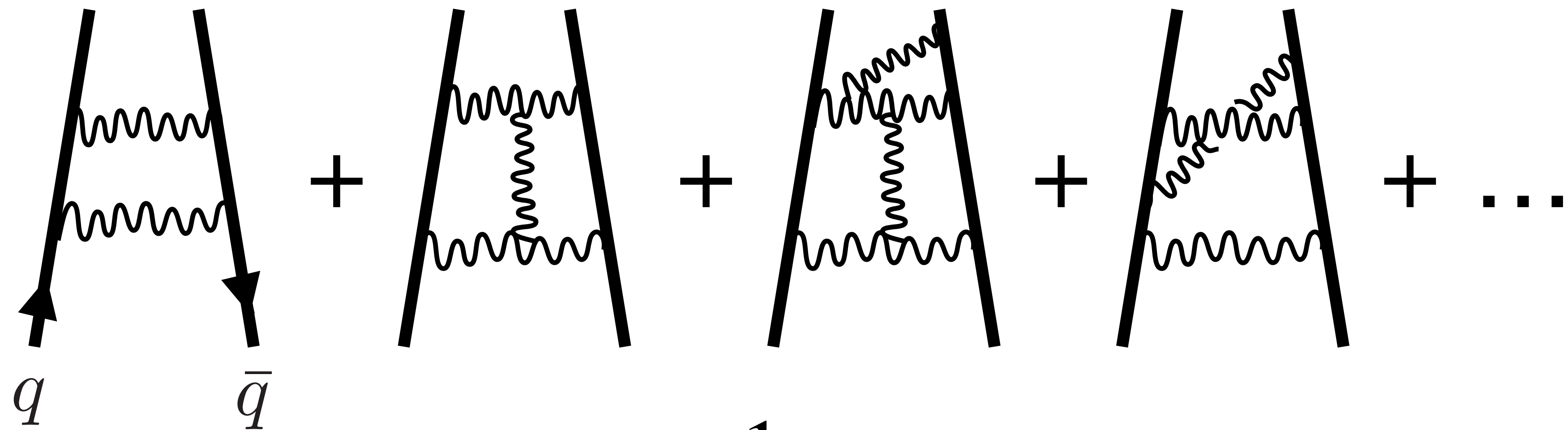


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$\lambda \equiv g_{\mathrm{YM}}^2 N$

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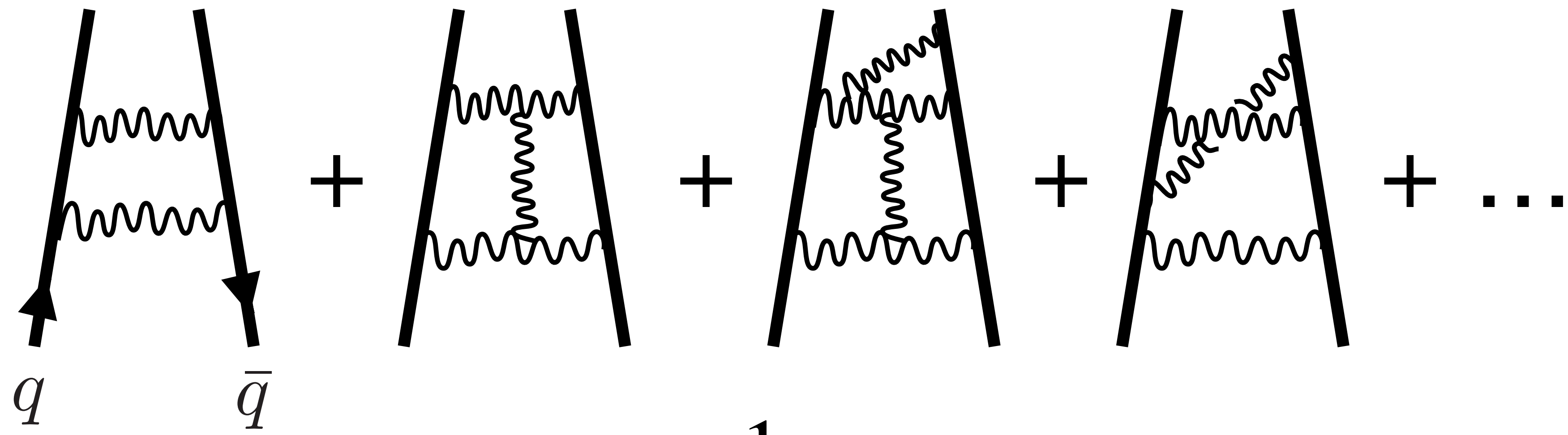


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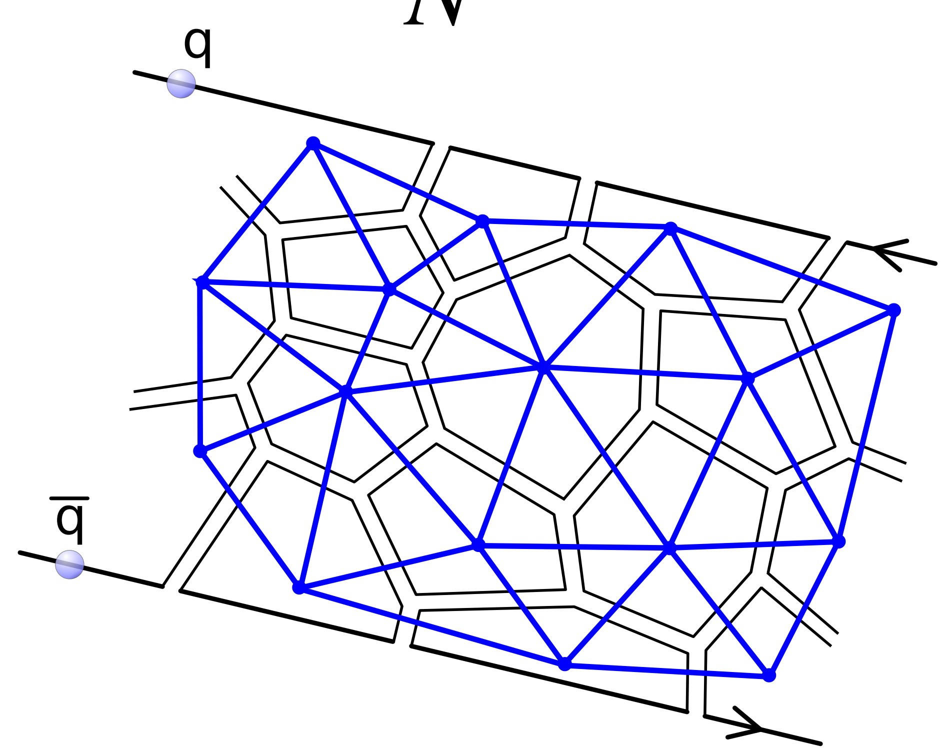

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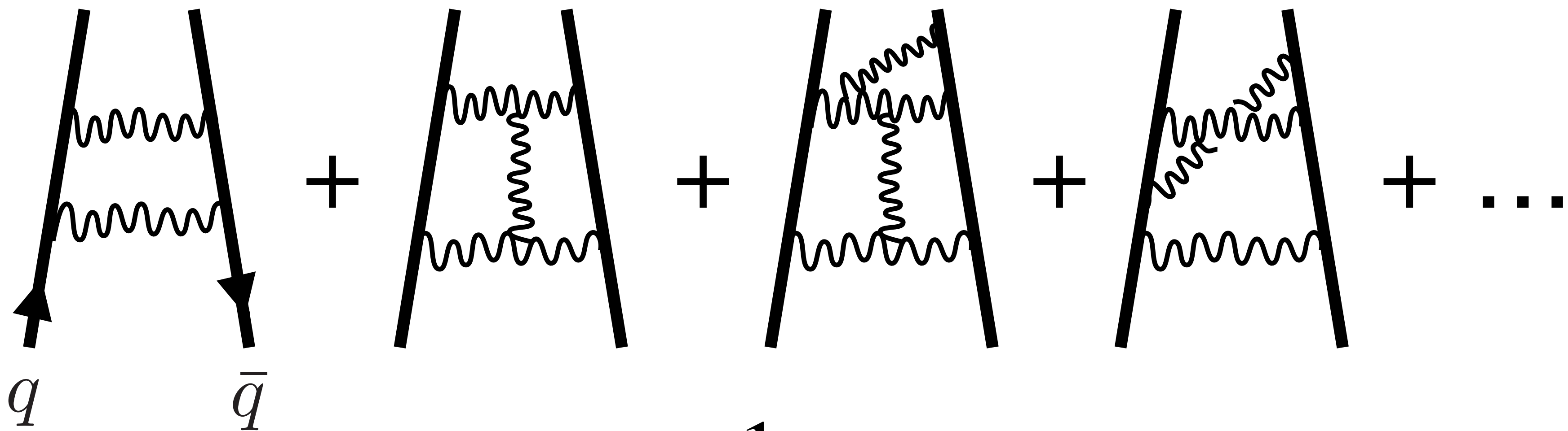


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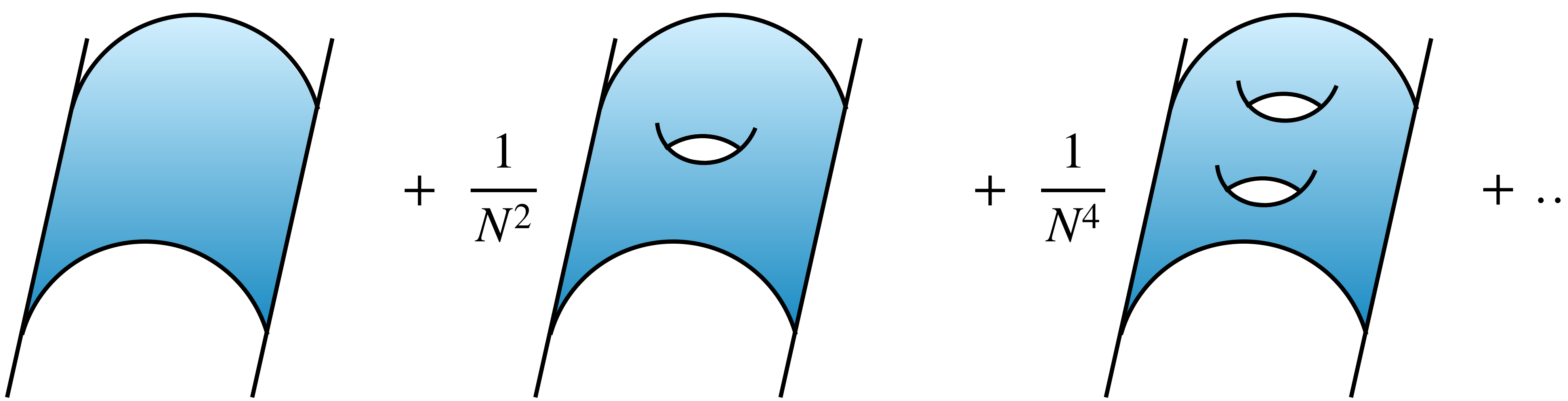

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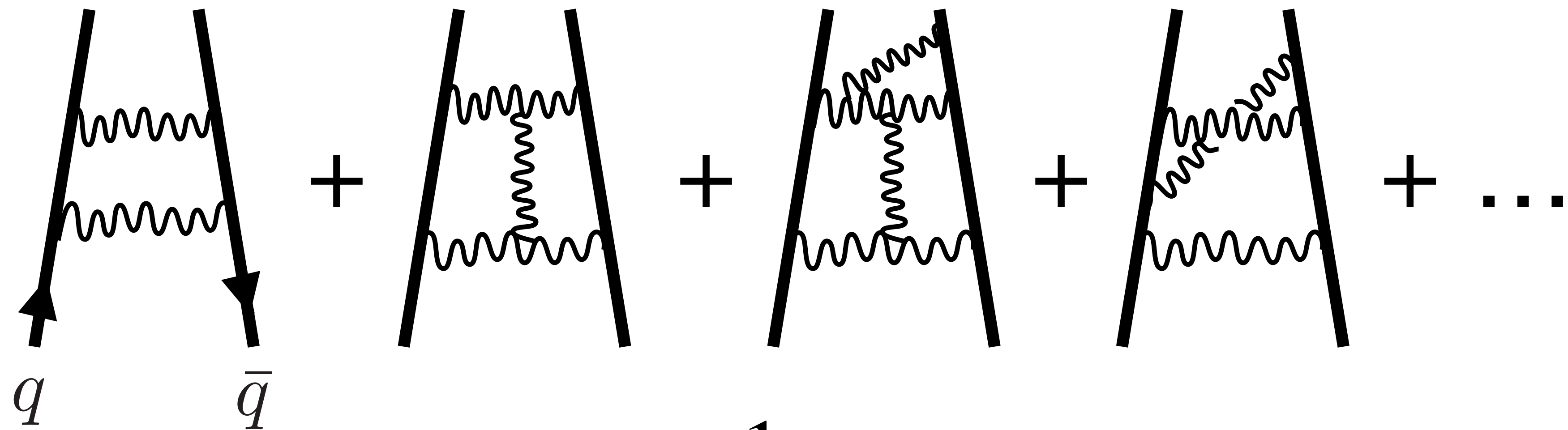


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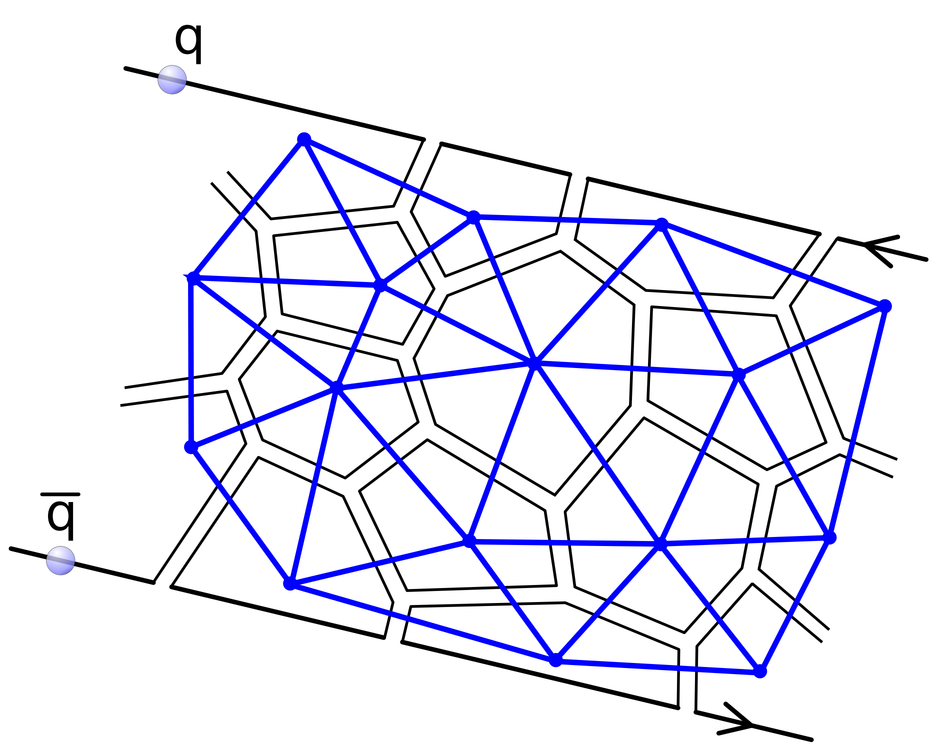

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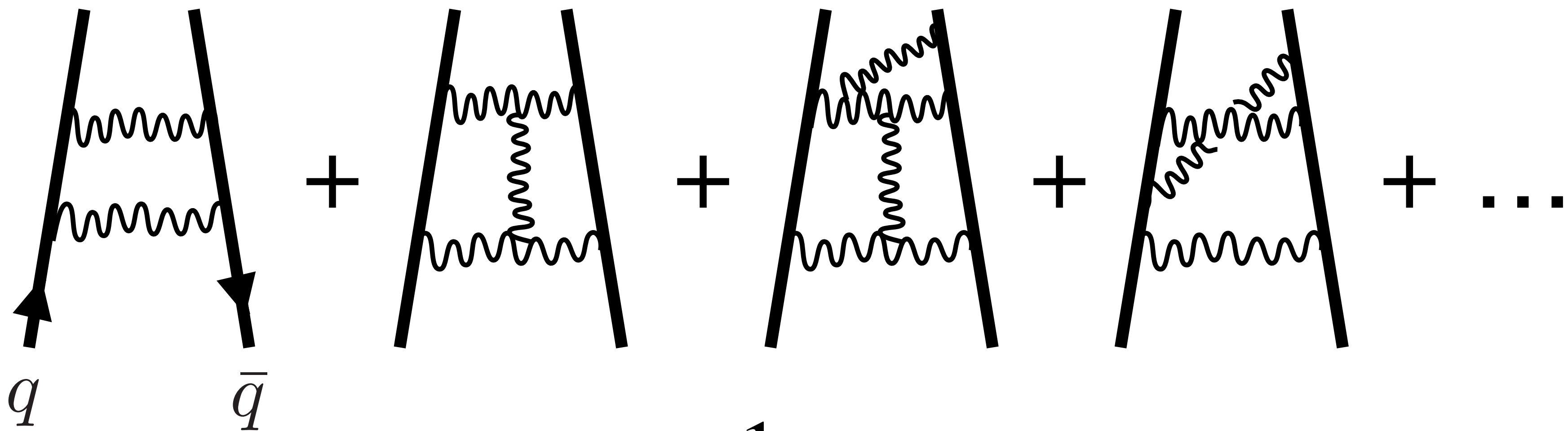


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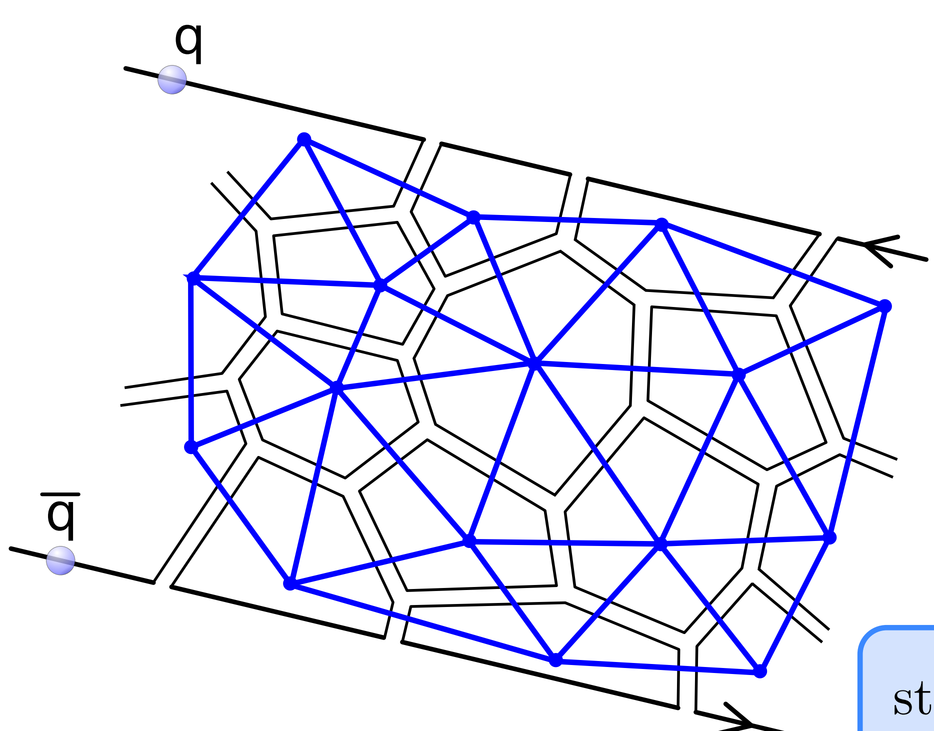


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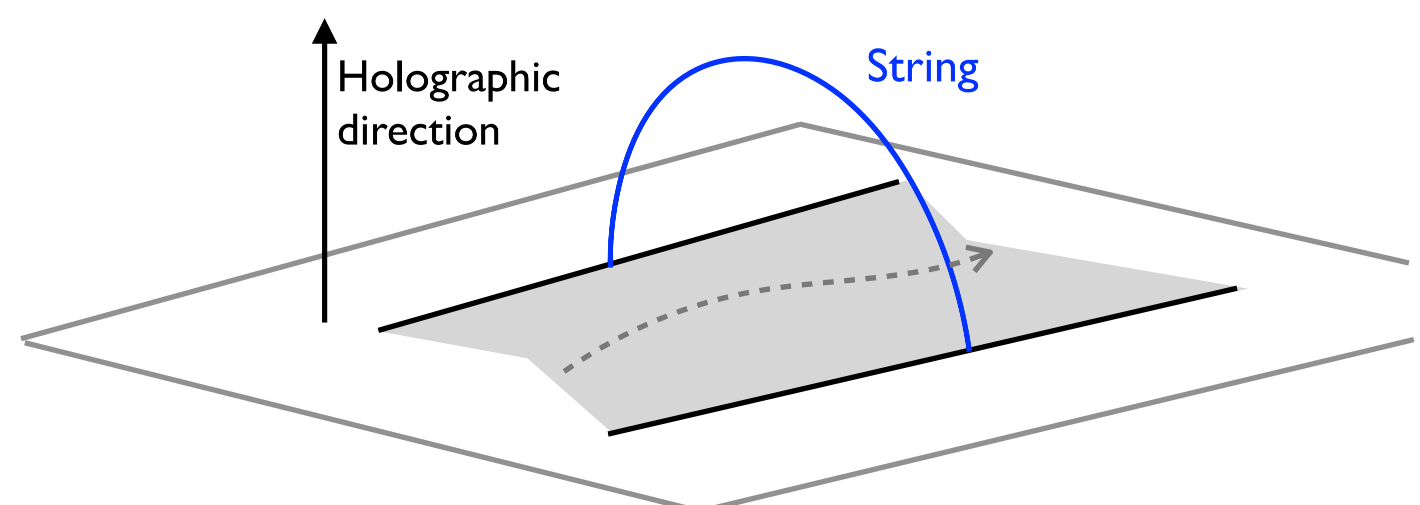


string tension $= \sqrt{\lambda}$

string coupling $= 1/N$

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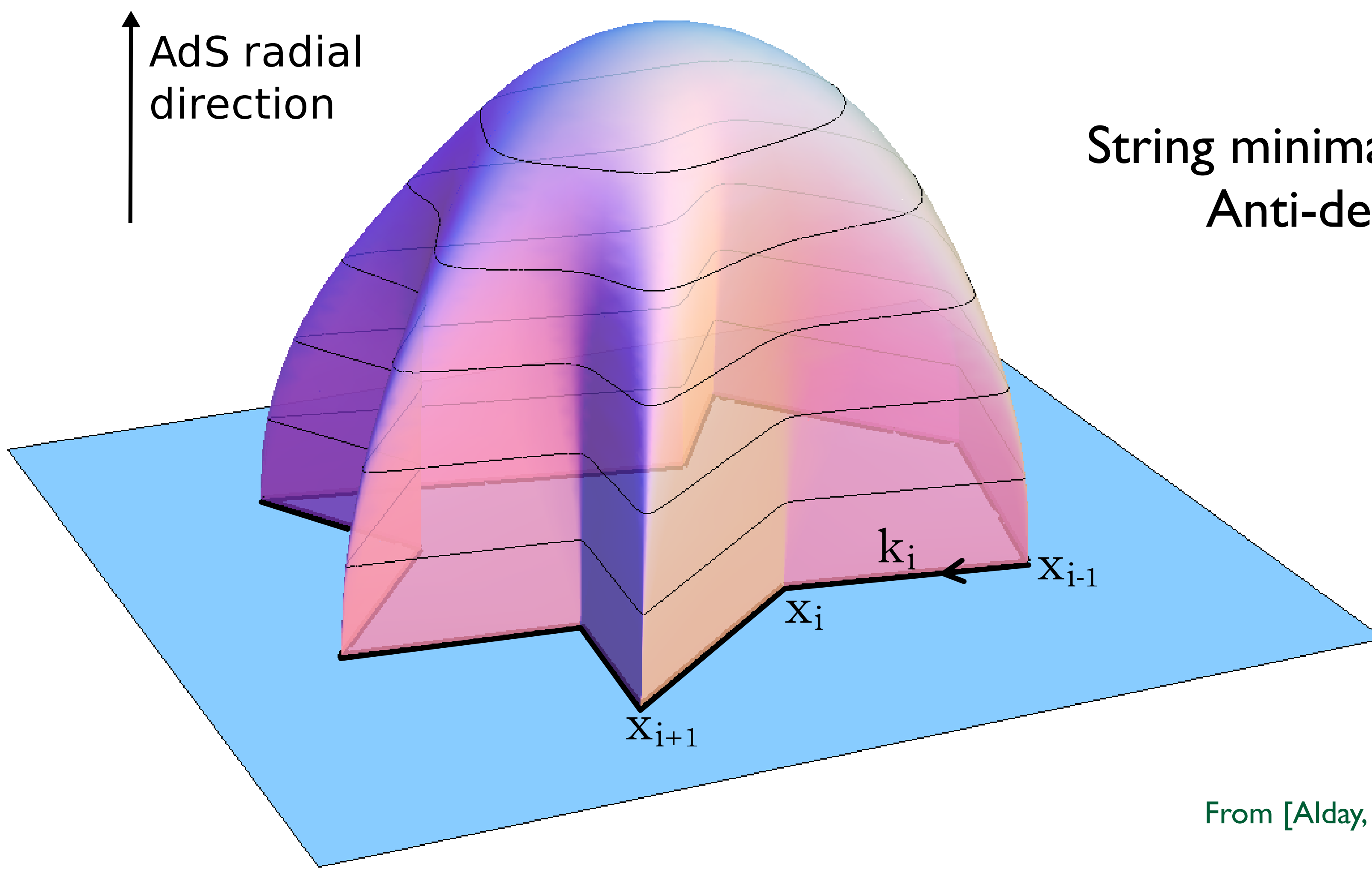
# At large 't Hooft coupling $\lambda$ string tension is large and classical string surfaces dominate




String minimal surface in Anti-de Sitter

From [Alday, Maldacena, Sever, Vieira'2010]

In these theories, life is simple(r) both at weak and strong coupling

# Concrete realization

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Maximal supersymmetric extension of Yang-Mills

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$$\mathcal{L} = \frac{1}{4g_{\rm YM}^2} \operatorname{tr} F_{\mu\nu} F^{\mu\nu} + \text{fermions} + \text{scalars}$$

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Maximal supersymmetric extension of Yang-Mills

$$\mathcal{L} = \frac{1}{4g_{\mathrm{YM}}^2} \mathrm{tr}\, F_{\mu\nu} F^{\mu\nu} + \mathrm{fermions} + \mathrm{scalars}$$

**More symmetries than QCD**

e.g. scale invariance…

# Conjecture of Maldacena (Gauge/Gravity duality or AdS/CFT)

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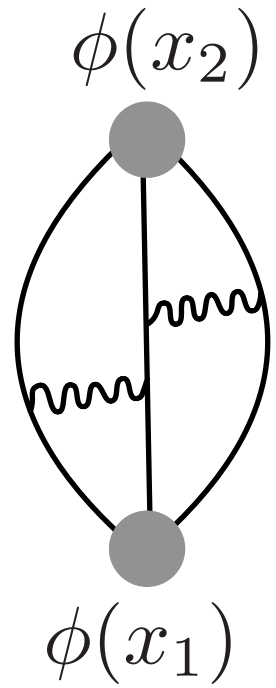

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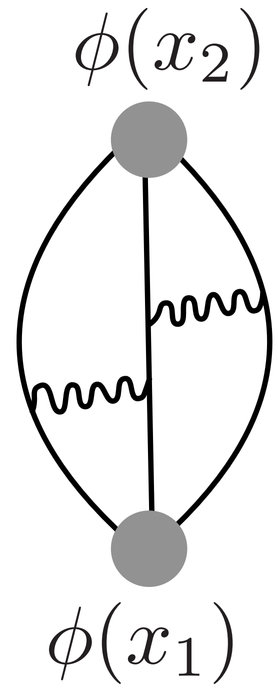


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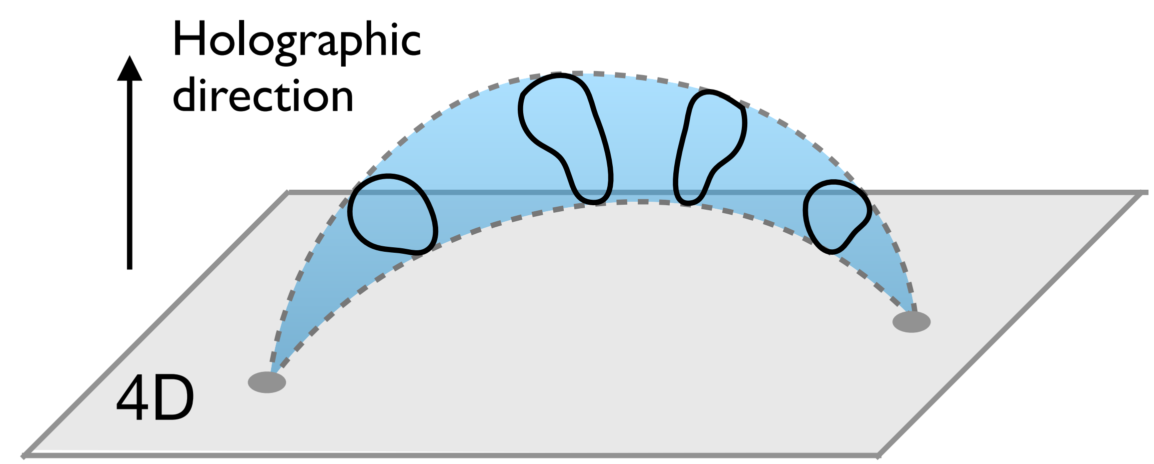

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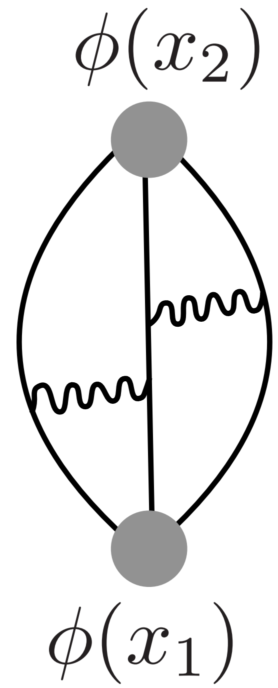


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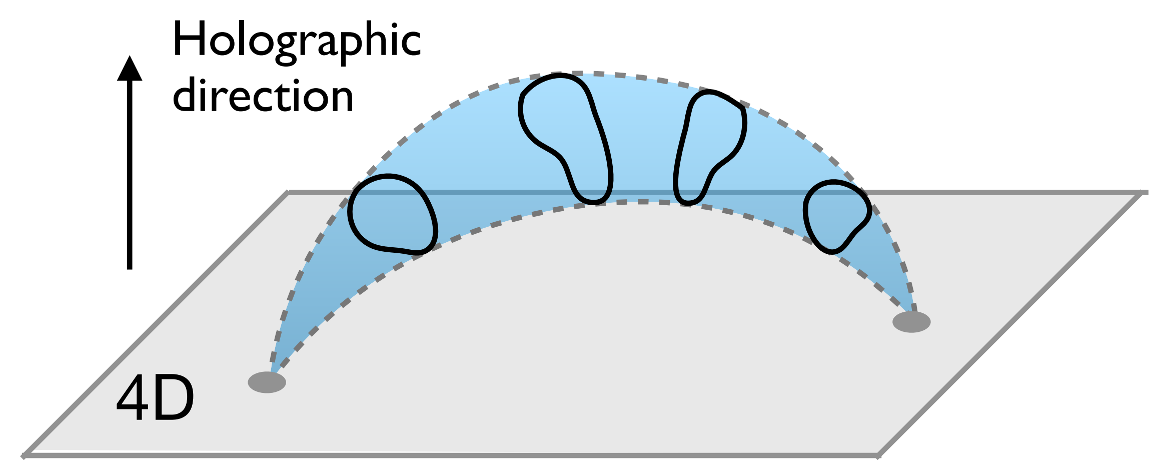


$\lambda \equiv g_{\rm YM}^2 N$

**weak coupling** → **strong coupling**

# Bonus

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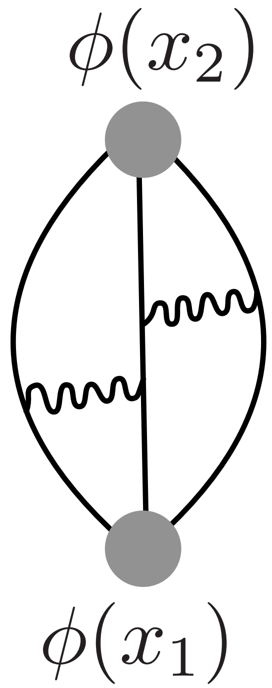


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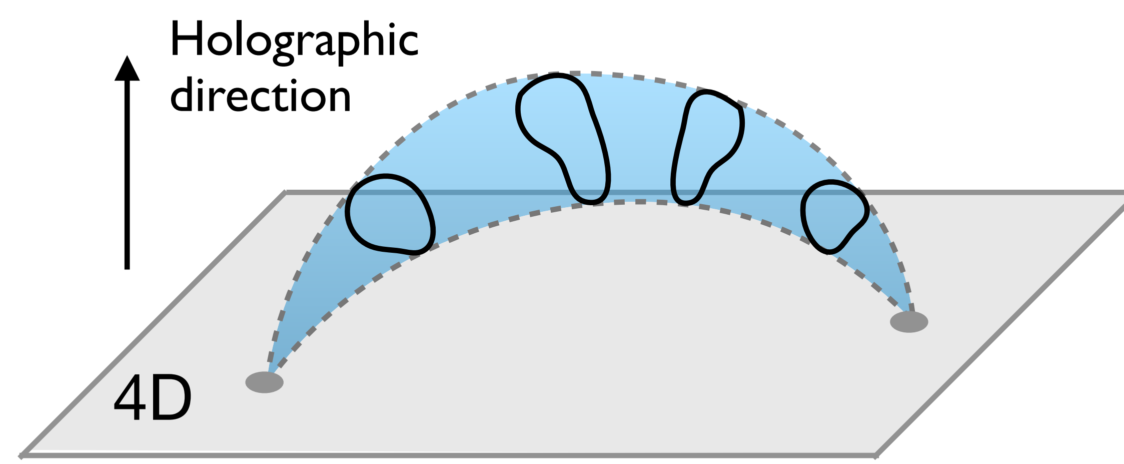


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# Integrability

The theory is **solvable** in the limit $N \to \infty$

# Integrability in N=4 SYM

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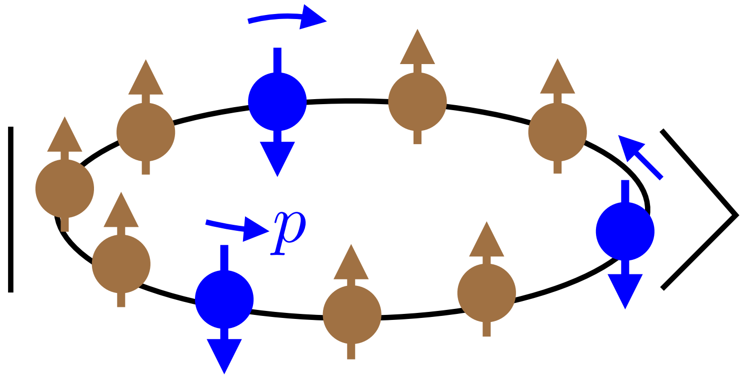
Local composite operators

$$\mathcal{O}(x) \sim \operatorname{tr} ZZZXZZXZZX$$

Elementary fields

# Integrability in N=4 SYM

**Spin Chain State**

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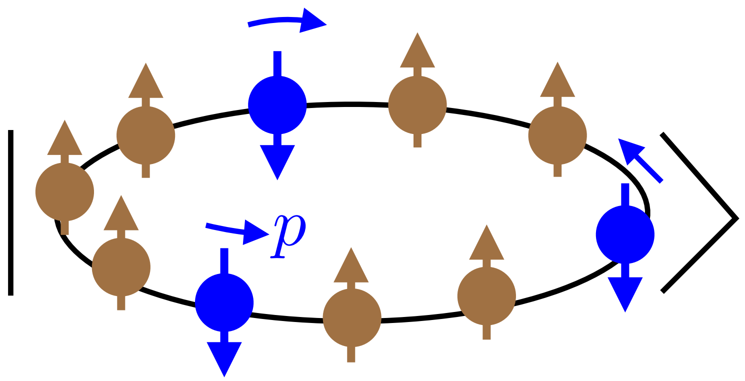


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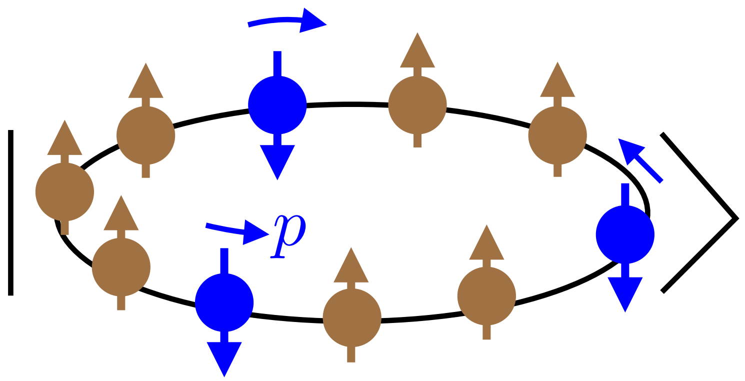


Energy of the chain

Spin Chain Hamiltonian $\mathbb{H}$

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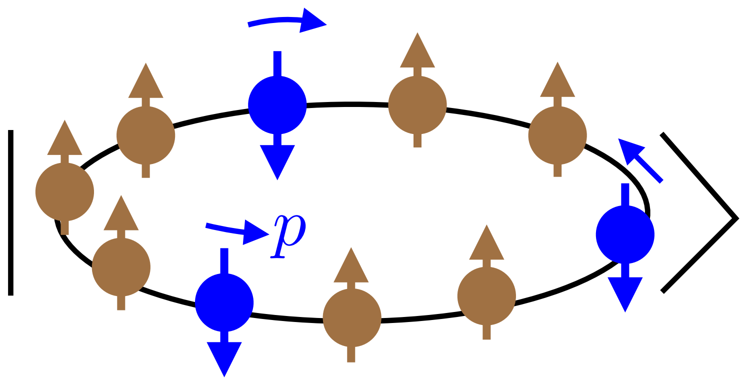


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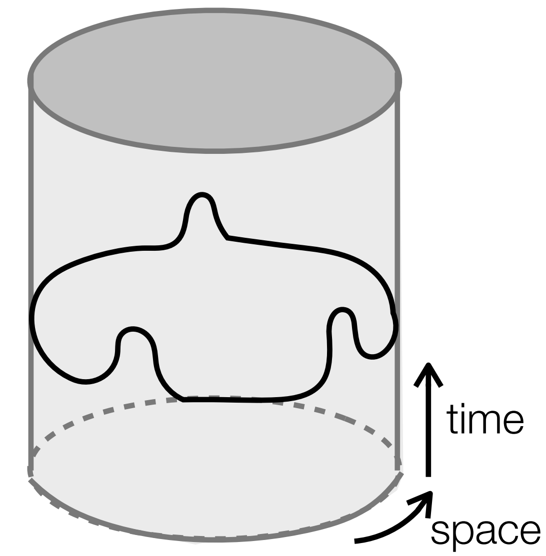

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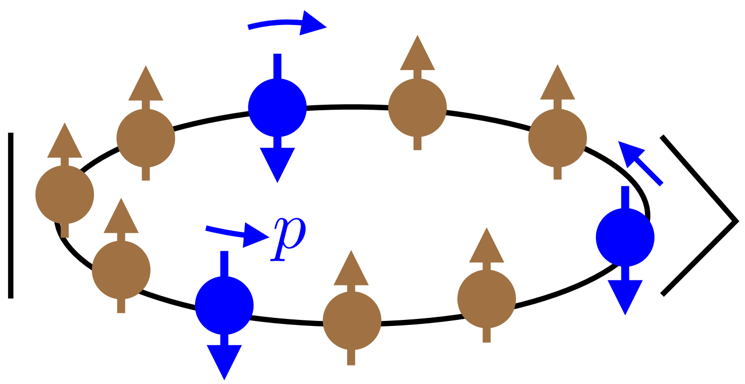


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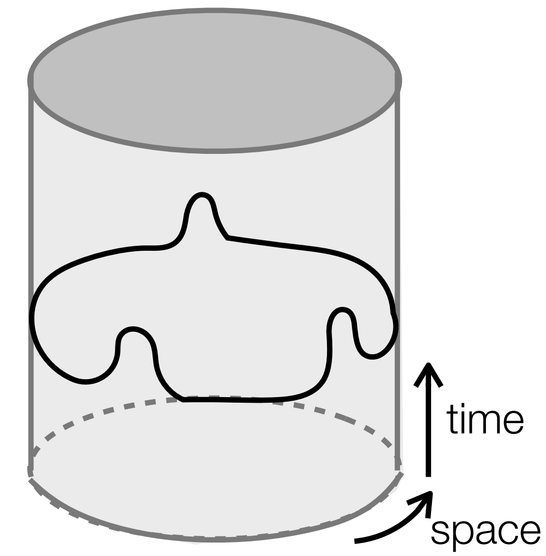


Energy of the string

String Hamiltonian

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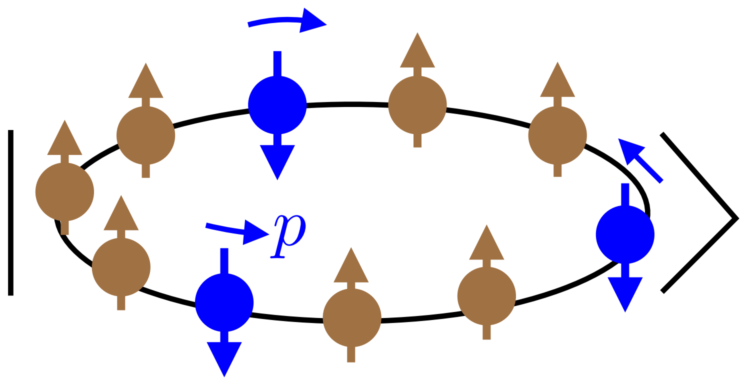


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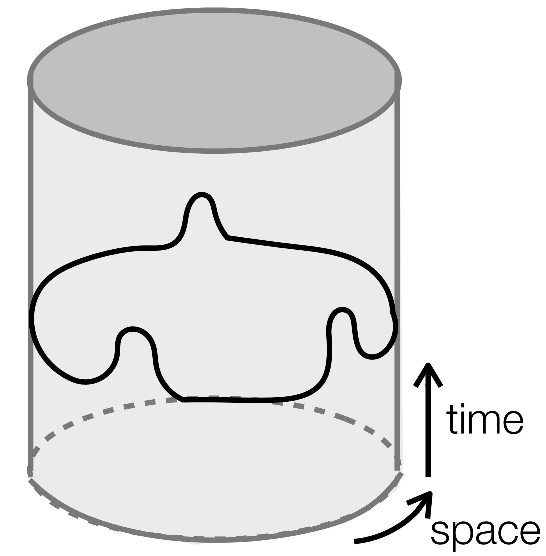


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String Hamiltonian

Hamiltonian is Integrable!

$$\mathbb{H}\,\psi = \Delta\,\psi$$

Integrable Classical String!

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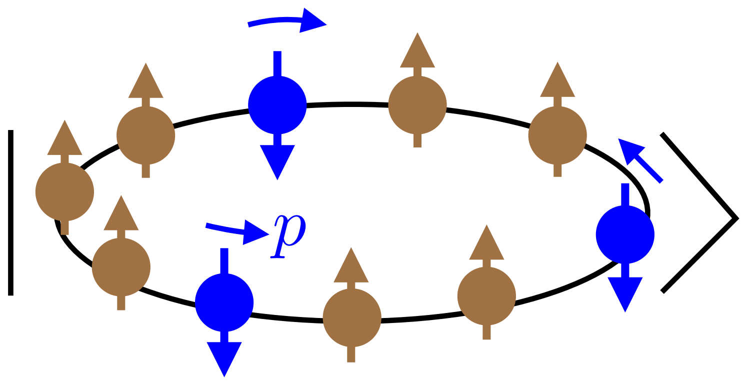


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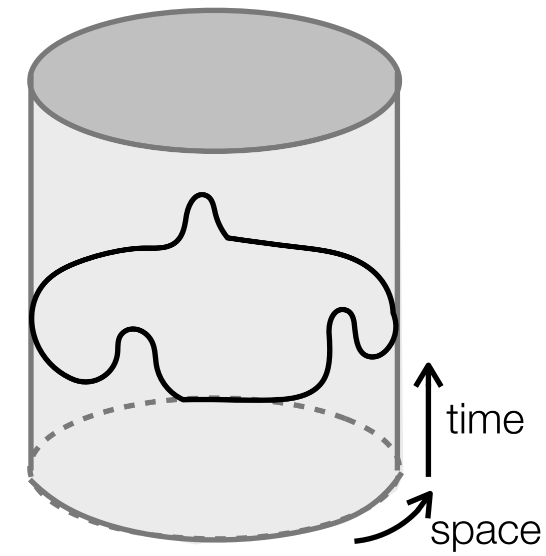


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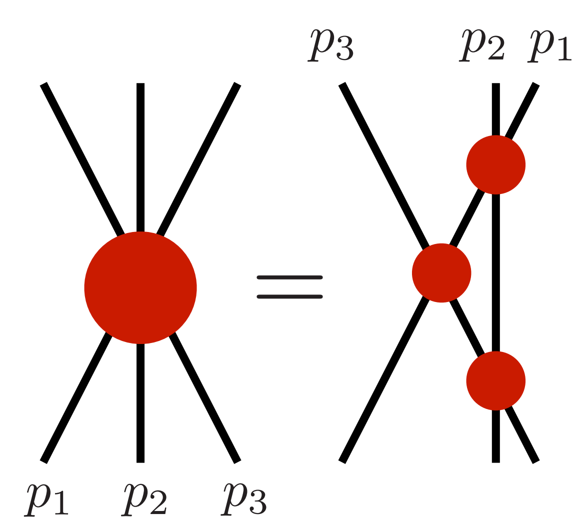


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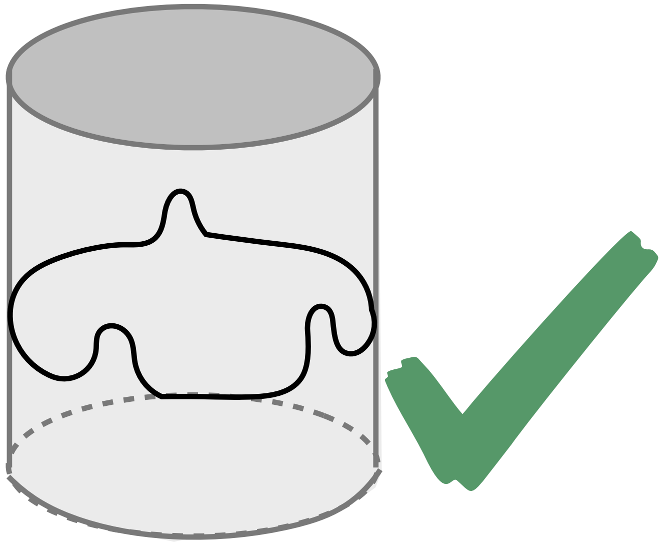
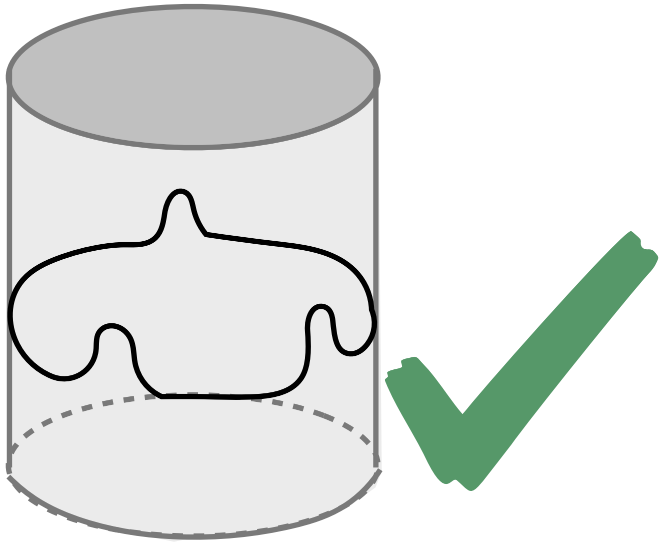


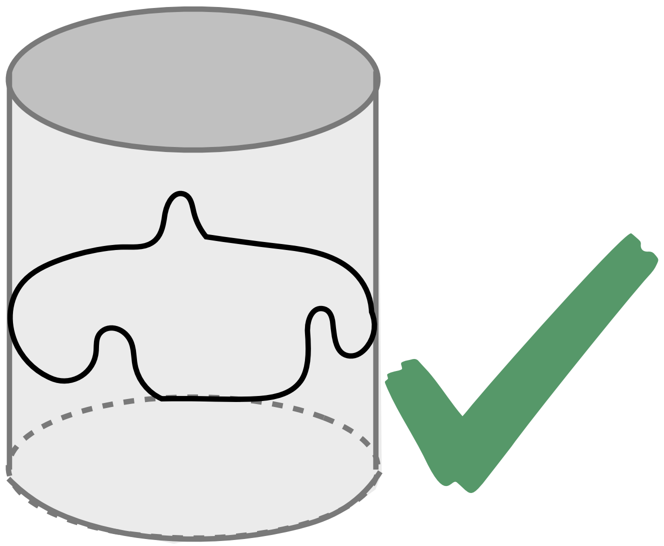
Cylinder = Spectral problem



Cylinder = Spectral problem

Standard 2D QFT (in finite volume)

# Other topologies?



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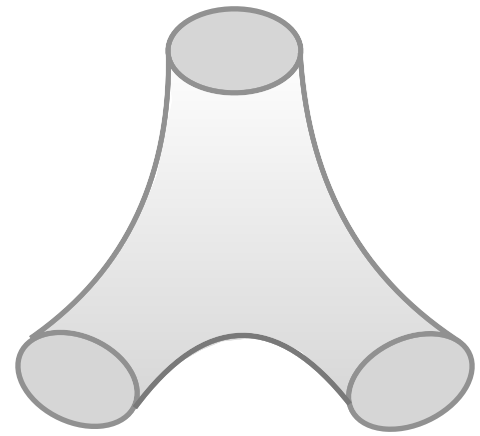
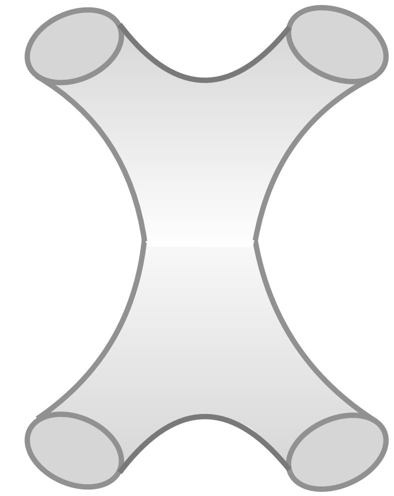


Pair of pants

$\langle \mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_3 \rangle$



Sphere with four punctures

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leading $N$

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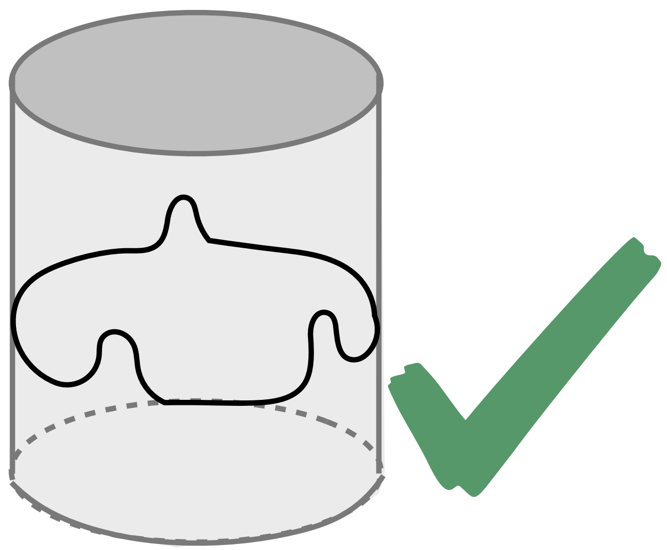

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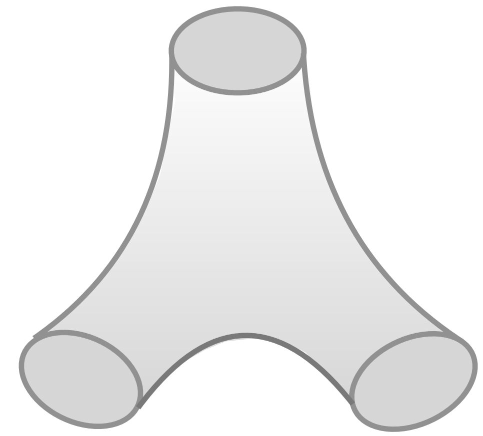

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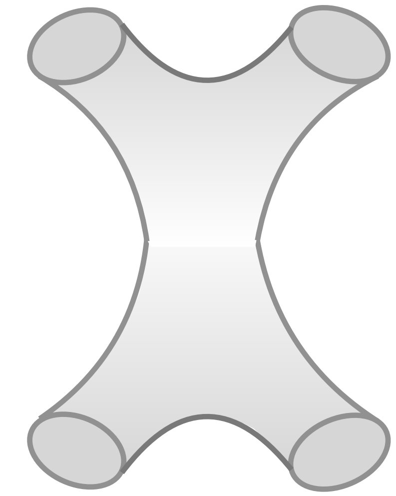

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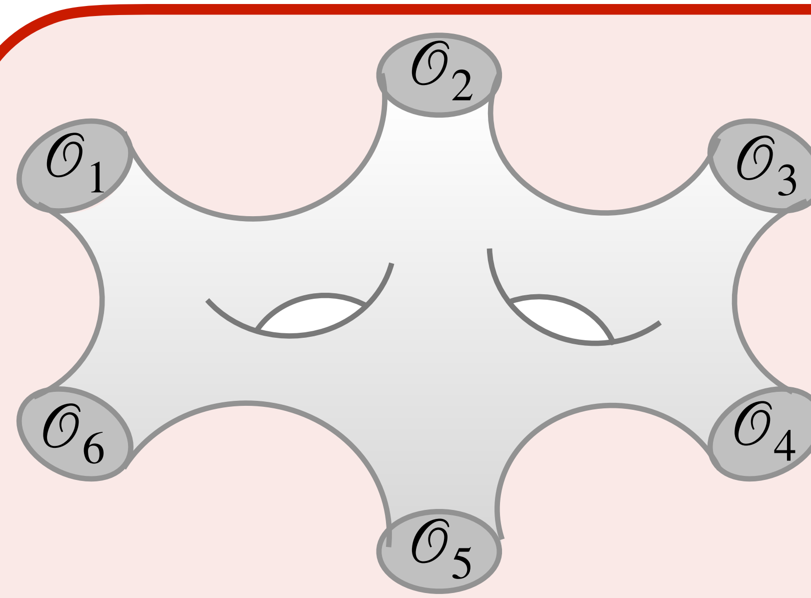
**General case:** Arbitrary number of operators (>2) and **beyond the large N limit**

e.g.




$\sim 1/N^8$

$\langle \mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_3 \mathcal{O}_4 \mathcal{O}_5 \mathcal{O}_6 \rangle$

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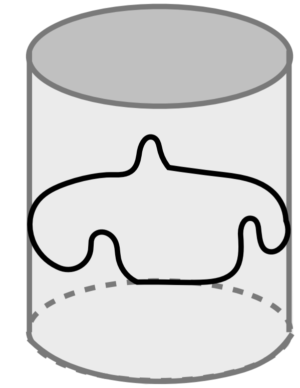

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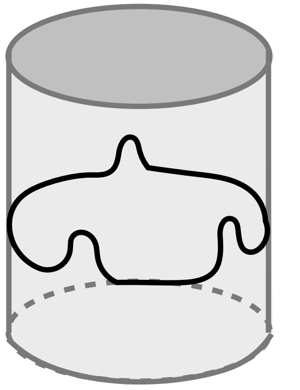

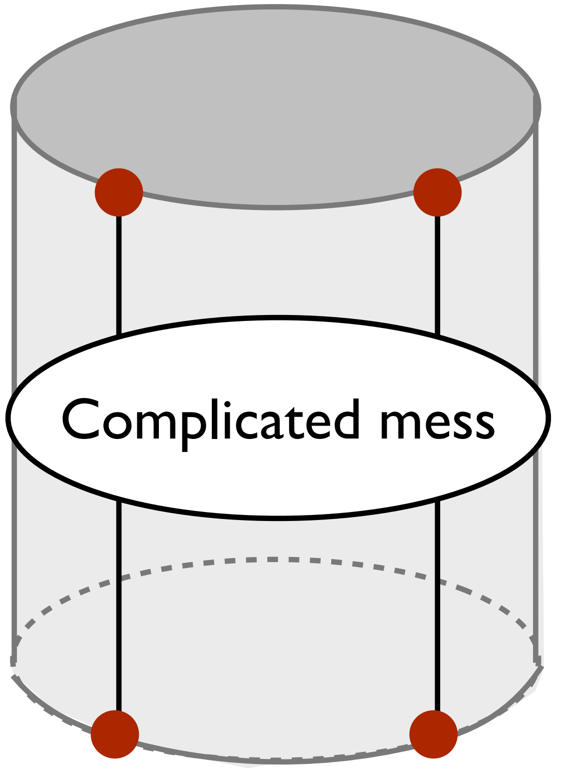

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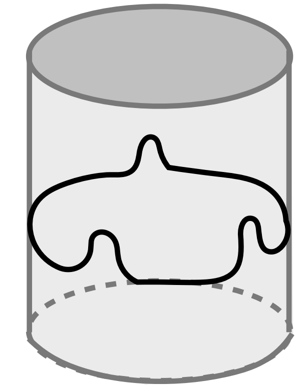

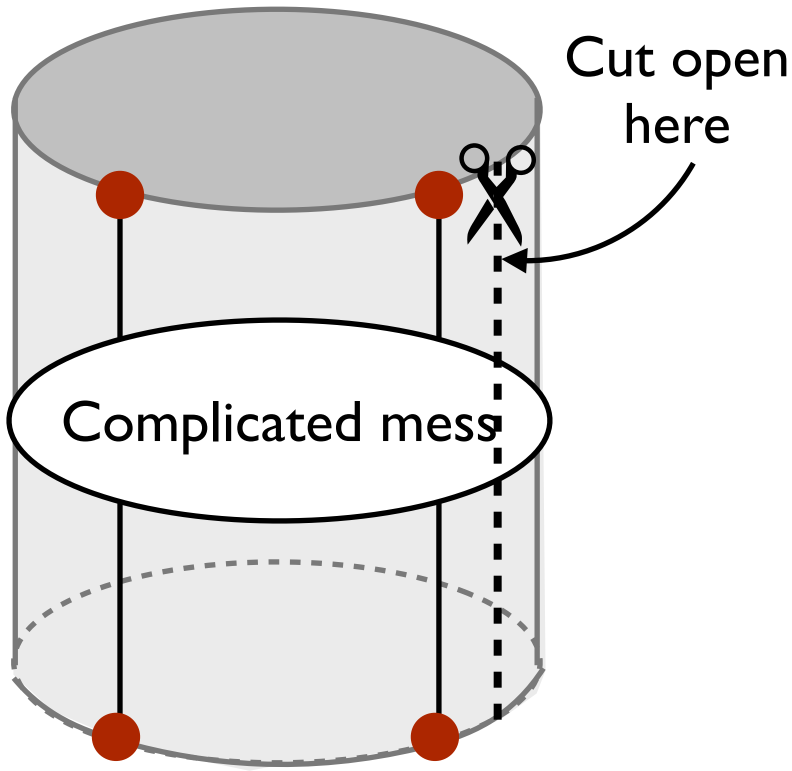

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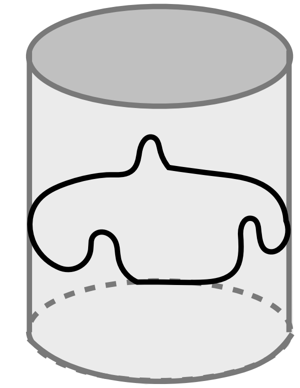

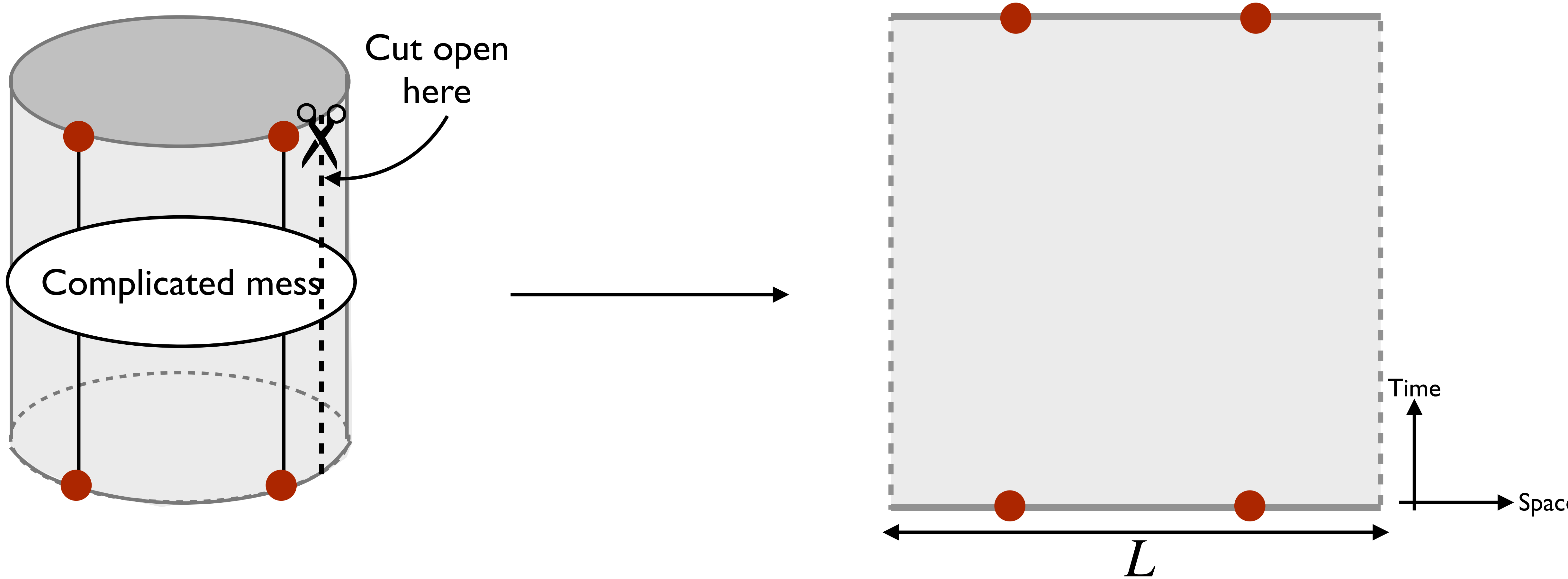

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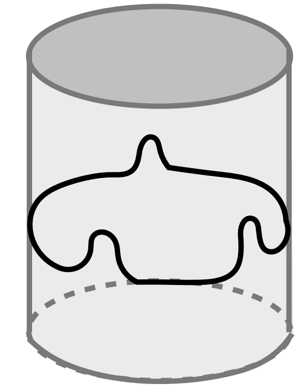
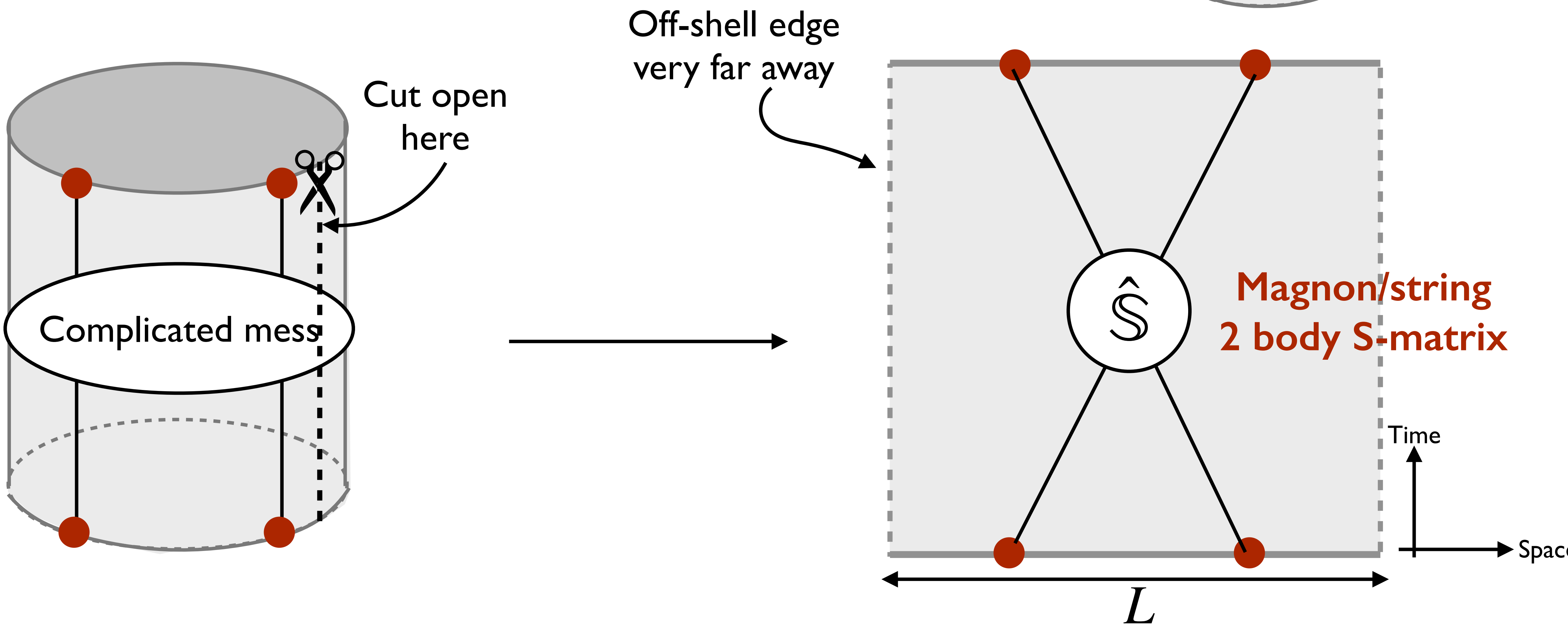
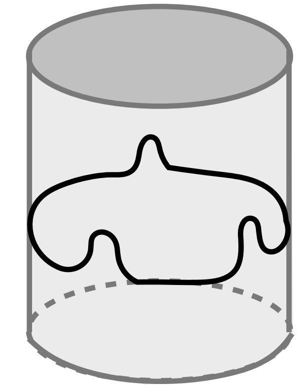
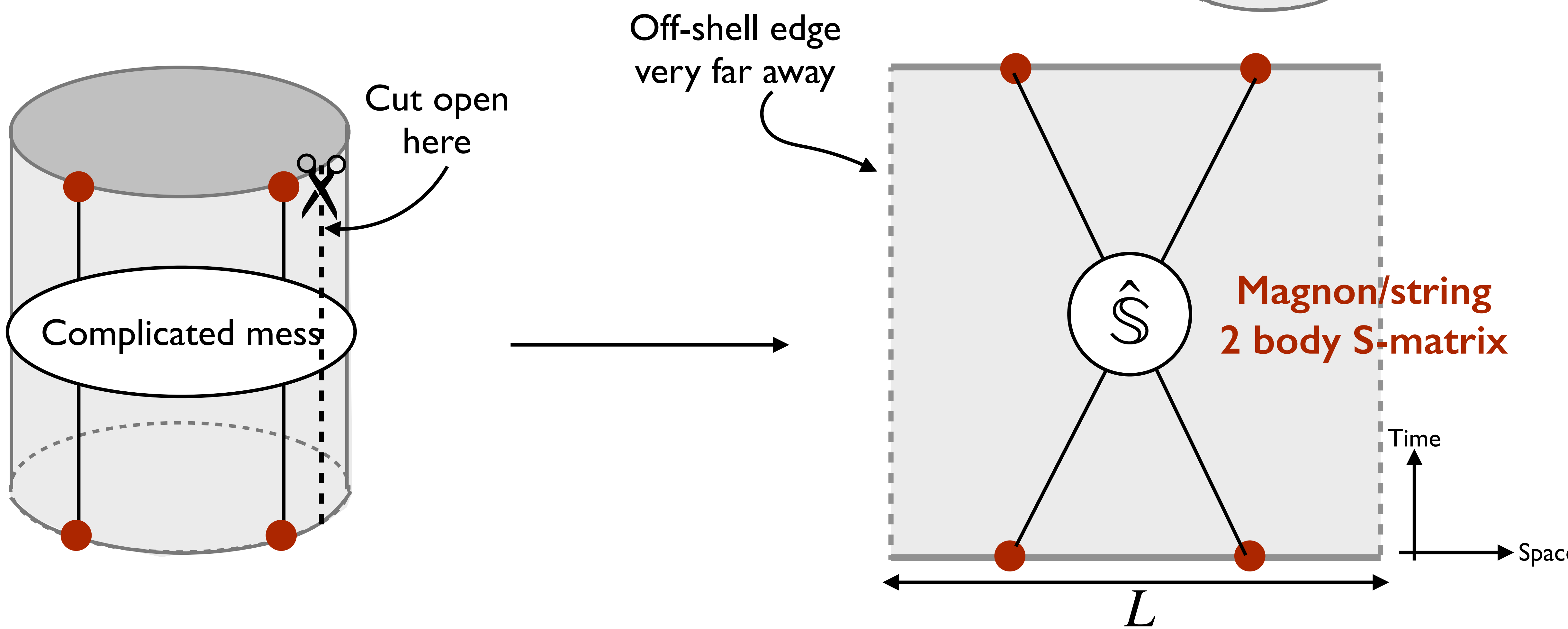





**Dilute gas approximation:** Complicated interactions replaced by 2 to 2 scattering events

# Cylinder = Spectral problem

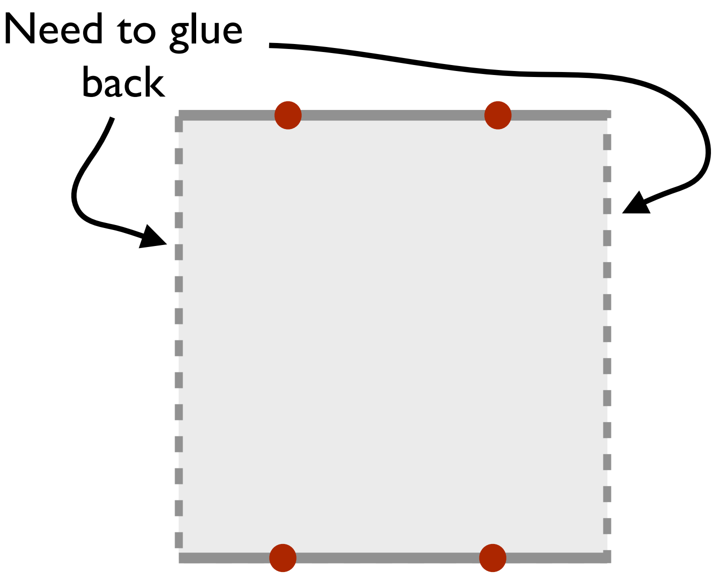




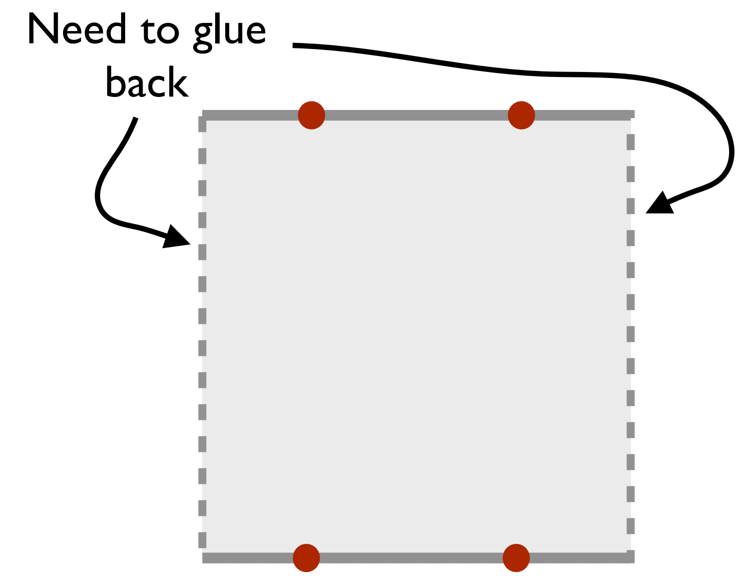

**Dilute gas approximation:** Complicated interactions replaced by 2 to 2 scattering events

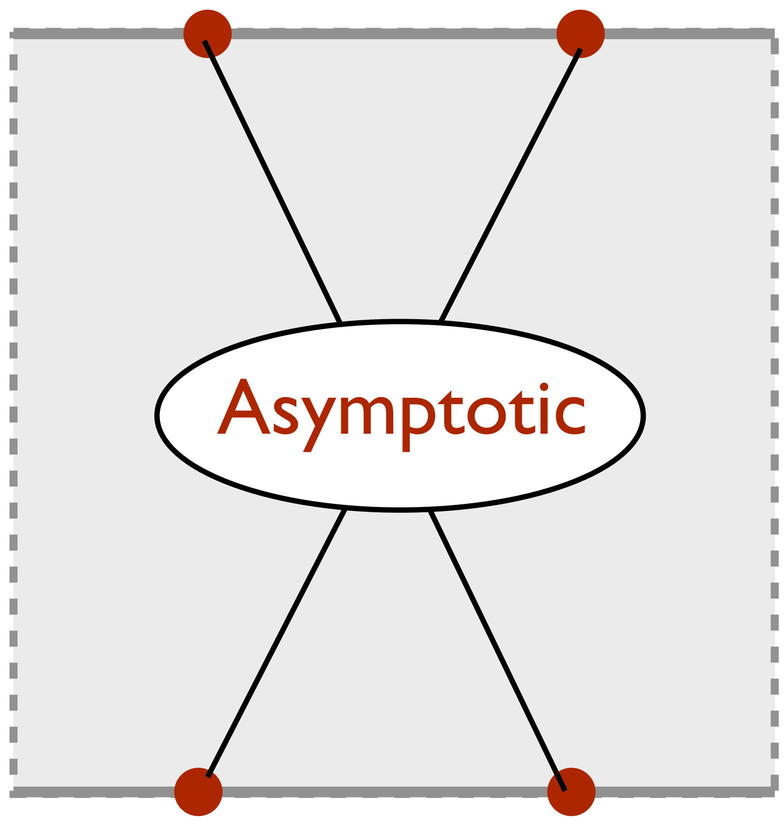
**Integrability:** Correct description up to exponentially small corrections in system size $e^{-LE} \sim \mathcal{O}(\lambda^L)$ (also known as **wrapping**)

# Glue back the cylinder



# Glue back the cylinder



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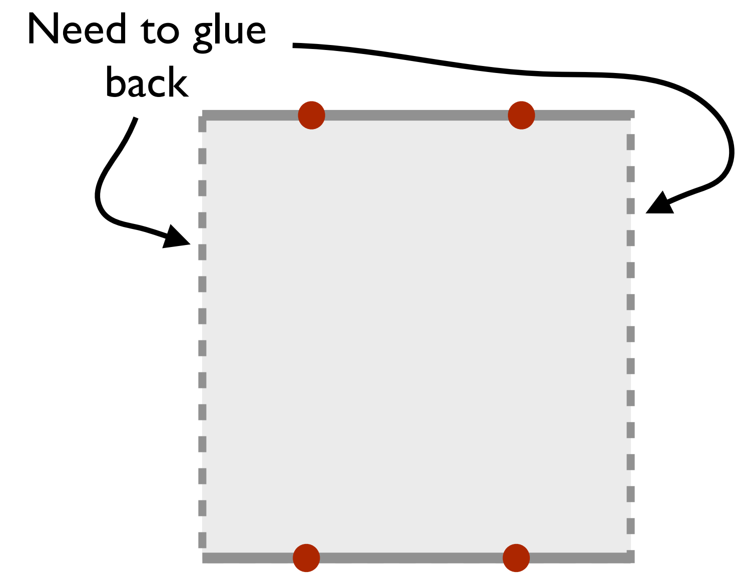


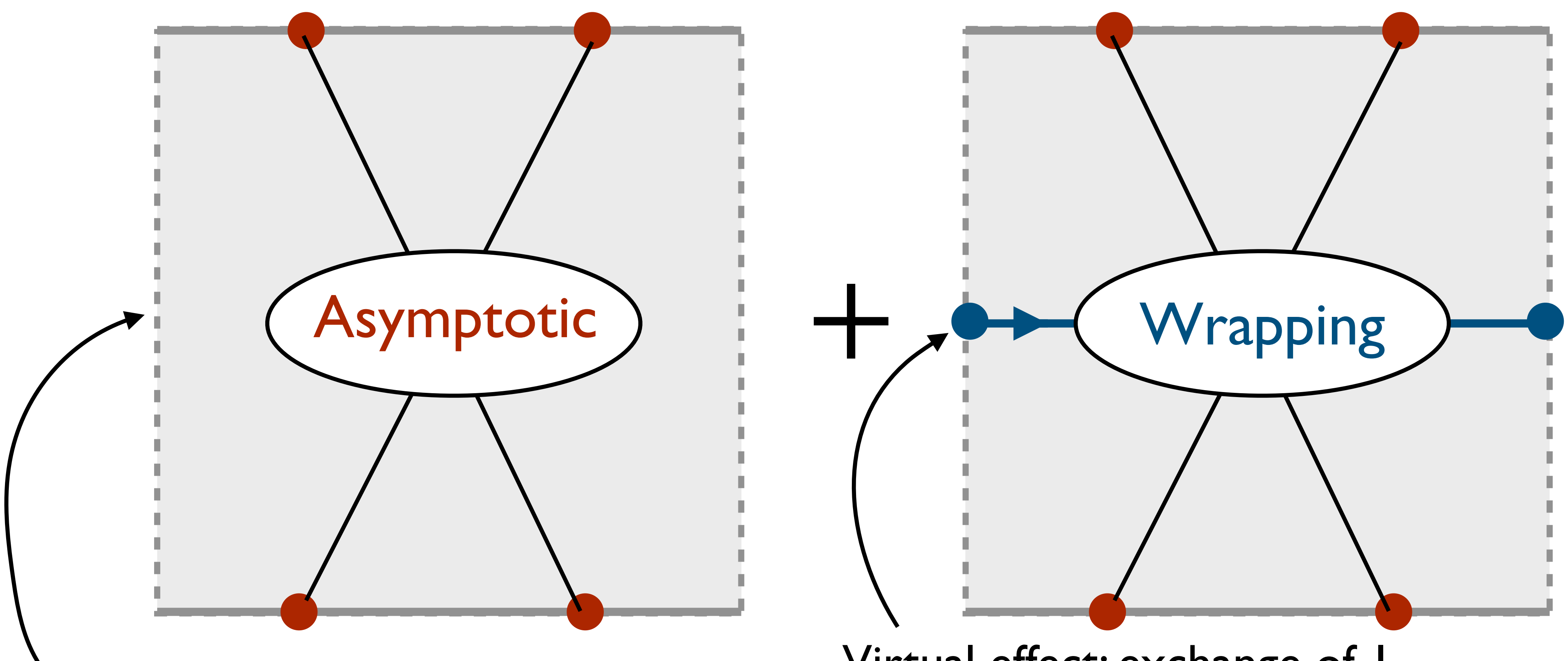


Vacuum (in mirror theory = double Wick rotated theory)

Virtual effect: exchange of 1 particle in mirror channel

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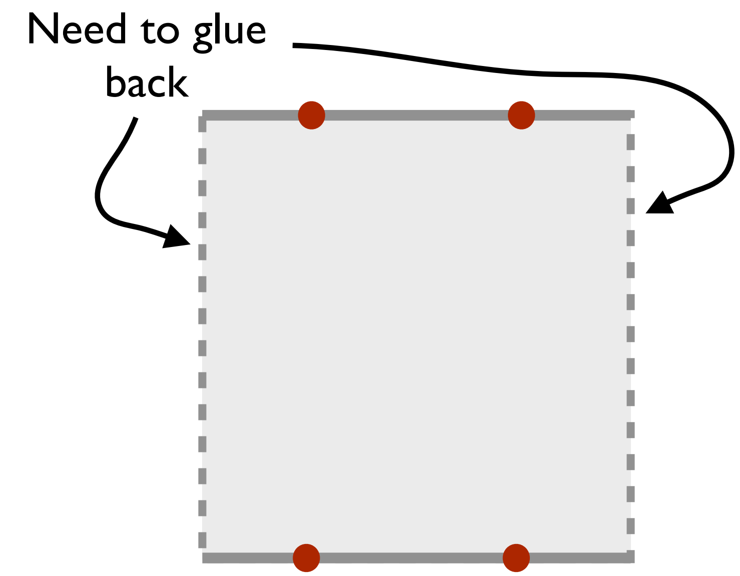


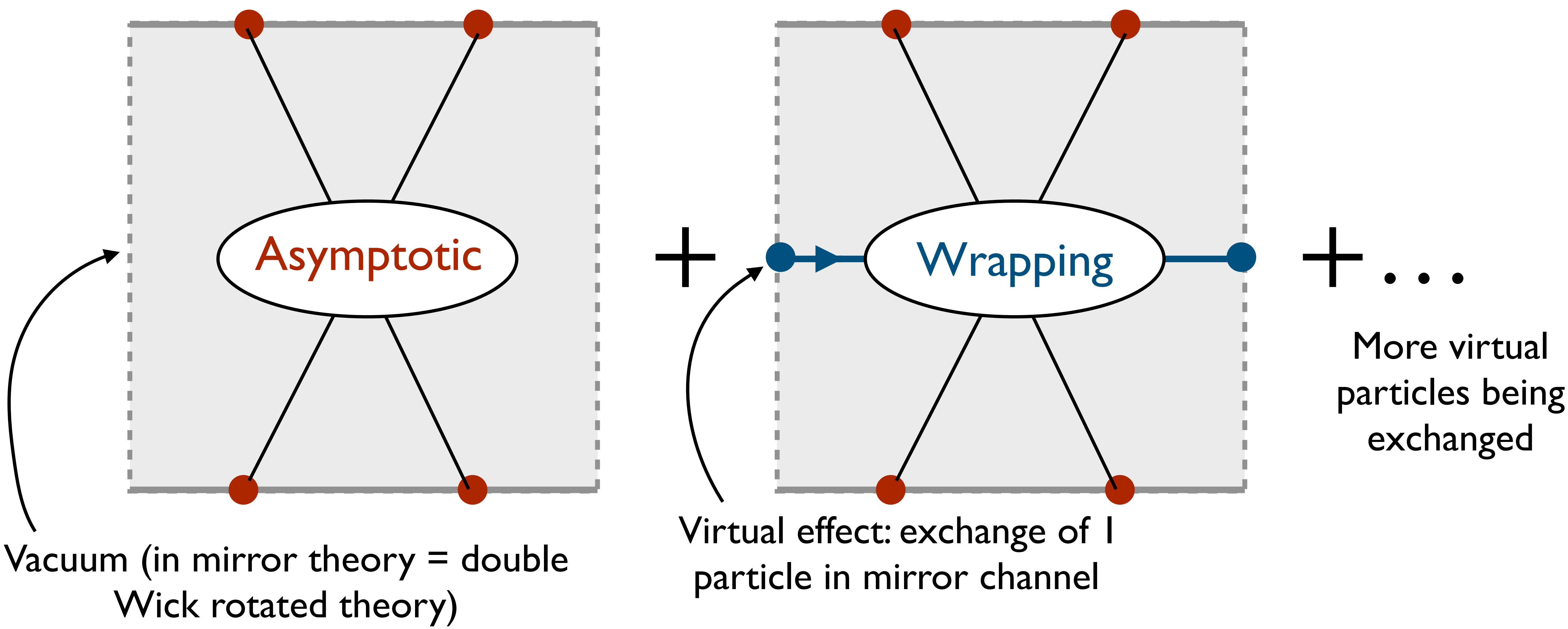

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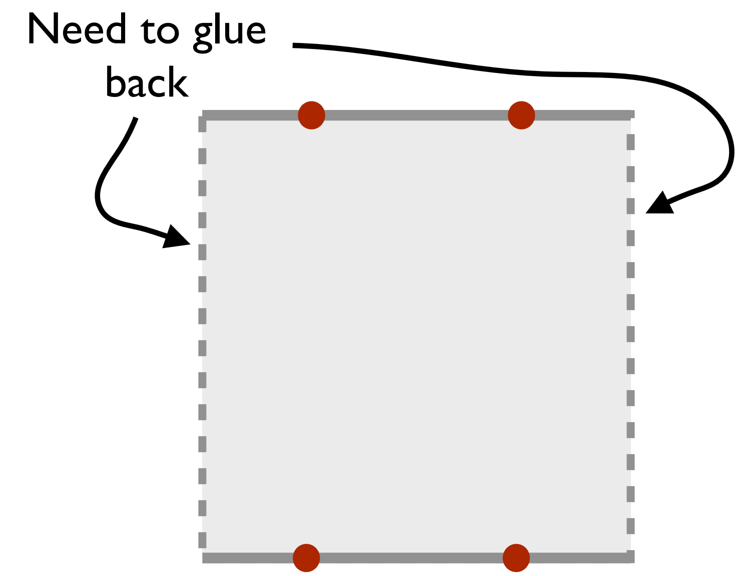


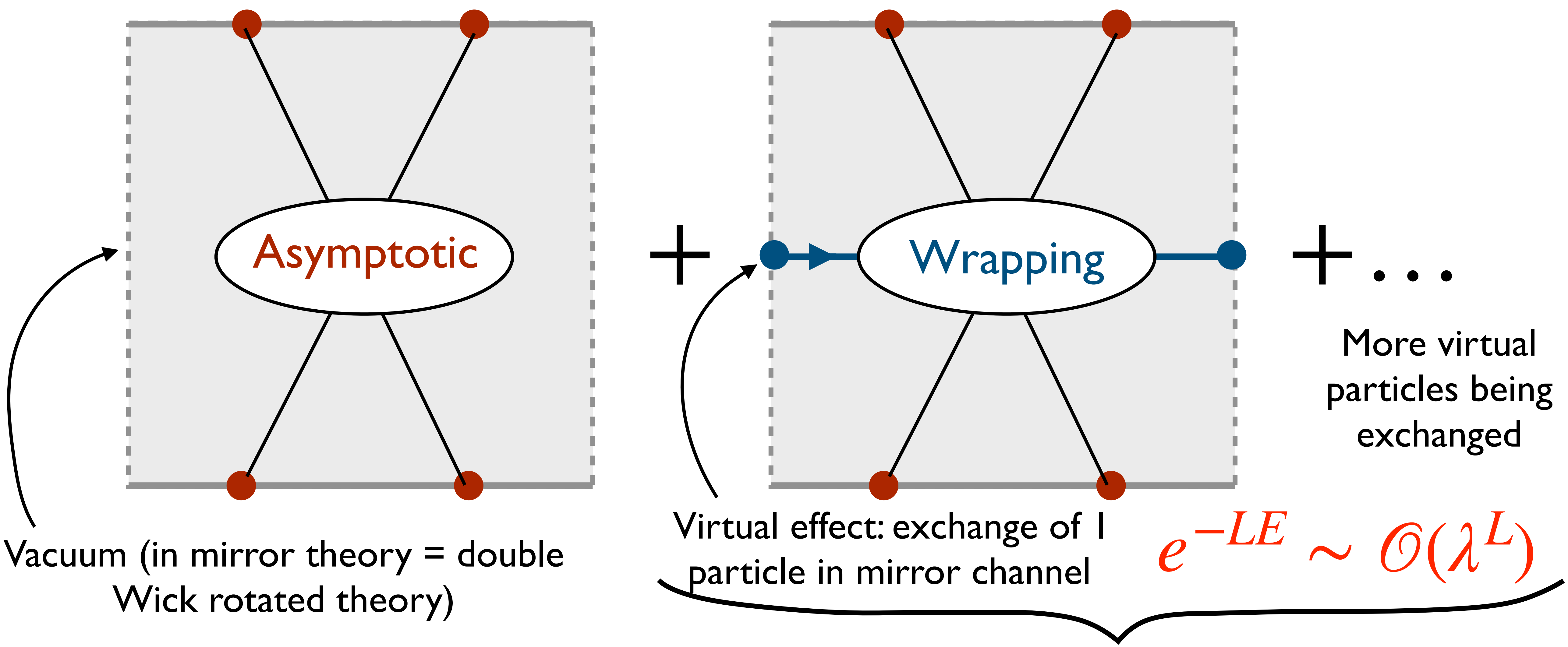


**Wrapping** corrections from 'mirror' excitations winding around the operators.
(Resummation of these corrections leads to TBA or Quantum spectral curve)
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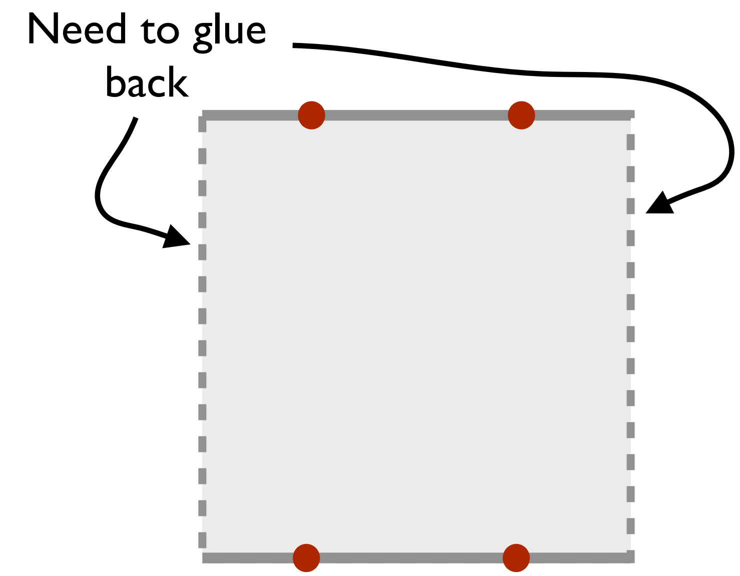


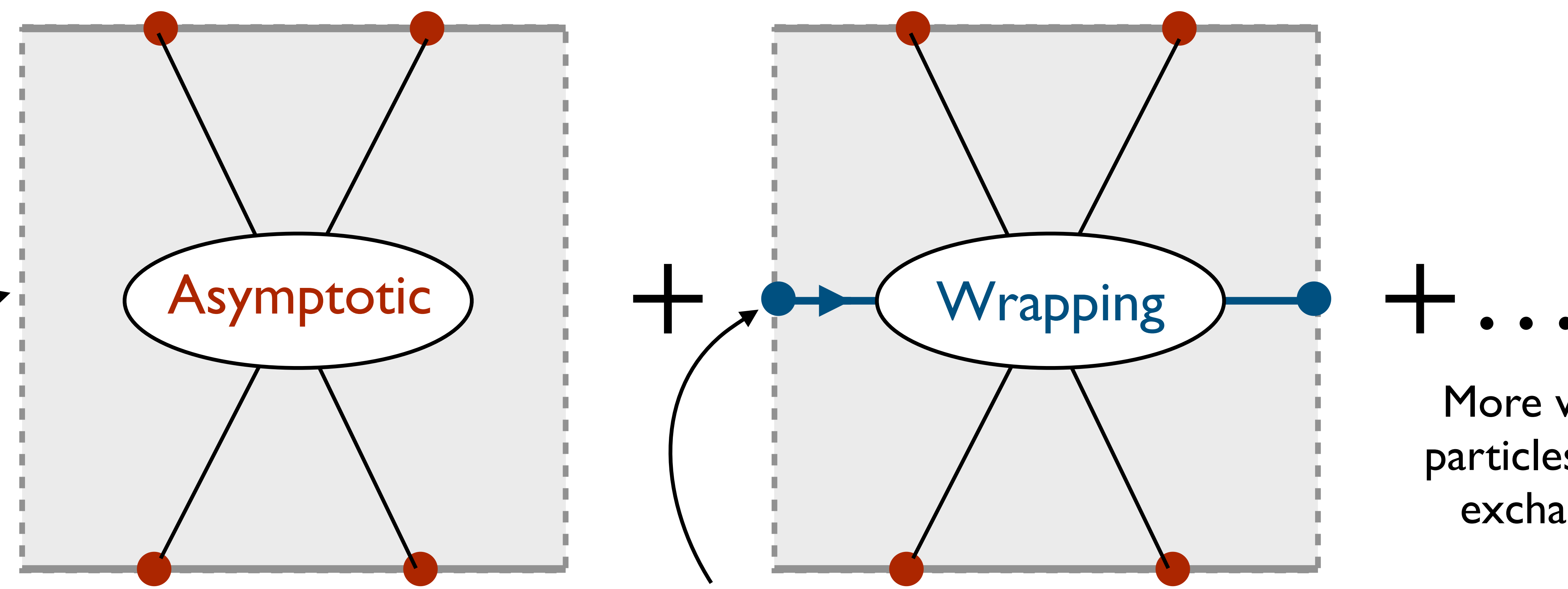


More virtual particles being exchanged

Vacuum (in mirror theory = double Wick rotated theory)

Virtual effect: exchange of 1 particle in mirror channel

$e^{-LE} \sim \mathcal{O}(\lambda^L)$

**Gluing edges = insert complete basis of states on those edges**

**Wrapping** corrections from 'mirror' excitations winding around the operators. (Resummation of these corrections leads to TBA or Quantum spectral curve)

[Gromov, Kazakov, Leurent, Volin'2013]

# Decompose the string into world-sheet patches

[Basso, Komatsu, Vieira'15]
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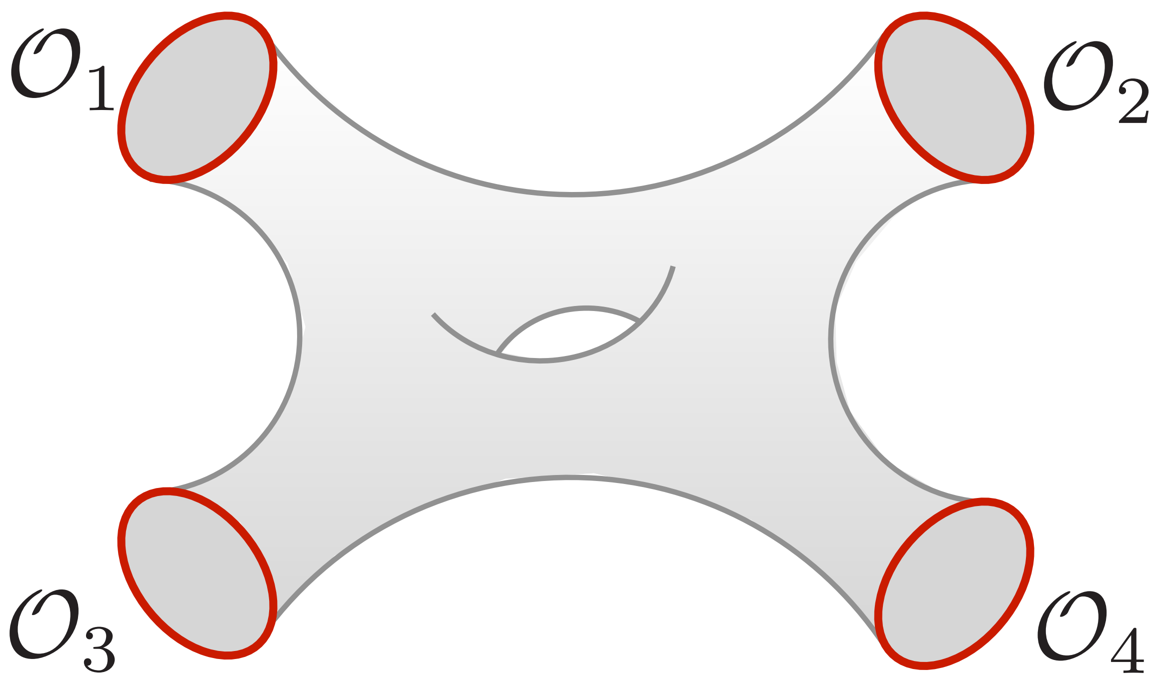

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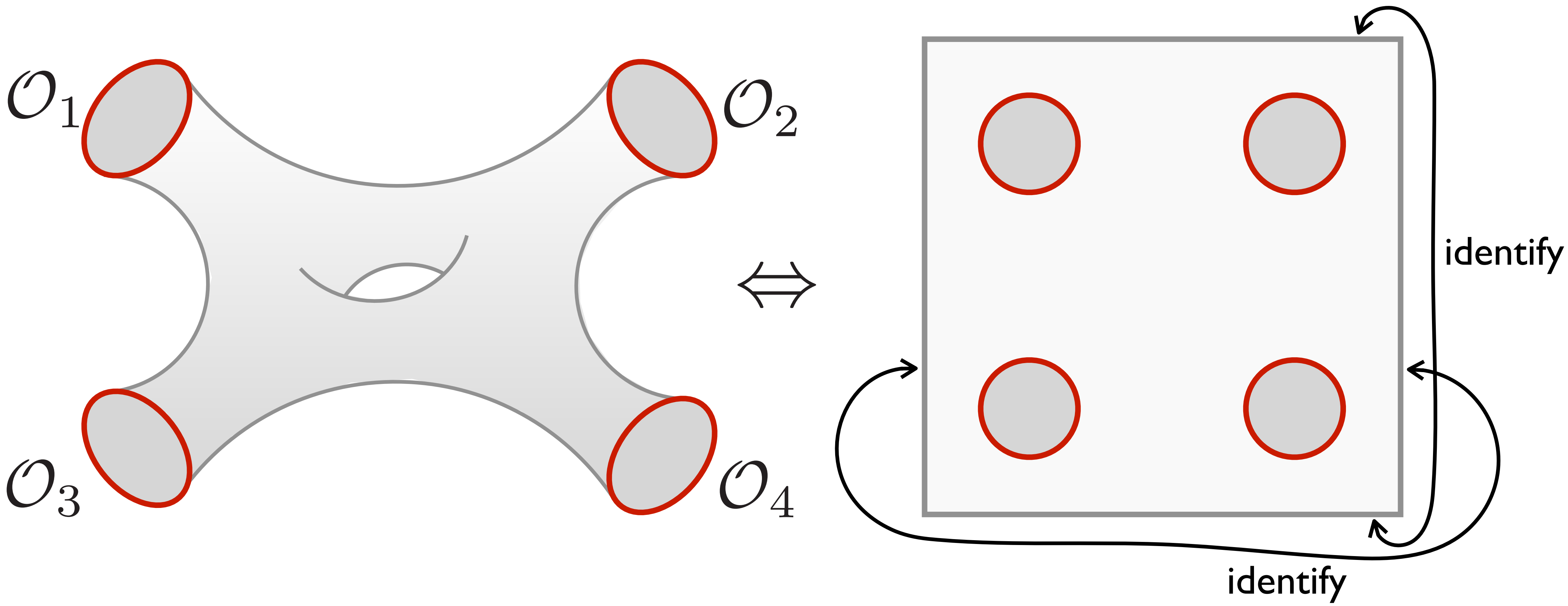

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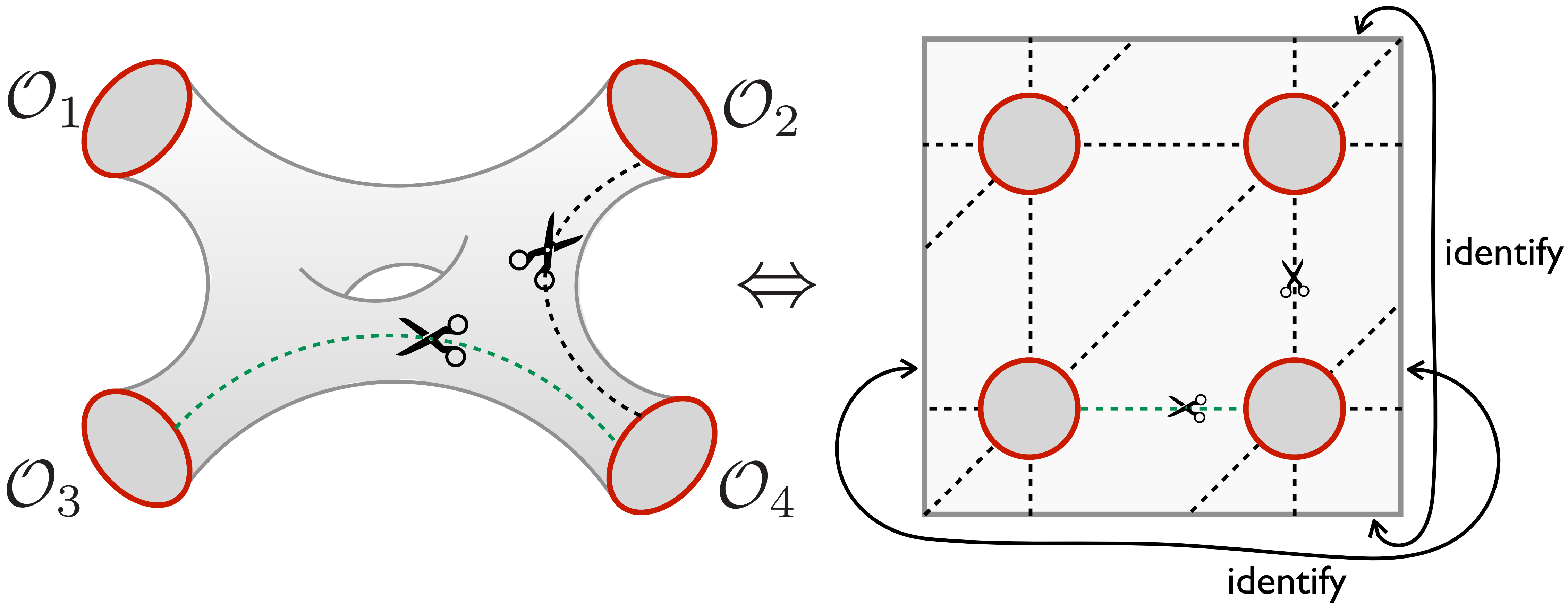

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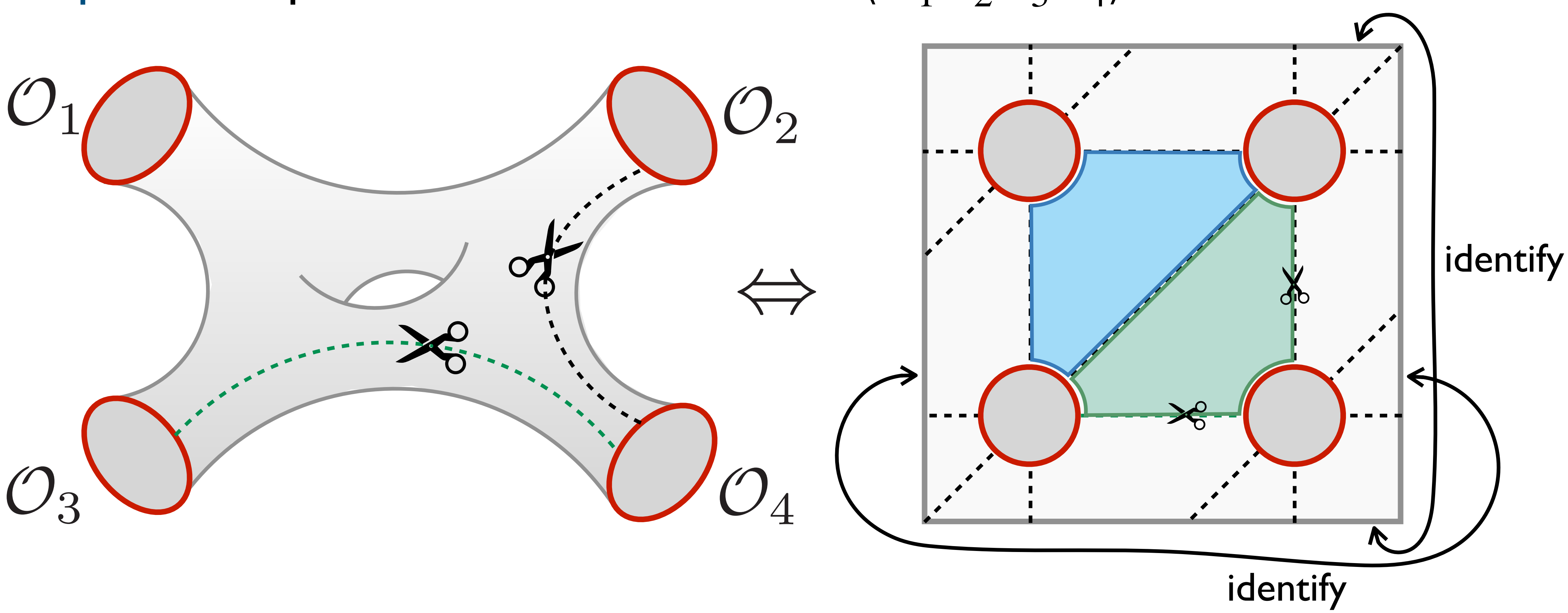

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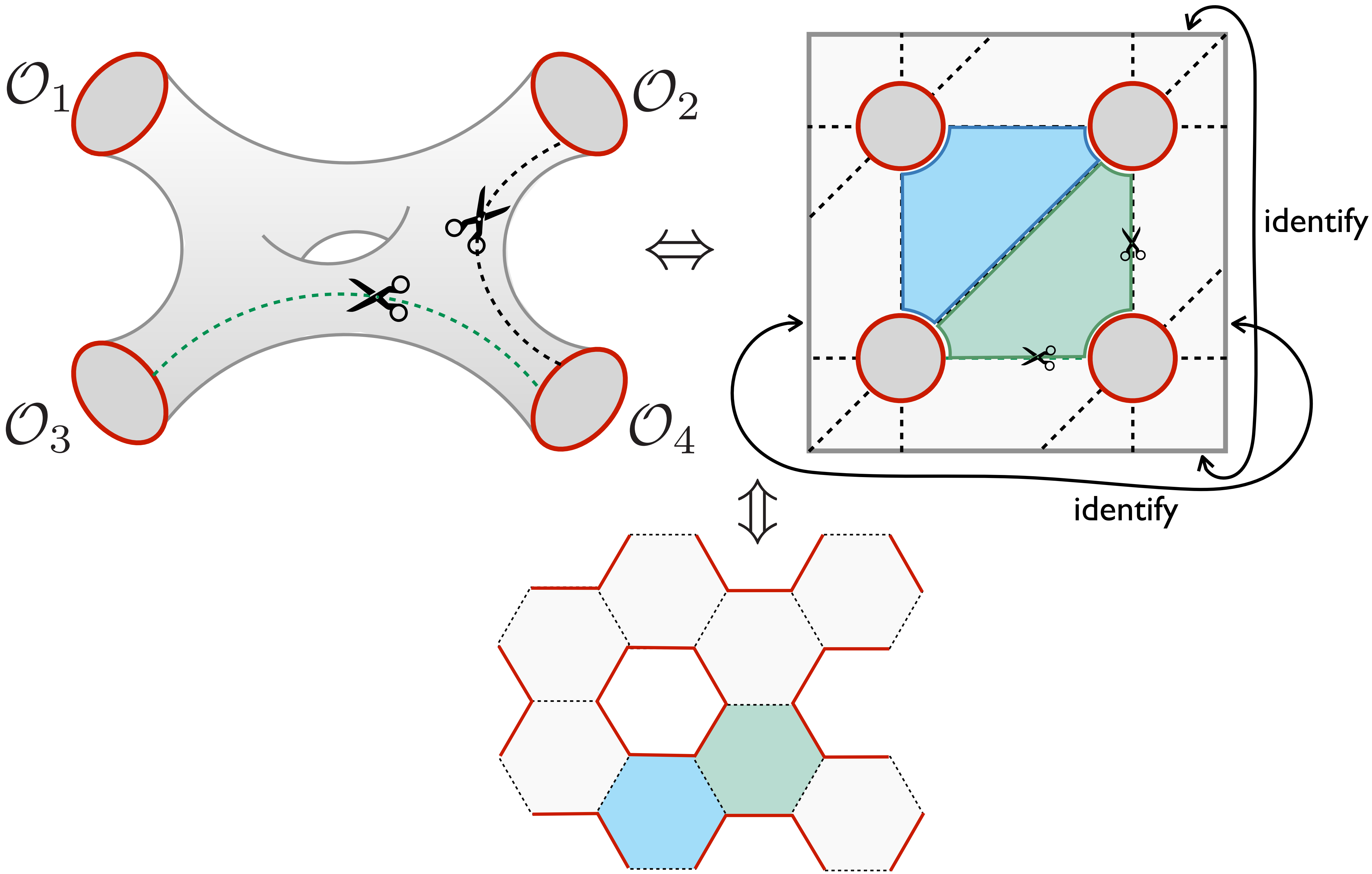

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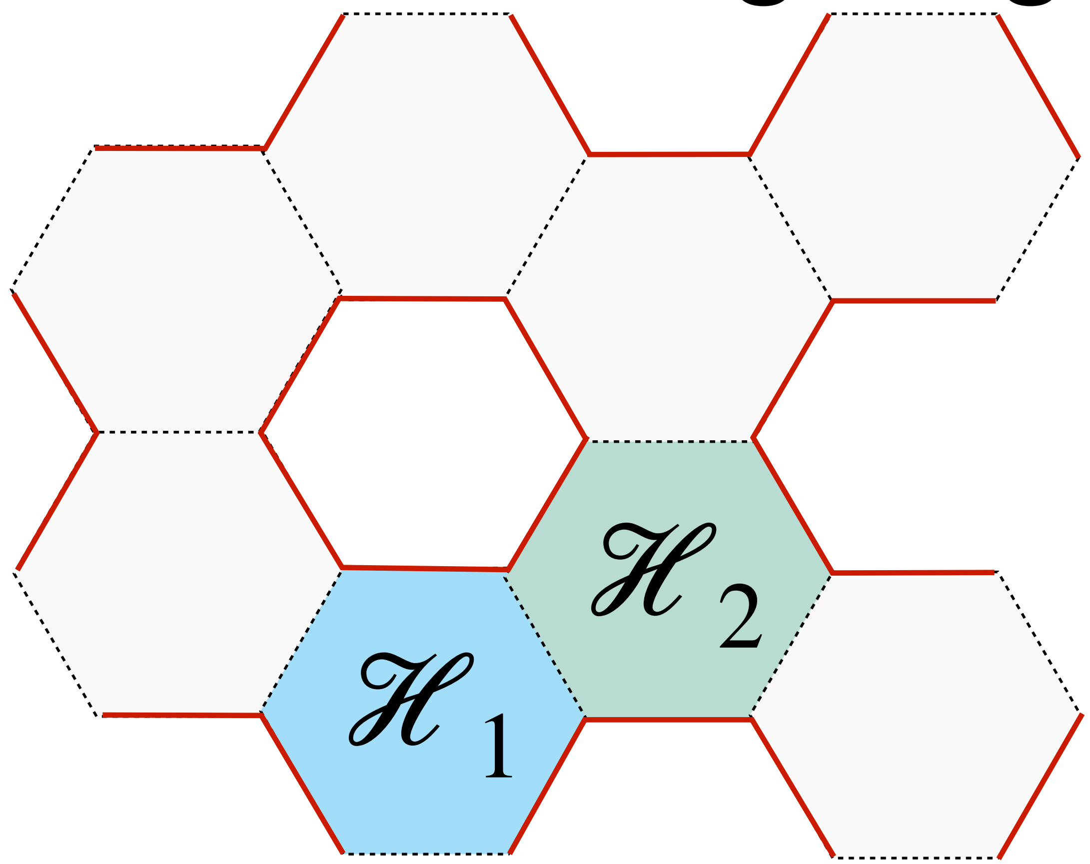

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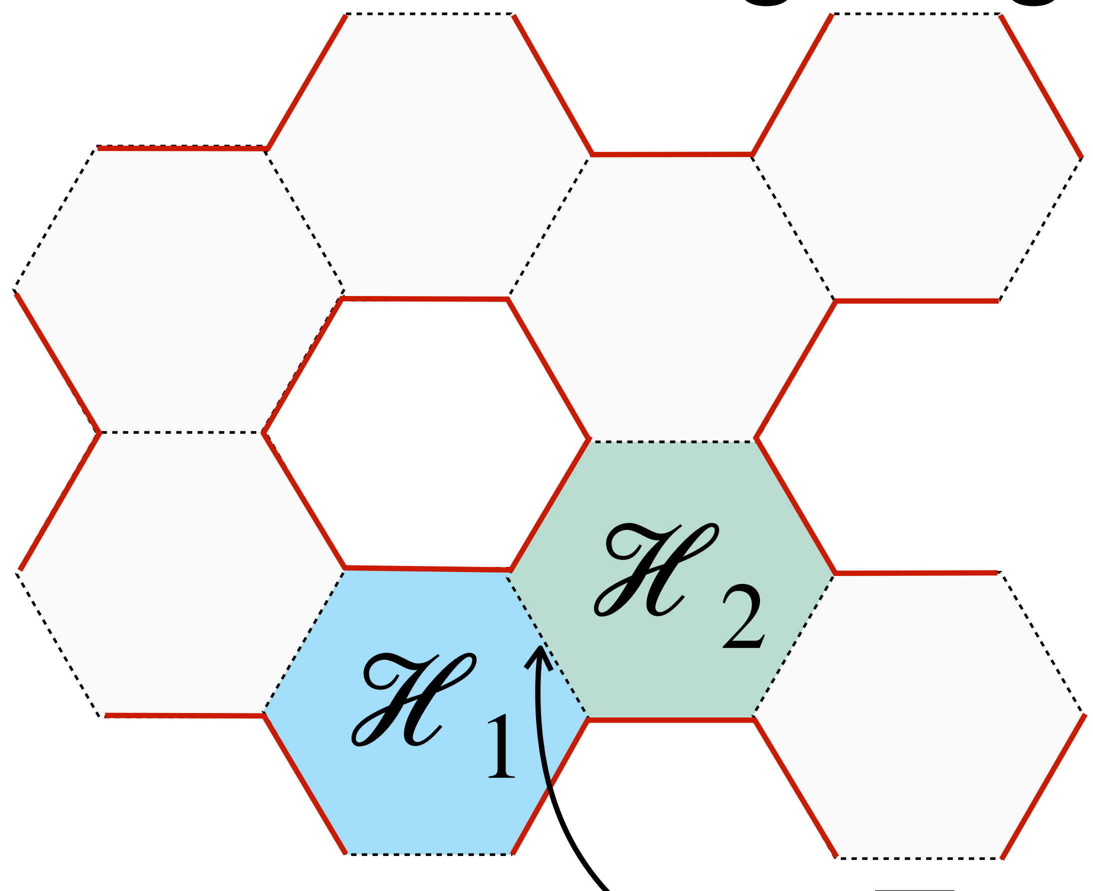


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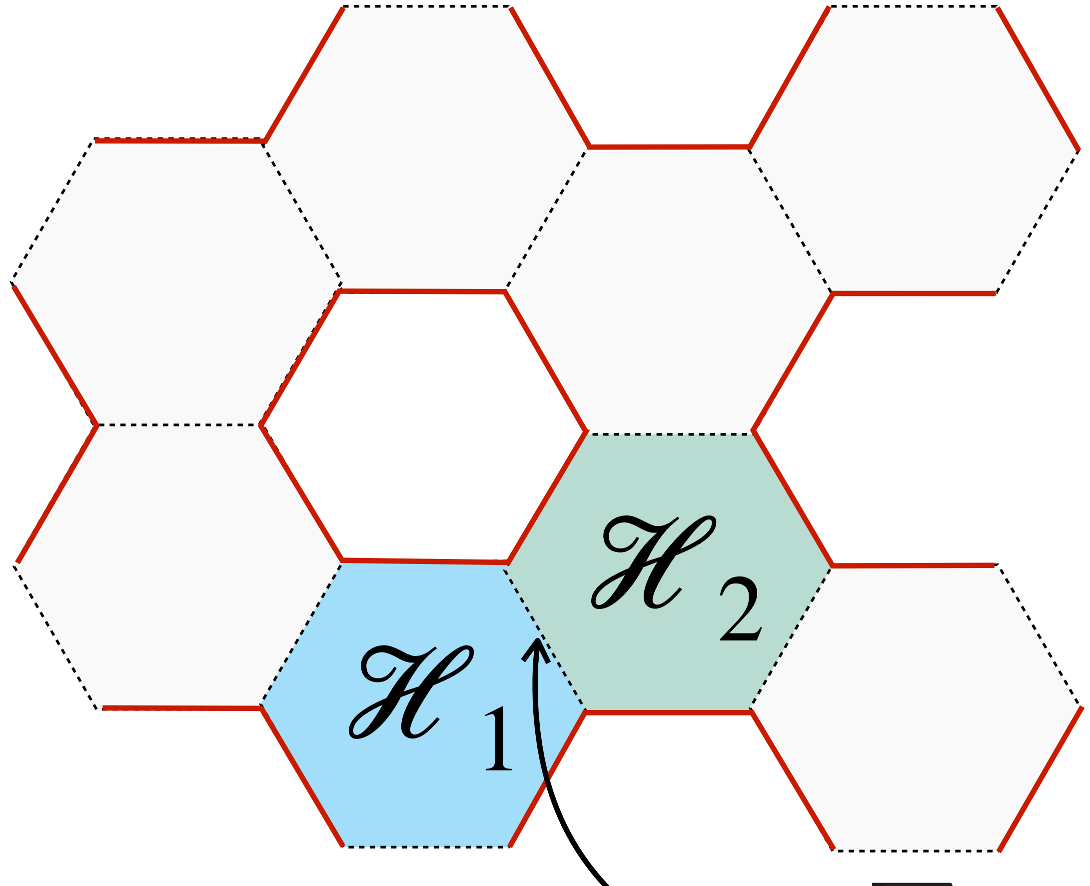


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$$\langle \mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_3 \mathcal{O}_4 \rangle \sim \frac{1}{N^4} \sum_{\substack{\text{complete} \\ \text{basis } \psi}} e^{-E_\psi \ell_{12} + \dots} \mathcal{H}_1(\dots, \psi) \mathcal{H}_2(\psi, \dots) \dots \mathcal{H}_8(\dots)$$

**This decomposition extends for any topology**

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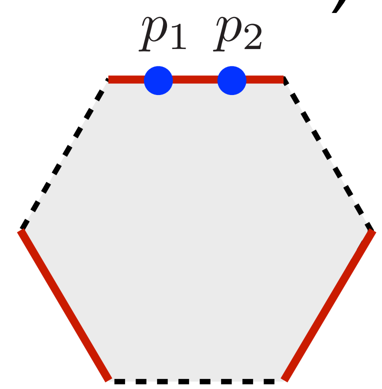
$$\mathscr{H}(p_1, p_2) =$$



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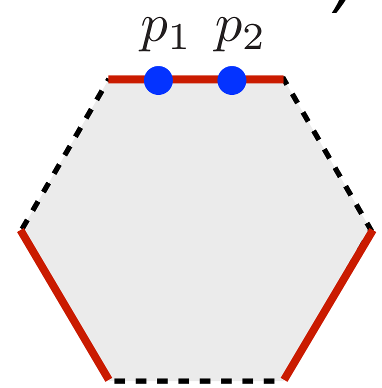


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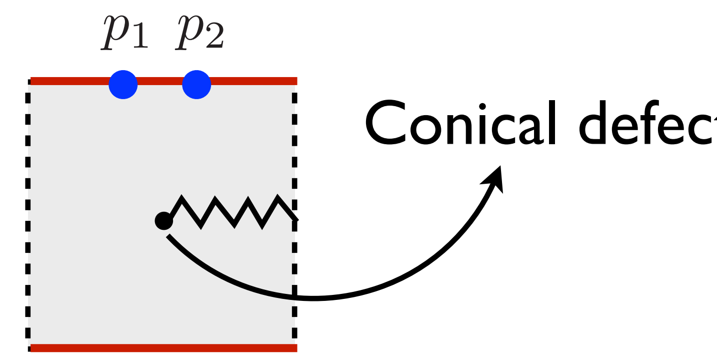

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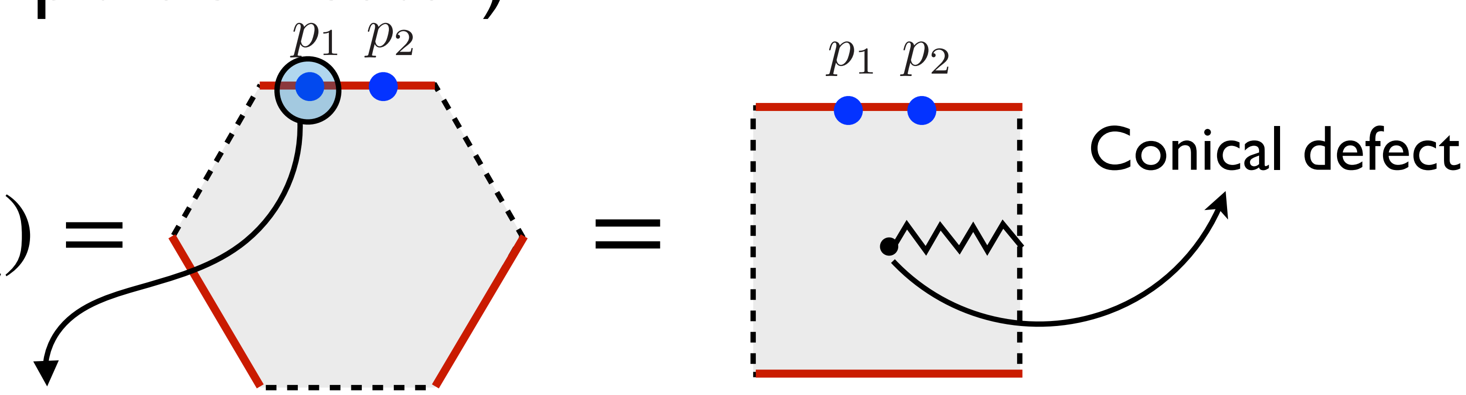
$\mathscr{H}(p_1, p_2) =$ 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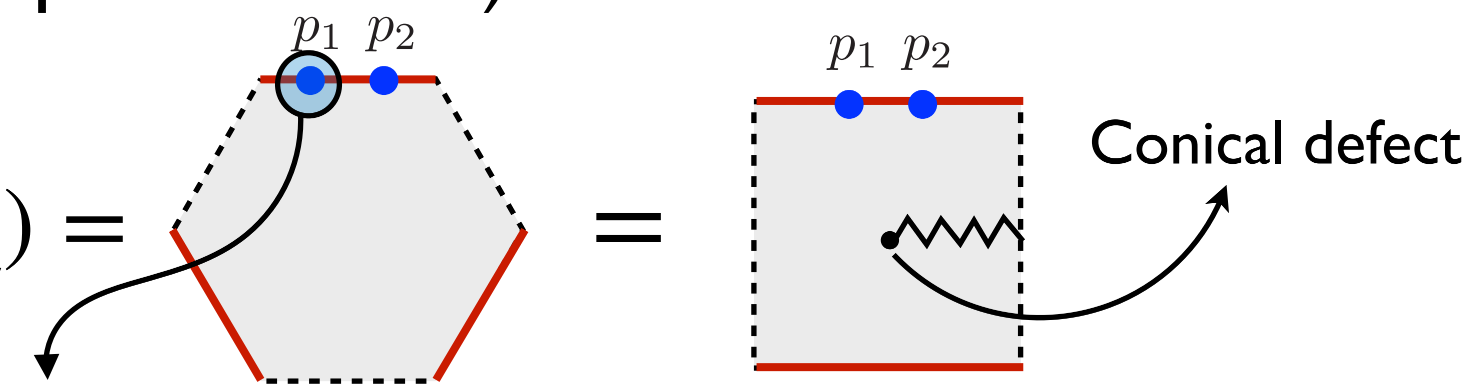


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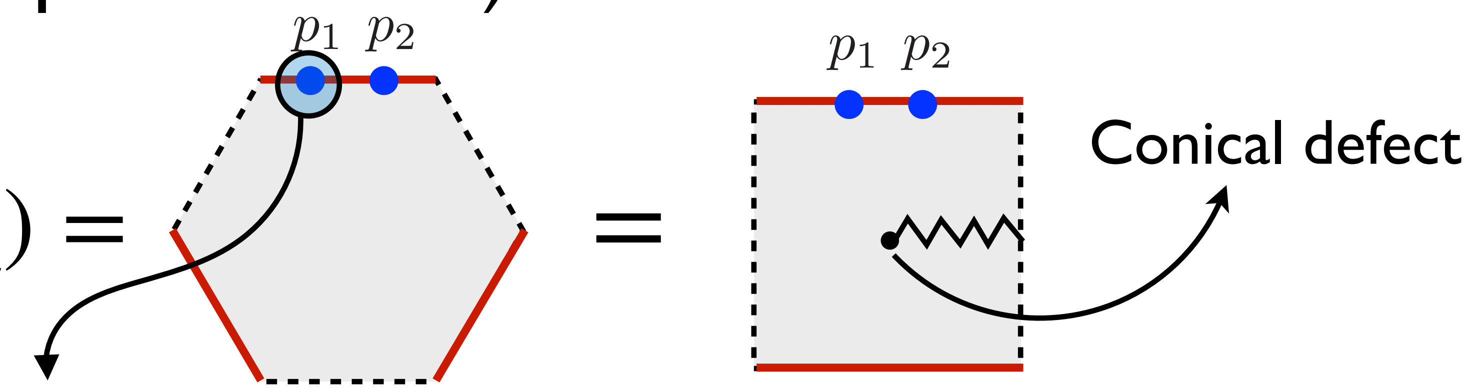


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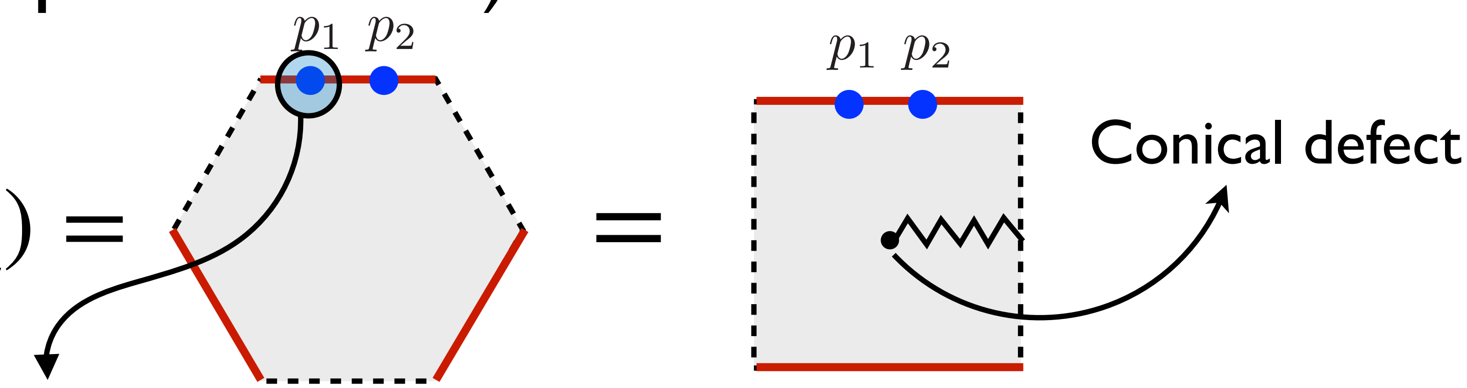


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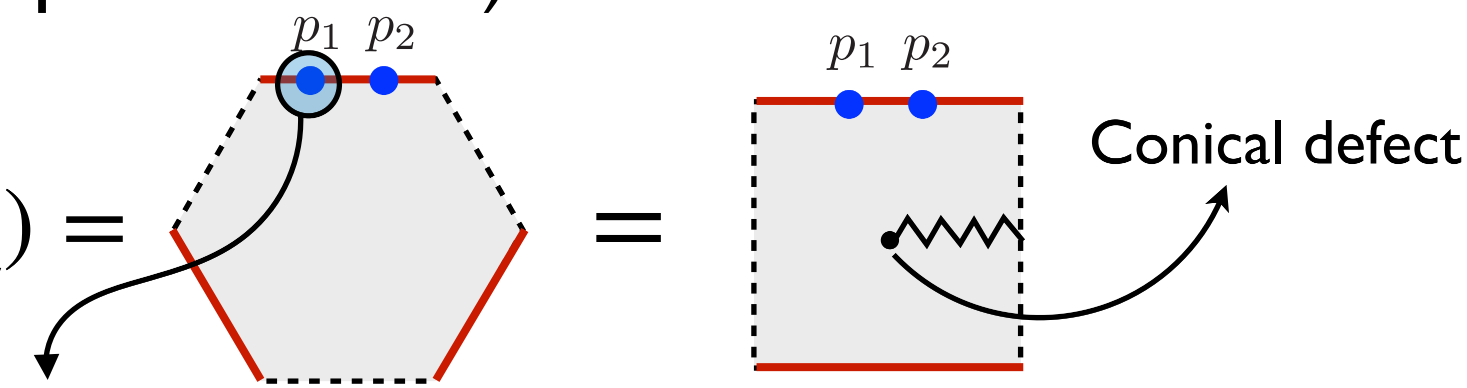


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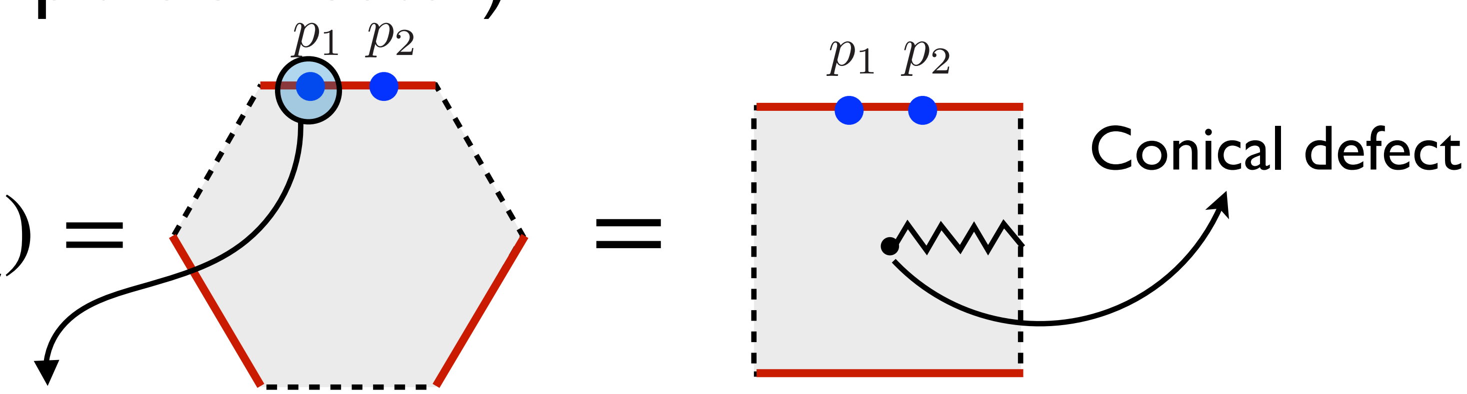


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$$\langle \mathcal{O}_1 \dots \mathcal{O}_4\rangle|_{\text{torus}} = -\frac{2k^6}{N_c^4}\Big\{$$

$$g^2\left[\tfrac{17}{6}r^4 - \tfrac{7}{4}r^2 + \tfrac{11}{32}\right]F^{(1)}$$ ✓ match

$$-\,2g^4\left[\left[\tfrac{17}{6}r^4 - \tfrac{7}{4}r^2 + \tfrac{11}{32}\right]F^{(2)} + \left[\tfrac{29}{6}r^4 - \tfrac{11}{4}r^2 + \tfrac{15}{32}\right]\tfrac{t}{4}\left(F^{(1)}\right)^2\right]$$ ✓ match

$$+\,g^6\left[[\dots]F^{(3)} + [\dots]\left(F^{(2)}\right)\left(F^{(1)}\right) + [\dots]\left(F^{(1)}\right)^3\right]$$ prediction!

$$+\,\mathcal{O}(g^8) + \mathcal{O}(1/k)\Big\}.$$

**Perfect match with known data!**

[Bargheer, **JC**, Fleury, Komatsu, Vieira'17 '18]

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**Area** of the minimal surface in AdS

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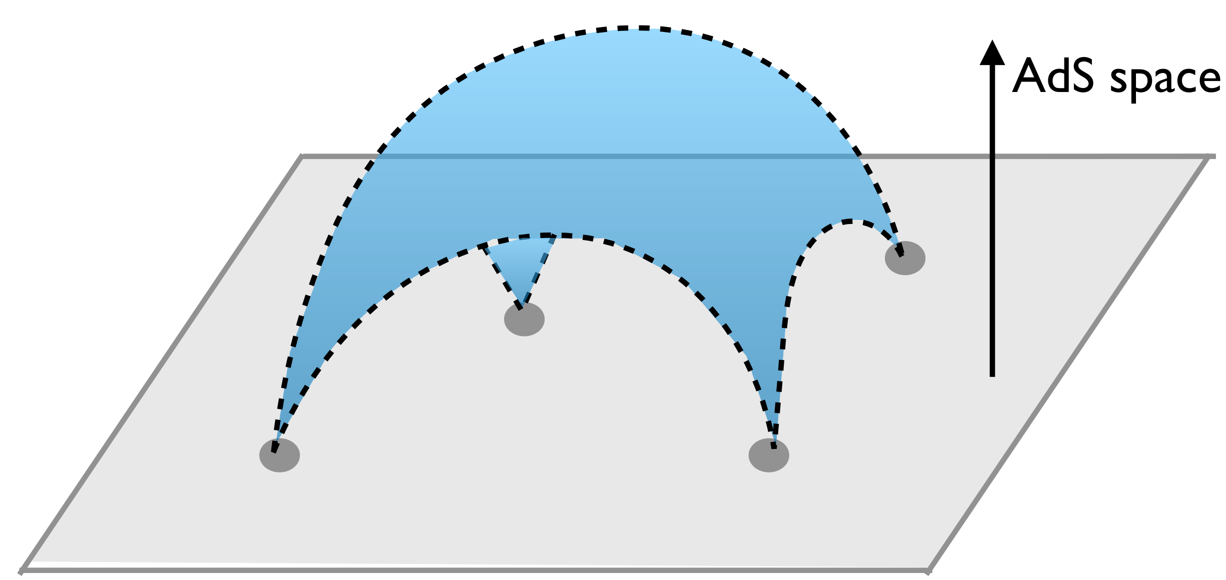

# Strong coupling correlation functions

**weak coupling** —— coupling $\lambda$ ——> **strong coupling**

## From Hexagons:

- Four point functions for 1/2 BPS operators in a special polarization [Coronado'18; Kostov, Petkova Serban'19; Belitsky, Korchemsky'19 '20; Bargheer Coronado, Vieira'19]
- Three point functions near the BMN limit [Basso, Zhong'19]
- Partial resummation of mirror excitations for 3 heavy operators [Jiang, Komatsu, Kostov, Serban'16]
- Hexagons for Fishnets and resummation [Basso, **JC**, Fleury '18]

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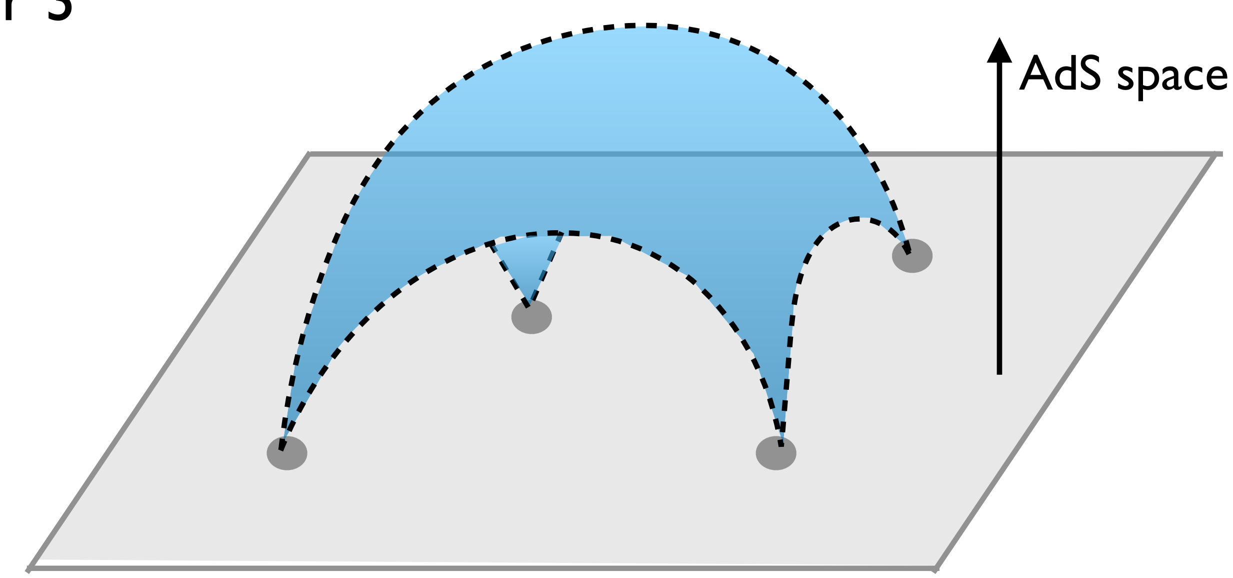

# Beyond AdS/CFT?

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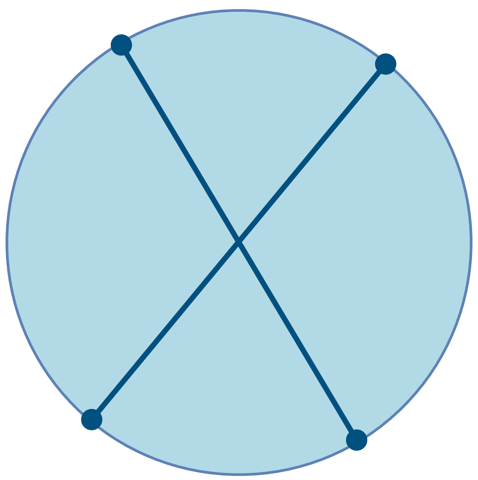
$\mathcal{N}$=4 SYM on $\mathbb{RP}^4$

[**JC**, Rastelli, to appear]
[**JC**, Komatsu, Rastelli, in progress]
[**JC**, Komatsu'21]

# Real projective space

$$\mathbb{RP}^4 = S^4 / \{X^\mu \sim -X^\mu\}$$



(simplest unorientable 4-manifold)

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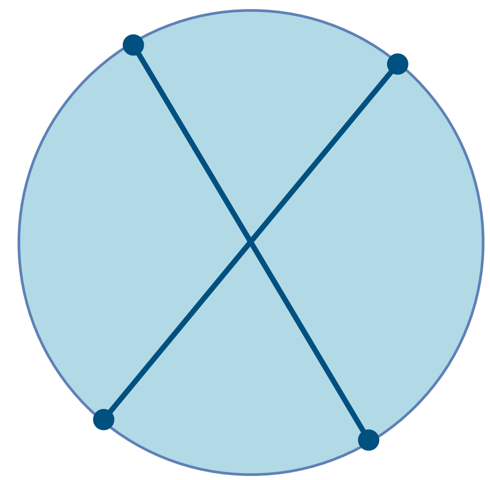

(simplest unorientable 4-manifold)

CFT$_d$ on $\mathbb{RP}^d$ $\quad \mathfrak{so}(d+1,1) \to \mathfrak{so}(d+1): \quad K_\mu - P_\mu\,,\ M_{\mu\nu}$

(Euclidean)

- Locally conformally flat, but not globally
- Same OPE structure as in flat space

# Why to study gauge theories on $\mathbb{RP}^4$?

- New setup of AdS/CFT, with exactly solvable tools like localization, integrability and bootstrap. New ingredients in holography.
- QFTs on unorientable manifolds: insight on time-reversal anomalies [Witten'16]
- CFT on $\mathbb{RP}^d$: conformal symmetry breaking
  - new observables $\langle \mathcal{O} \rangle$ satisfying bootstrap constraints
  - similar to the boundary setup but much more rigid.

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- **Color part?**

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- $\mathcal{N} = 4$ SYM: Antipodal map preserves 16 supercharges **1/2-BPS configuration**

$$\mathfrak{osp}(4^*|4) \supset \underbrace{\mathfrak{so}(5)}_{\text{spacetime}} \times \underbrace{\mathfrak{so}(3) \times \mathfrak{so}(2,1)}_{\text{R-symmetry}}$$

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Spacetime and R-symmetry transformations fixed by SUSY

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(At least) **two** choices, for $\mathcal{N} = 4$ SYM on $\mathbb{RP}^4$ depending whether we **gauge** $\tau$ or not

# No charge conjugation

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$$\Phi_I^a(x')\,T_a = \hat{\Phi}_I^a(x)\,T_a$$

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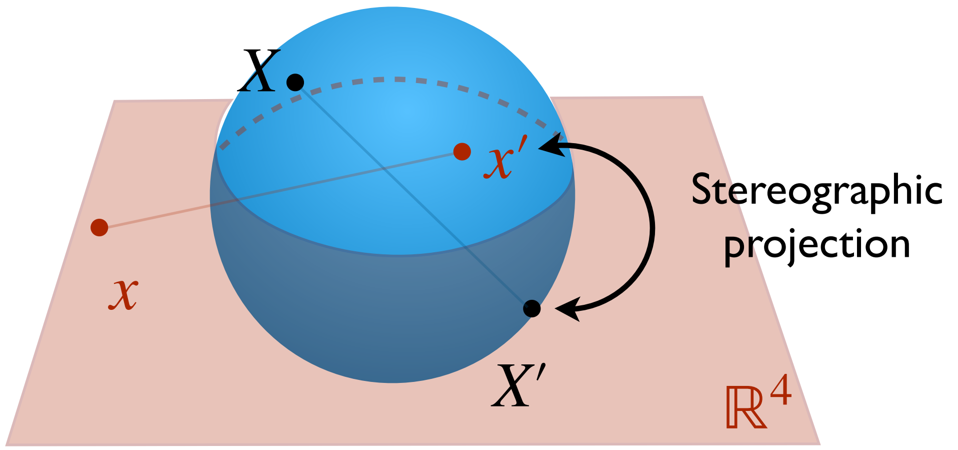


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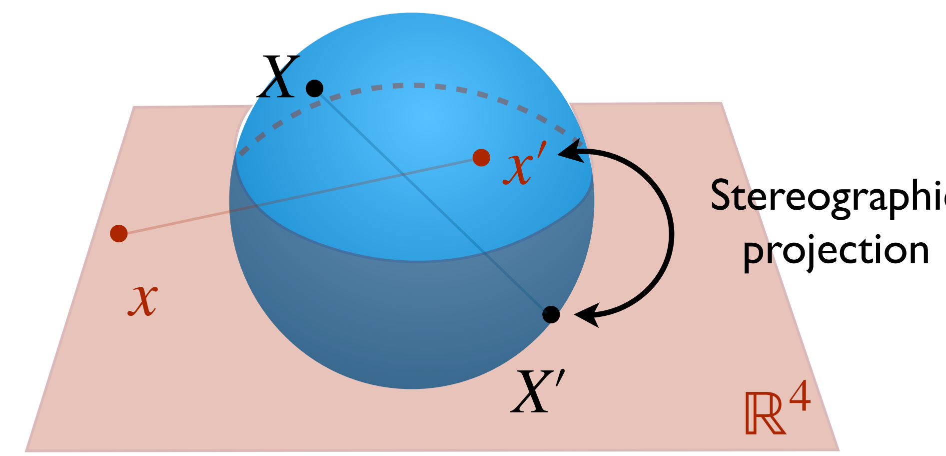


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Inversion tensor

$$\mathscr{R} = -\,\Gamma_{790}$$

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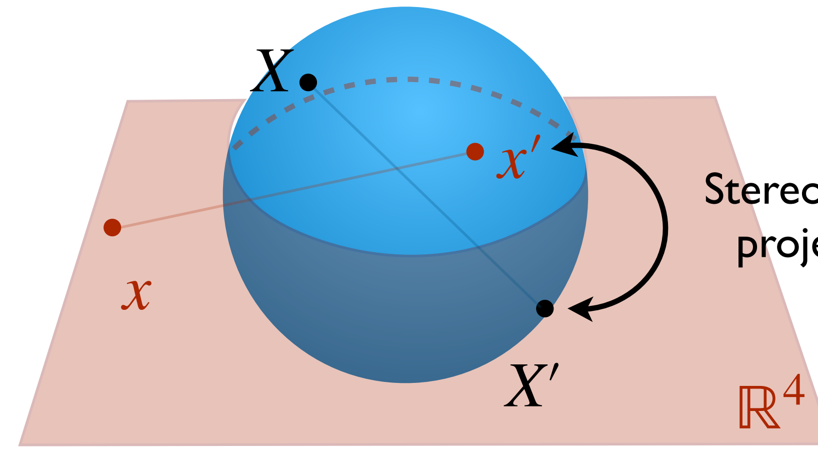


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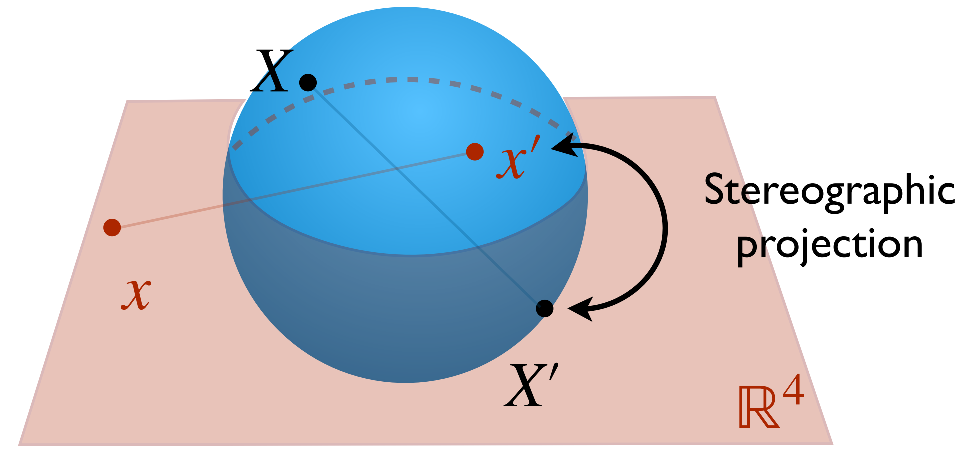


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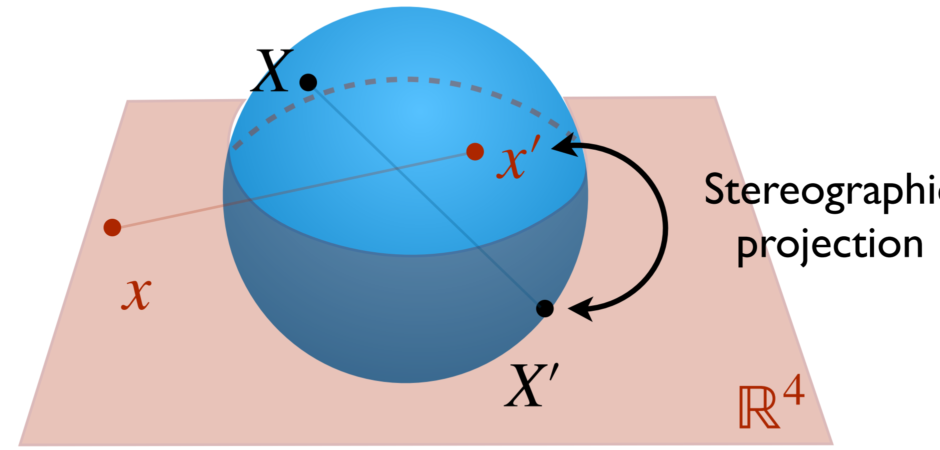


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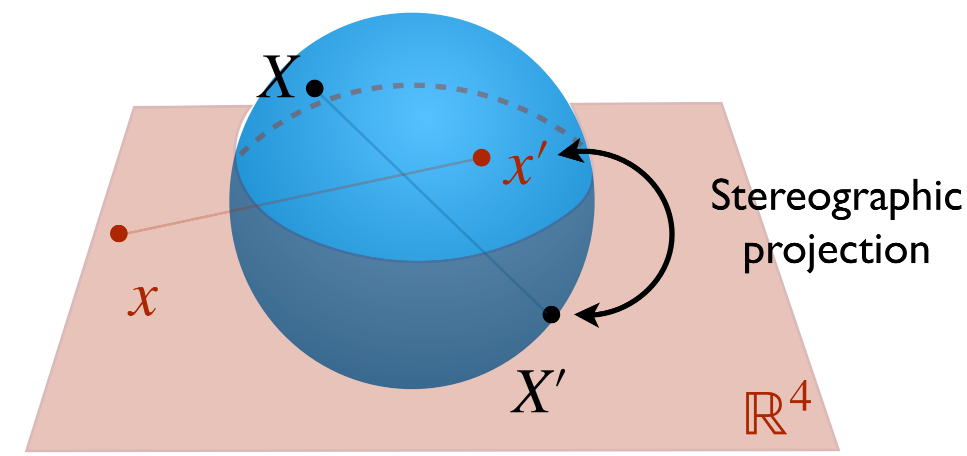


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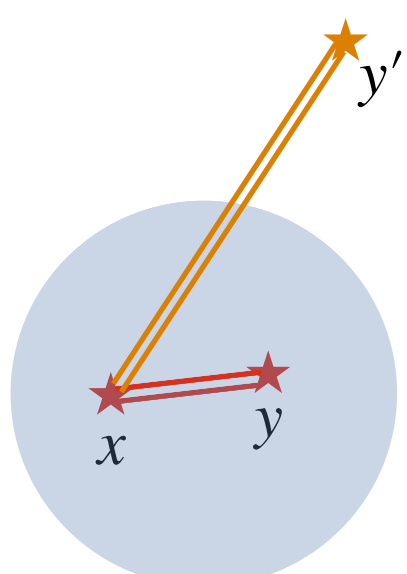

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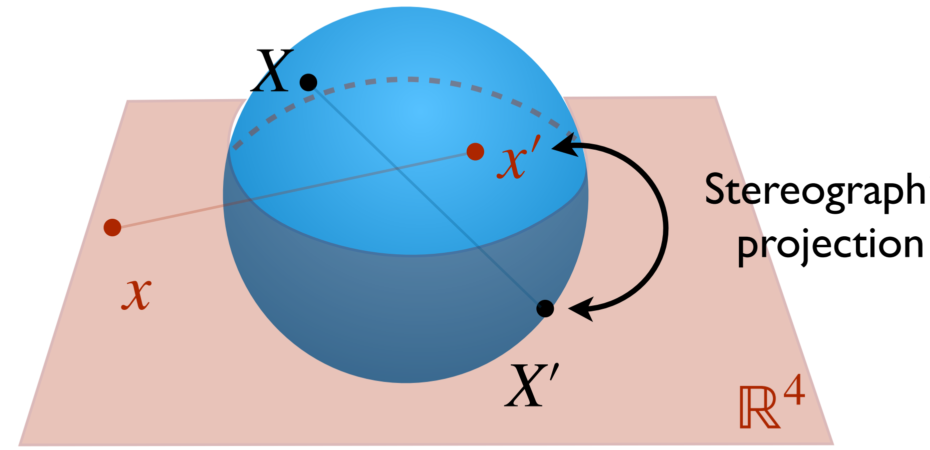


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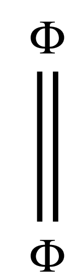


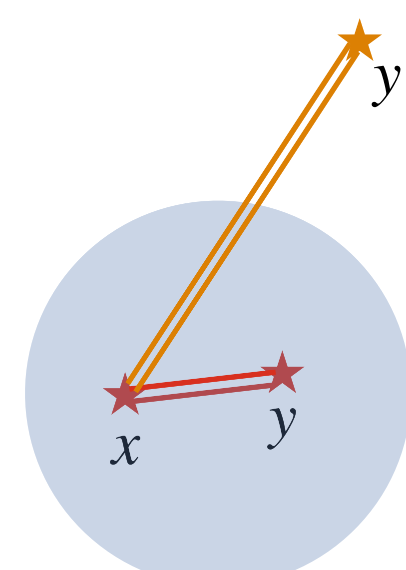

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Conformal symmetry breaking $\langle \mathcal{O} \rangle \neq 0$

Lorentz scalar and SO(3)xSO(3) singlet

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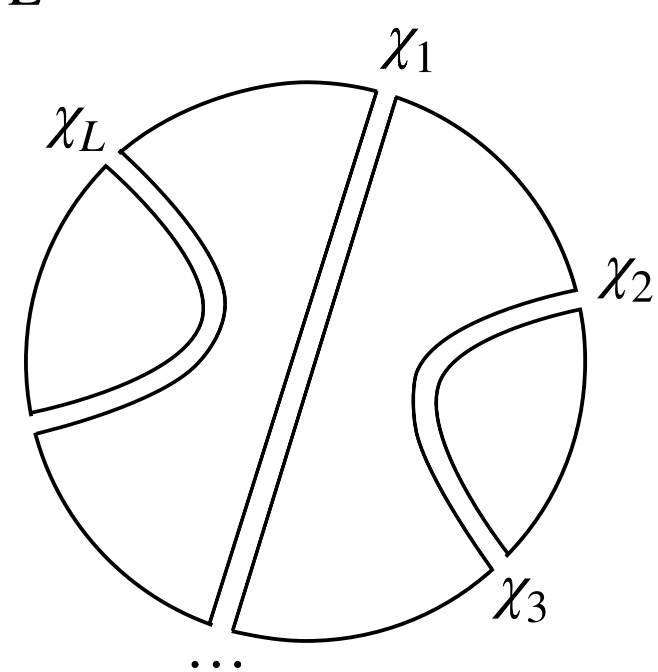
Take a single trace $\mathcal{O} \sim \mathrm{Tr}[\chi_1 \dots \chi_L]$

$$\langle \mathcal{O} \rangle \sim \frac{1}{N^{L/2}}$$




$$\sim N$$

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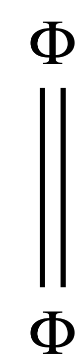
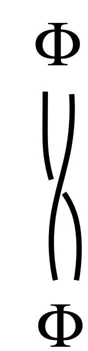
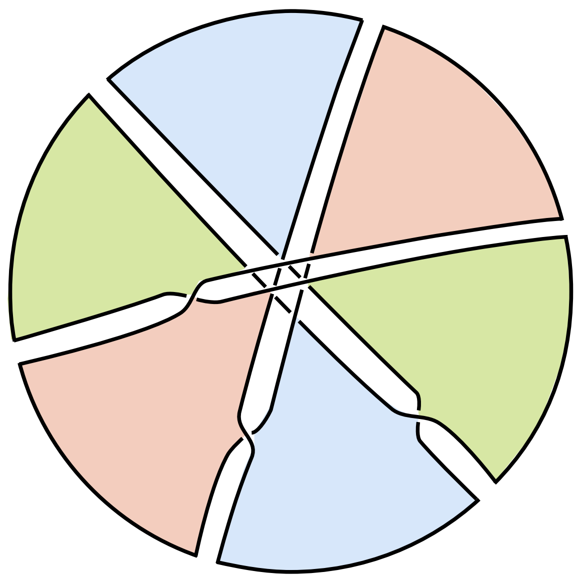


# With charge conjugation

$$\langle \mathcal{O} \rangle \sim \frac{1}{N^{L/2}}$$



$$\sim N^0$$

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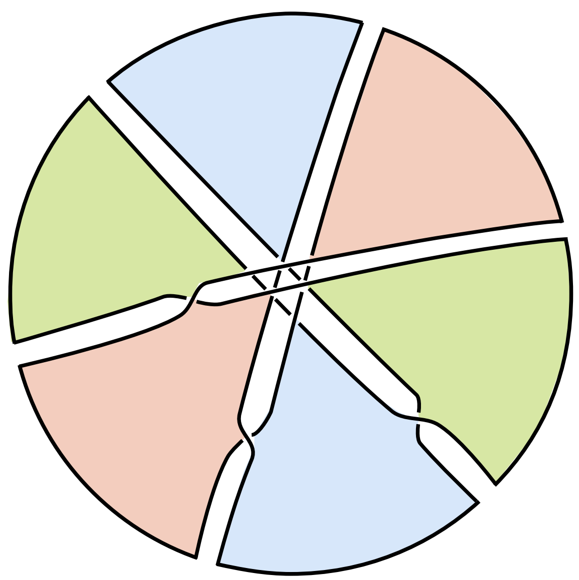

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Identification of antipodal points on the spin chain

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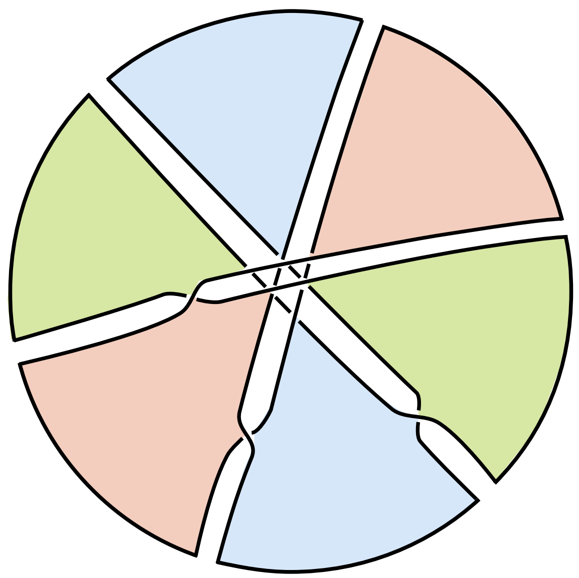

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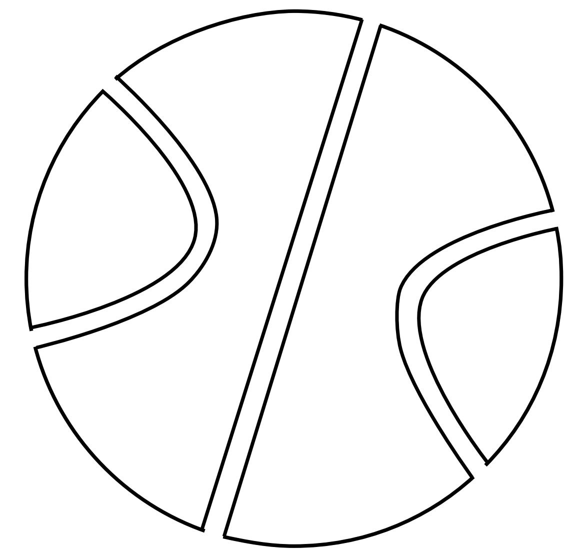

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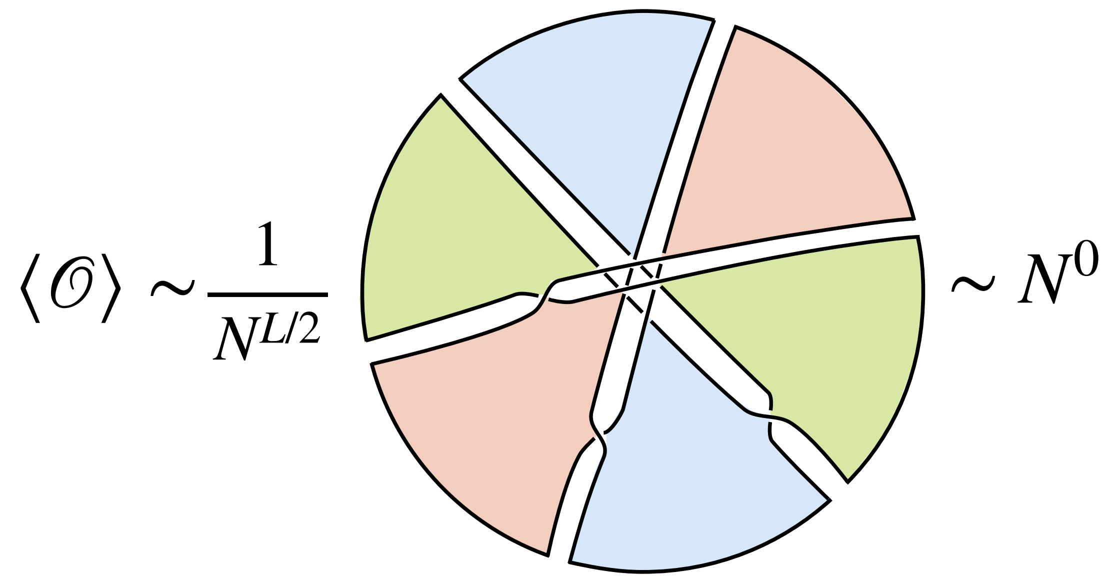
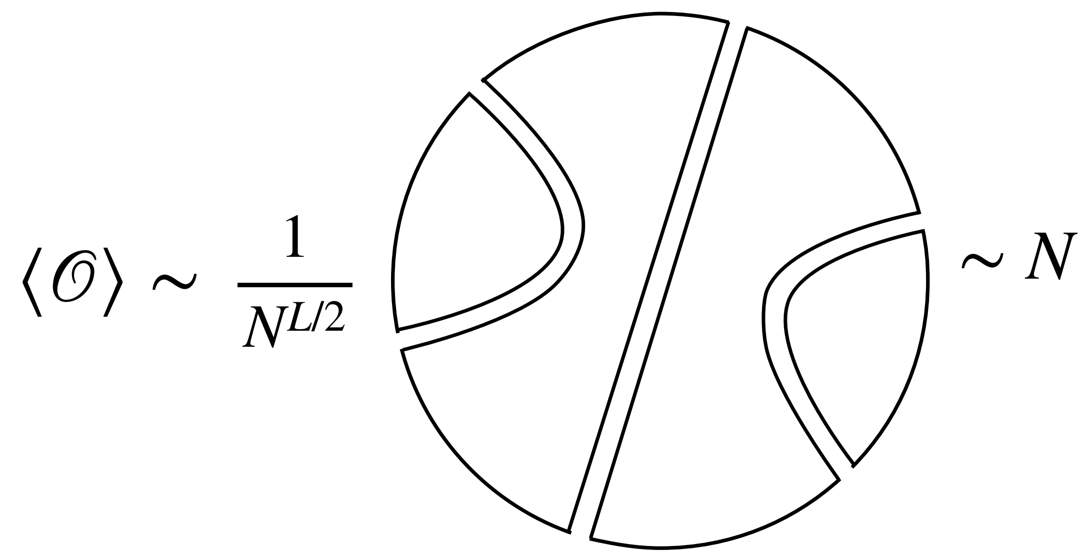



Identification of antipodal points on the spin chain

**Background unchanged to leading order** (apart from orientifold projection). **Integrable setup!**

# No charge conjugation




**New classical background! Asymptotic to $AdS_5 \times S^5$**

[**JC**, Rastelli, to appear]

# $\mathcal{N} = 4$ SYM on $\mathbb{RP}^4$

(with charge conjugation)

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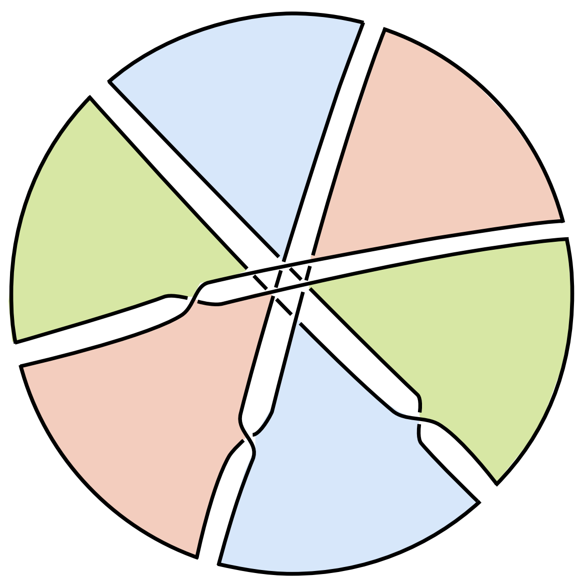

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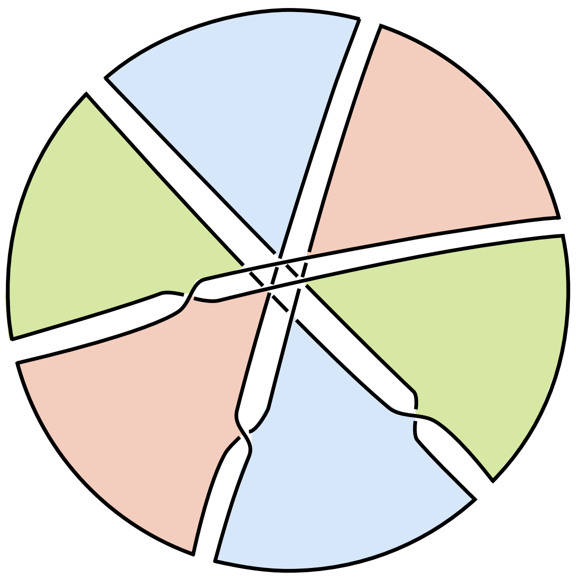
(with charge conjugation)

$\langle \mathscr{O} \rangle \sim$ 

Take a SU(2) spin chain state

$$\mathscr{O}(x) = \sum_{\text{perms}} \psi \, \mathrm{tr}\,(Z\,Z\,X\,Z\,Z\,X)$$

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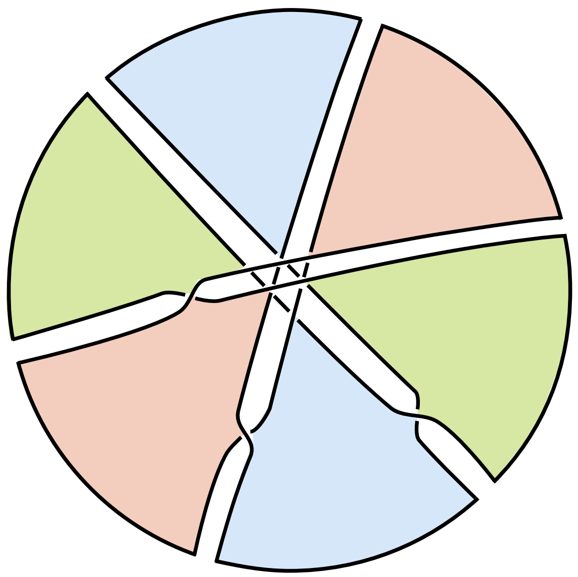
$\langle \mathcal{O} \rangle \sim$ 

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$$\sim \sum_{\text{perms}} \psi \,|\uparrow\uparrow\downarrow\uparrow\uparrow\downarrow\rangle$$

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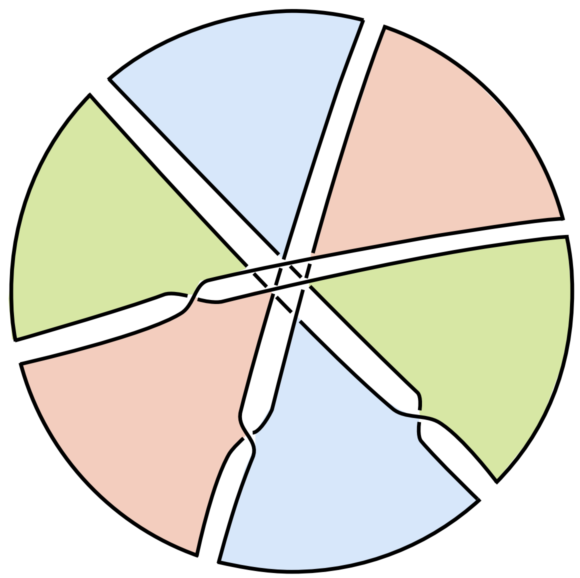

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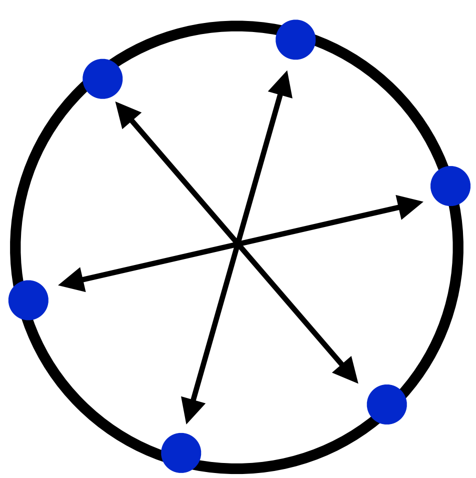

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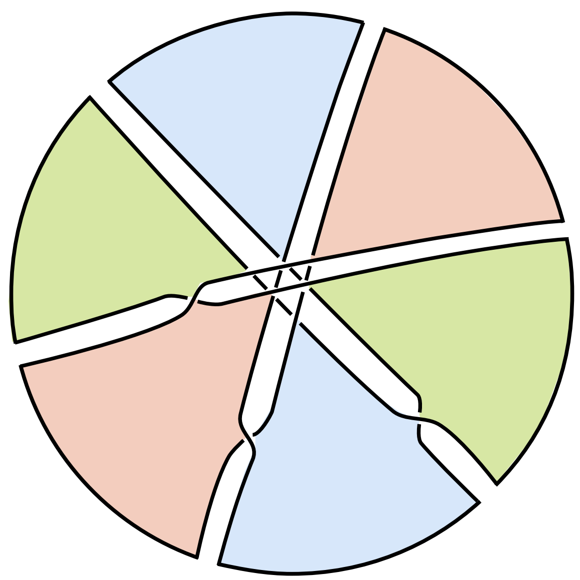
$\langle \mathcal{O} \rangle \sim$ 

Take a SU(2) spin chain state

$$\mathcal{O}(x) = \sum_{\text{perms}} \psi \, \mathrm{tr}\,(\color{red}{Z}\,\color{red}{Z}\,\color{blue}{X}\,\color{red}{Z}\,\color{red}{Z}\,\color{blue}{X})$$

$$\sim \sum_{\text{perms}} \psi \, |\, \color{red}{\uparrow\,\uparrow}\,\color{blue}{\downarrow}\,\color{red}{\uparrow\,\uparrow}\,\color{blue}{\downarrow}\, \rangle$$

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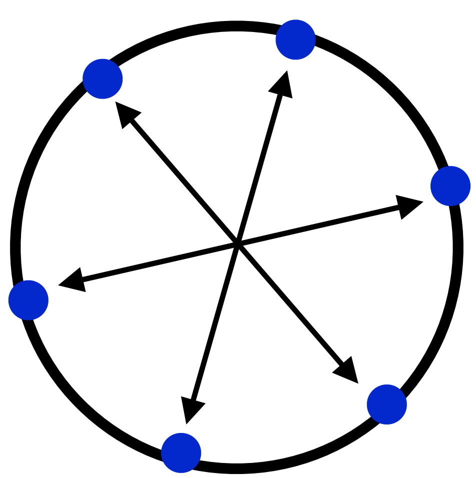

$$|c\rangle\rangle_j \equiv |\uparrow\,\rangle_j \otimes |\uparrow\,\rangle_{j+\frac{L}{2}} + |\downarrow\,\rangle_j \otimes |\downarrow\,\rangle_{j+\frac{L}{2}}$$

$$|\mathscr{C}\rangle \equiv \prod_{j=1}^{\frac{L}{2}} \Big( |c\rangle\rangle_j \Big)^{\otimes}$$

**Crosscap state**

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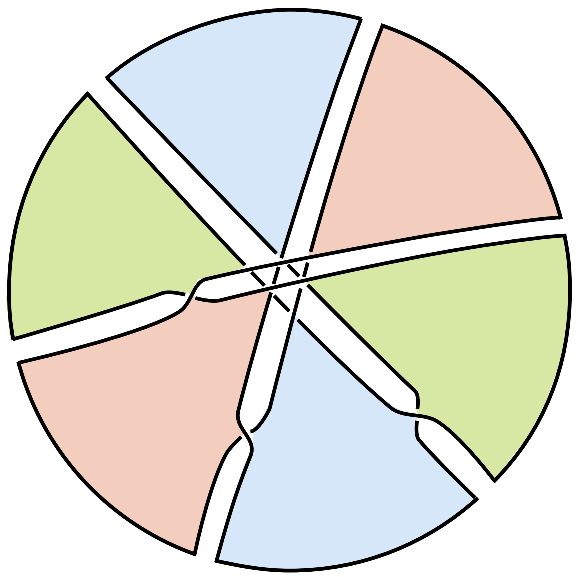
$\langle \mathcal{O} \rangle \sim$ 

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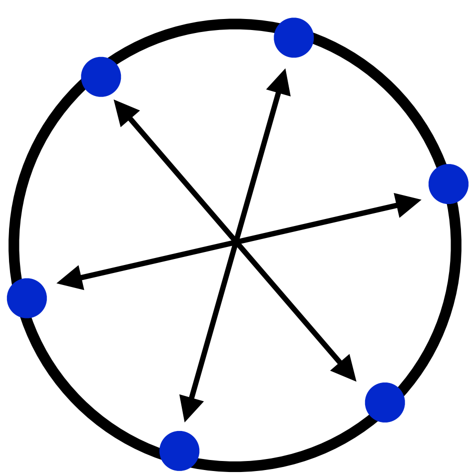

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**Crosscap state**

$$\langle \mathcal{O} \rangle \Leftrightarrow \langle \mathscr{C} | \mathcal{O} \rangle$$

# Crosscap states in Integrable Theories (2d)

[**JC**, Komatsu'21]

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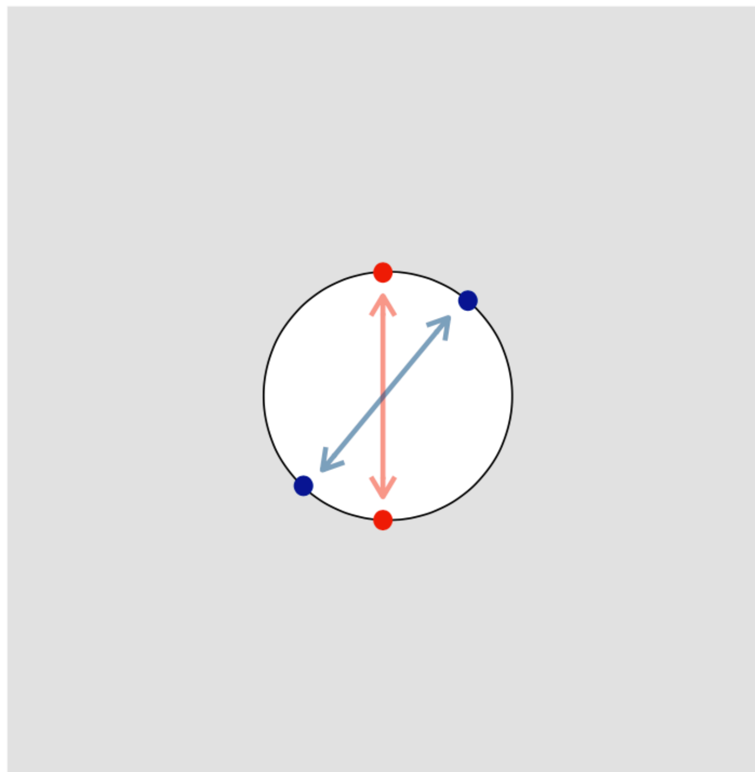
$$z \sim -1/\bar{z}$$



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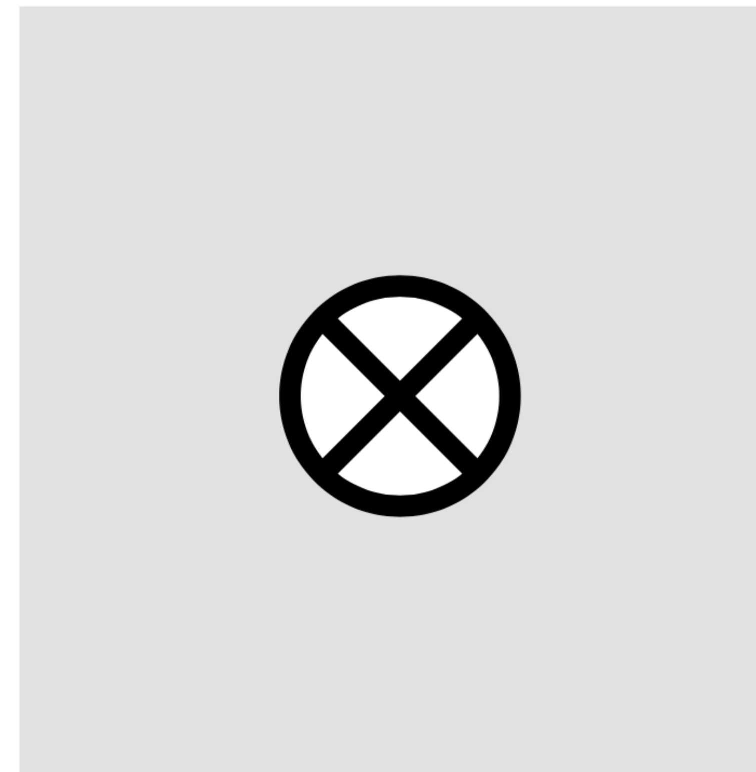

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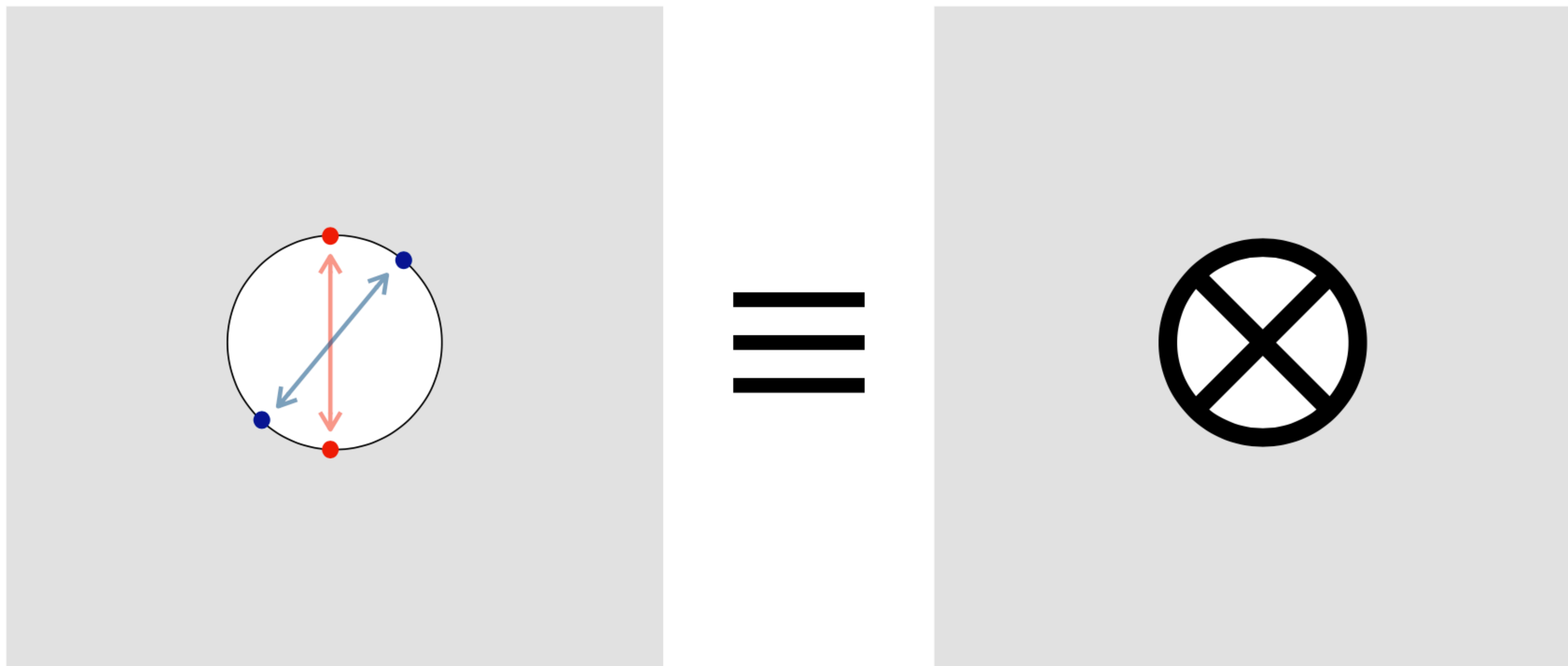

Cut out a disk from a 2d surface + identify points at the boundary of the disk

The state created by this procedure is the **crosscap state**

# Exact crosscap overlaps in Integrable Theories

[**JC**, Komatsu'21]

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$$|\langle \mathscr{C} | \Psi \rangle| = \sqrt{\left(1 + \sqrt{\frac{Y(0)}{1+Y(0)}}\right) \frac{\det\left[1 - \hat{G}^{\bullet}_{-}\right]}{\det\left[1 - \hat{G}^{\bullet}_{+}\right]}}$$

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**Y-function** $$0 = LE(u) + \log Y(u) - \log(1+Y) \star \mathscr{K}_{+}(u)$$

# Exact crosscap overlaps in Integrable Theories

[**JC**, Komatsu'21]

- A crosscap preserves integrability of the bulk theory.
- Simplest integrable model: **single type** of particle (massive) (e.g sinh-Gordon model)

$$|\langle \mathscr{C} | \Psi \rangle| = \sqrt{\left(1+\sqrt{\frac{Y(0)}{1+Y(0)}}\right)\frac{\det\left[1-\hat{G}^{\bullet}_{-}\right]}{\det\left[1-\hat{G}^{\bullet}_{+}\right]}}$$

**Y-function** $0 = LE(u) + \log Y(u) - \log(1+Y) \star \mathscr{K}_{+}(u)$

Dispersion relation

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S-matrix

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$$\hat{G}^{\bullet}_{\pm} \cdot f(u) = \sum_k \frac{i\mathscr{K}_{\pm}(u,u_k)}{\partial_u \log Y(\tilde{u}_k)} f(\tilde{u}_k) + \int_0^{\infty} \frac{dv}{2\pi} \frac{\mathscr{K}_{\pm}(u,v)}{1+1/Y(v)} f(v)$$

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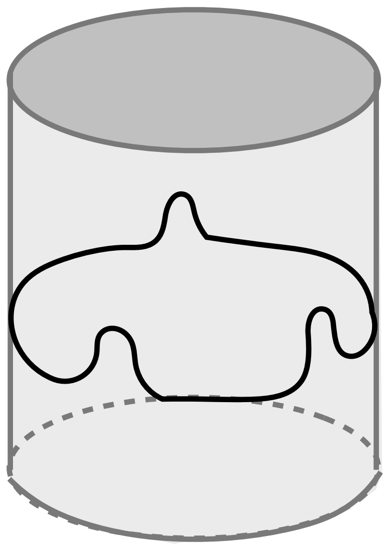
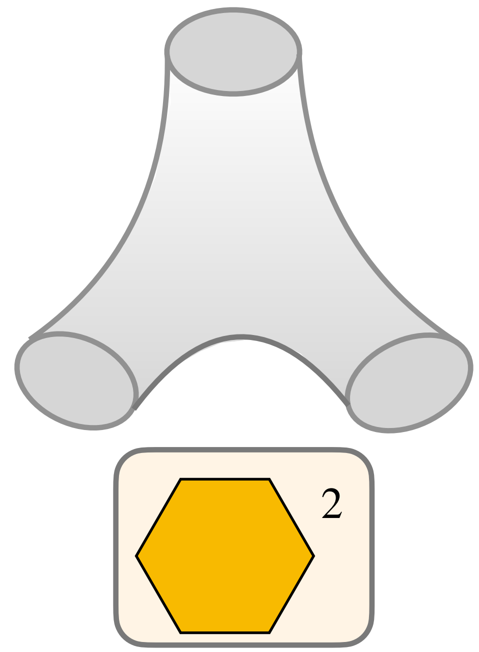
- In $\mathcal{N} = 4$ SYM, the model is "nested", i.e. several levels ⇔ several Y-functions
- The corresponding formula is a generalization of this one.
- We therefore compute the exact one-point functions $\langle \mathcal{O} \rangle$!

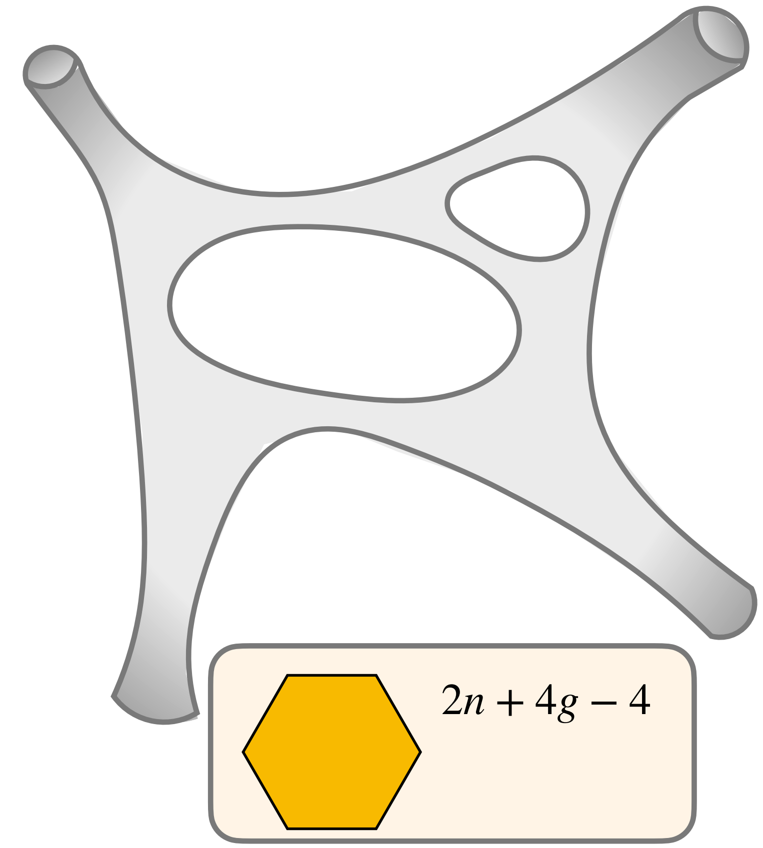
[**JC**, Komatsu, Rastelli, in progress]

# Unified picture emerging



Quantum
Spectral Curve



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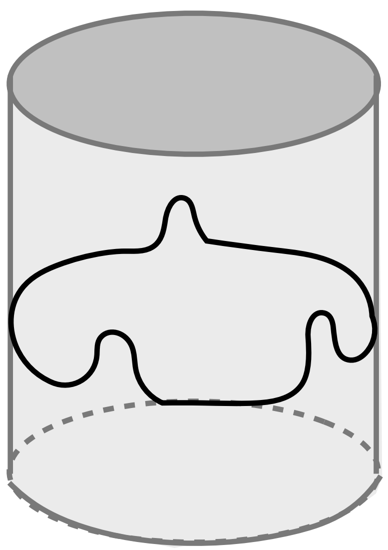

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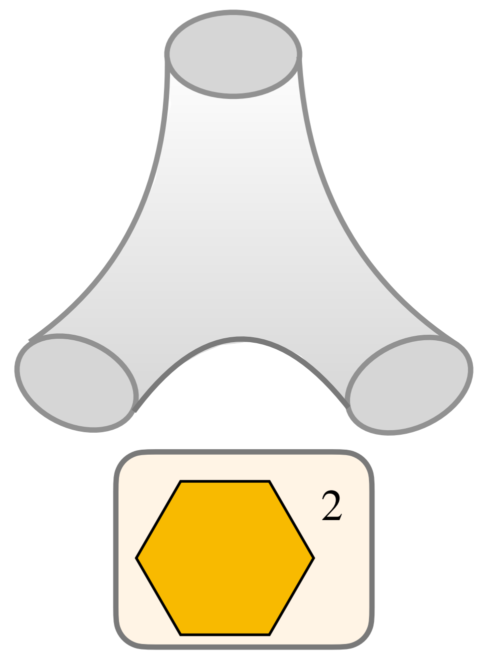


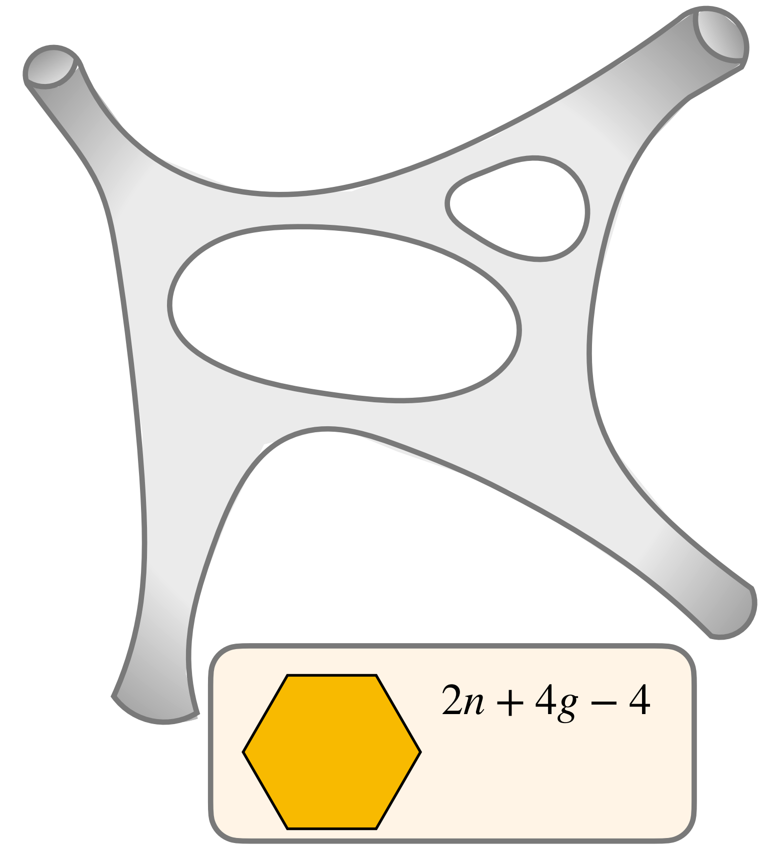


- Is there a quantum spectral curve for higher point correlation functions?
- Explore holography: bulk locality, Regge limit, compare with results from conformal bootstrap etc.
- Finite N physics (resummation in N)?

# Conclusions

- $\mathcal{N} = 4$ SYM on $\mathbb{RP}^4$ provides new rich setups of AdS/CFT
- Without charge conjugation: new supergravity background.
- Matrix model from supersymmetric localization
- With charge conjugation: orientifold in AdS and integrable setup
- New matrix model from supersymmetric localization
- Studied crosscap states in integrable theories
- Lessons for orientifolds in curved backgrounds? Non-perturbative definition?

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Thank you!

# Backup slides

# Holographic Dual of $\mathcal{N} = 4$ SYM on $\mathbb{RP}^4$ (without charge conjugation)

- New (euclidean) 1/2-BPS solution of 10D IIB supergravity (asymptotically AdS)

$$ds^2_{\text{10D}} = \Delta^{1/4}\left(\underbrace{ds^2_{\text{5D}}}_{dr^2 + e^{2A}ds^2_{\mathbb{RP}^4}} + \frac{4}{g^2}\left(d\theta^2 + \frac{\cos^2\theta}{1+\mathscr{K}_+\cos^2\theta}\underbrace{d\Omega^2_{S^2}}_{SO(3)} + \frac{\sin^2\theta}{1+\mathscr{K}_-\sin^2\theta}\underbrace{d\Omega^2_{dS_2}}_{SO(2,1)}\right)\right)$$

Explicit functions of $r$ and $\mathcal{J}$

$$e^{2A} = \frac{\mathcal{J}^3}{4}\sinh 2r + \frac{1}{4}(2 - \mathcal{J}^3)\cosh 2r - \frac{1}{2}$$

Parametric family of backgrounds. Needs to be fixed by comparison to the gauge theory.

$\mathcal{J} \to 0$ Standard (euclidean) $AdS_5 \times S^5$

- Solution contains explicit expressions for non-trivial dilaton, $B_2$, $C_2$ and $C_4$