Quantum dynamics with attractive densities

A unified quantum dynamics including the Schrödinger evolution and the von Neumann spontaneous collapse

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INTRODUCTION

Macroscopic uniqueness is not contained in the Schrödinger evolution

Two possible attitudes: interpretation vs dynamics

QUANTUM DYNAMICS WITH ATTRACTIVE DENSITIES

1. dBB theory, brief reminder: empty waves, coordination between Bohmian trajectories

2. GRW (Ghirardi-Rimini-Weber) and CSL (Continuous Spontaneous Localization) theories: a unified dynamics

3. Combining the two theories: quantum dynamics with attractive densities

4. Consequences of the dynamics: microscopic systems, macroscopic systems, measurements, decoherence

The usual quantum dynamics (Schrödinger equation, von Neumann, path integrals, etc.) does not predict macroscopic uniqueness: macroscopic objects can be at the same time in different positions of space.

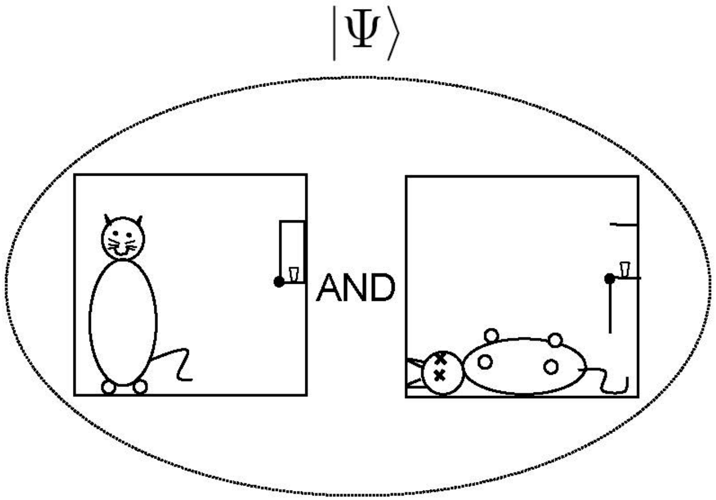
QSMDS : Quantum Superpositions of Macroscopically Distinct States (Leggett)

In particular, the Schrödinger equation does not predict that experiments provide well-defined results: in general, all possible results are obtained at the same time.

Bell: there is a difference between « and » and « or ».

« It is a pity that the world is given to us only once.. »

The famous parable of the Schrödinger cat



Schrödinger calls this a « ridiculous case ». The question is: to what extent should be take the Schrödinger equation seriously?

Can we explain by which mechanism potential properties of quantum systems become actual?

Three possible attitudes:

1. The difficulty is ignored in the dynamics (the equations), but considered as a problem of interpretation.

Many interpretations have been proposed : Copenhagen, textbooks (von Neumann reduction), information, statistical interpretation, modal, etc.

2. The problem does not exist: macroscopic uniqueness is just a mental delusion, a consequence of the way our « memory register » store the observations (Everett, multiverse, etc.).

3. The problem is considered as sufficiently serious to justify a change of the dynamical equations or of the formalism.

Interpretation or equations? A matter of personal preference.

1. DE BROGLIE-BOHM (dBB) INTERPRETATION

The standard wave function is:

$$\Psi\left(\mathbf{r}_1, \mathbf{r}_2, ..., \mathbf{r}_N\right) = R\left(\mathbf{r}_1, \mathbf{r}_2, ..., \mathbf{r}_N\right) \; e^{i\xi\left(\mathbf{r}_1, \mathbf{r}_2, ..., \mathbf{r}_N\right)}$$

It evolves according to the standard Schrödinger equation.

Additional variables are introduced, the dBB positions $\mathbf{q}_i\left(t\right)$

The positions are « guided » by the wave function according to:

$$\frac{d}{dt}\mathbf{q}_i\left(t\right) = \frac{\hbar}{m} \overrightarrow{\nabla}_{\mathbf{r}_i}\xi\left(\mathbf{q}_1, \mathbf{q}_2, ., \mathbf{r}_i, ..., \mathbf{q}_N\right)\Big|_{\mathbf{r}_i=\mathbf{q}_i}$$

The positions are considered as real physical quantities, which evolve in ordinary 3D space; but they are driven by a wave that propagates in configuration space (hence non-local effects in ordinary space).

DBB INTERPRETATION (2)

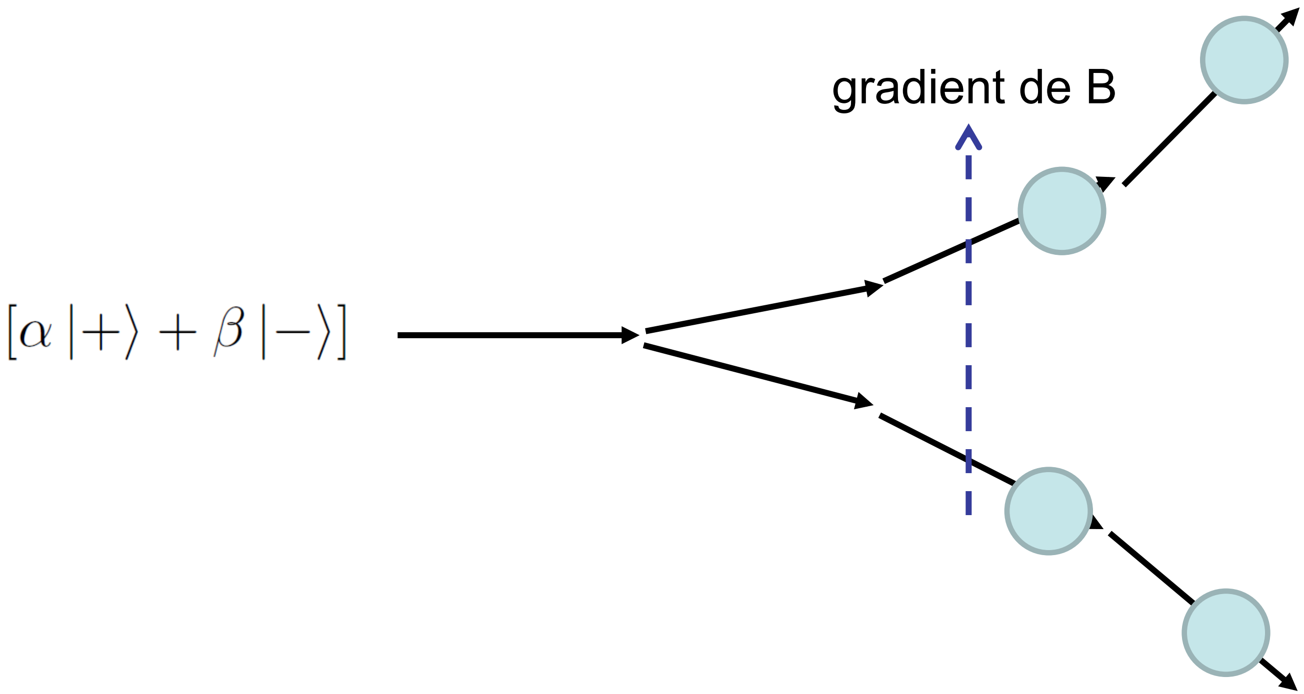
The initial distribution at t=0 of the dBB positions is random; in configuration space, it is defined by a probability distribution standard wave function is:

$$D_{dBB} = \left| R\left(\mathbf{r}_1, \mathbf{r}_2, ..., \mathbf{r}_N\right) \right|^2$$

This distribution is equal to the quantum distribution at t=0. One can then show that the two distributions remain equal at all times.

The dBB time evolution is completely deterministic (but starts with completely random positions). The dBB positions can be considered as directly representing physical reality.

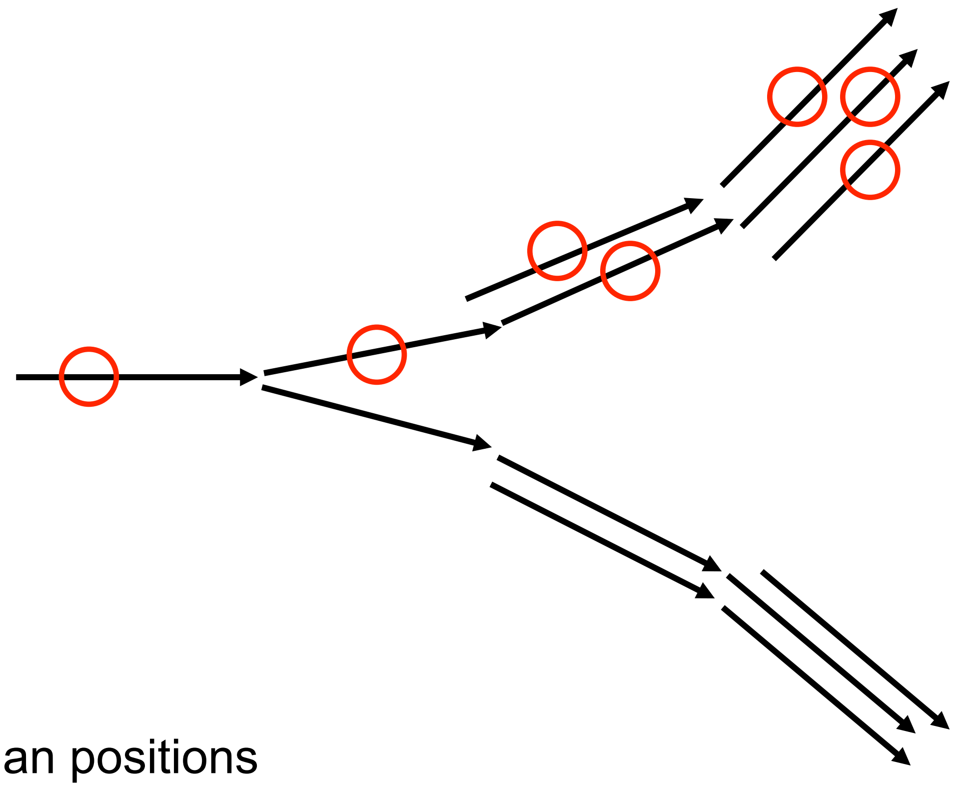
A Stern-Gerlach experiment in dBB theory



$$|\Phi_0\rangle = [\alpha\,|+\rangle + \beta\,|-\rangle]\;\; |\varphi_0\rangle\,|\chi_0\rangle\;\; |\zeta_0\rangle\,|\theta_0\rangle\,\ldots$$

$$\begin{aligned}|\Phi\rangle = {} & \alpha\,|+\rangle\,|\varphi_+\rangle\,|\chi_0\rangle\,|\zeta_+\rangle\,|\theta_0\rangle\,\ldots \\ & + \beta\,|-\rangle\,|\varphi_0\rangle\,|\chi_-\rangle\,|\zeta_0\rangle\,|\theta_-\rangle\,\ldots\end{aligned}$$

The Bohmian positions remain grouped together



 : Bohmian positions

For a single realization of the experiment, the Bohmian positions of the « pointer » remain bunched together. They cannot spread in different positions in space because of the cohesion forces inside the pointer.

A strong mismatch between the spatial density associated with the state vector:

$$D_\Phi(\mathbf{r}) = \frac{\langle \Phi | \, \Psi^\dagger(\mathbf{r}) \, \Psi(\mathbf{r}) \, | \Phi \rangle}{\langle \Phi \, | \Phi \rangle}$$

is a signature of Schrödinger's « ridiculous cases », or Leggett's QSMDS.

2. MODIFIED SCHRÖDINGER DYNAMICS

GRW (Ghirardi, Rimini, Weber)

The wave function is subject to randoms « hits » at all points of space:

$$\left|\Psi'(t)\right\rangle = \frac{F_j\,|\Psi(t)\rangle}{\langle\Psi(t)|\left(F_j\right)^2|\Psi(t)\rangle} \qquad\qquad F_j = c\; e^{-\alpha\left(\mathbf{R}-\mathbf{r}_j\right)^2/2}$$

The probability P per unit time of a hit obeys the « probability rule »:

$$P = \lambda\,\langle\Psi(t)|\left(F_j\right)^2|\Psi(t)\rangle$$

The Schrödinger wave propagating in configuration space is considered as a field that is physically real. The theory depends on two parameters, a rate λ and a localization length $\alpha^{-1/2}$

MODIFIED SCHRÖDINGER DYNAMICS (2)

CSL (Continuous Spontaneous Localization); P. Pearle

The evolution of the wave function is continuous, but contains random additional terms (Wiener processes) :

$$|\psi(t)\rangle_w = \mathcal{T} e^{-i\int_0^t dt' H(t') - \frac{1}{4\lambda}\int_0^t dt' \int d\mathbf{x}' [w(\mathbf{x}',t') - 2\lambda G(\mathbf{x}')]^2} |\psi(0)\rangle$$

$$G(\mathbf{x}) \equiv \sum_n \frac{m_n}{M} \frac{1}{(\pi a^2)^{3/4}} \int d\mathbf{z} e^{-\frac{1}{2a^2}[\mathbf{x}-\mathbf{z}]^2} \xi_n^\dagger(\mathbf{z}) \xi_n(\mathbf{z})$$

A probability rule is also postulated.

These theories are not strictly equivalent to standard quantum mechanics. New effects are predicted. For instance, the evolution of mesoscopic systems should be different. Experiments are possible to test these theories (Markus Arndt in Vienna; European MAQRO project).

3. Combining the two theories: quantum dynamics with attractive densities

Bohmian density:

$$D_B(\mathbf{r},t) = \sum_{n=1}^{N} \delta(\mathbf{r} - \mathbf{q}_n)$$

Averaged Bohmian density:

$$N_B(\mathbf{r},t) = \int d^3r' \; e^{-(\mathbf{r}-\mathbf{r}')^2/(a_L)^2} D_B(\mathbf{r}',t)$$

Localization operator:

$$L(t) = \int d^3r \; N_B(\mathbf{r},t') \;\; \Psi^\dagger(\mathbf{r})\,\Psi(\mathbf{r})$$

$$\boxed{i\hbar\frac{d}{dt}|\Phi(t)\rangle = [H + i\hbar\gamma_L \; L(t)]\,|\Phi(t)\rangle}$$

Norm-conserving version of the dynamics:

$$i\hbar\frac{d}{dt}\left|\overline{\Phi}\left(t\right)\right\rangle = \left[H + \overline{H}_L\right]\left|\overline{\Phi}\left(t\right)\right\rangle$$

$$\overline{H}_L = i\hbar\gamma_L \int d^3r\ \left[\Psi^\dagger\left(\mathbf{r}\right)\Psi\left(\mathbf{r}\right) - D_\Phi\left(\mathbf{r}\right)\right] N_B\left(\mathbf{r},t\right)$$

Microscopic systems, macroscopic systems, measurements, decoherence

A choice of the constants is necessary. It results from a compromize:

- on one hand, do not introduce a contradiction with the enormous body of experimental data that agree with the predictions of quantum mechanics, sometimes with a fantastic precision (for microscopic systems).

- on the other hand, ensure that a sufficient fast collapse occurs when needed: quantum superpositions of macroscopically distinct states (cats) must be cancelled rapidly. This is the cat hunting game.

A standard choice in GRW and CSL theories is:

$$\gamma_L = 10^{-16}\ s^{-1}$$

$$a_L \simeq 10^{-6}\ m$$

Microscopic system

Consider an atom, molecule, nucleus, etc. with wave function extending over distance:

$$a_0 \ll a_L$$

In the limit $a_0/a_L \to 0$, $\qquad L(t) \to N\widehat{N}$

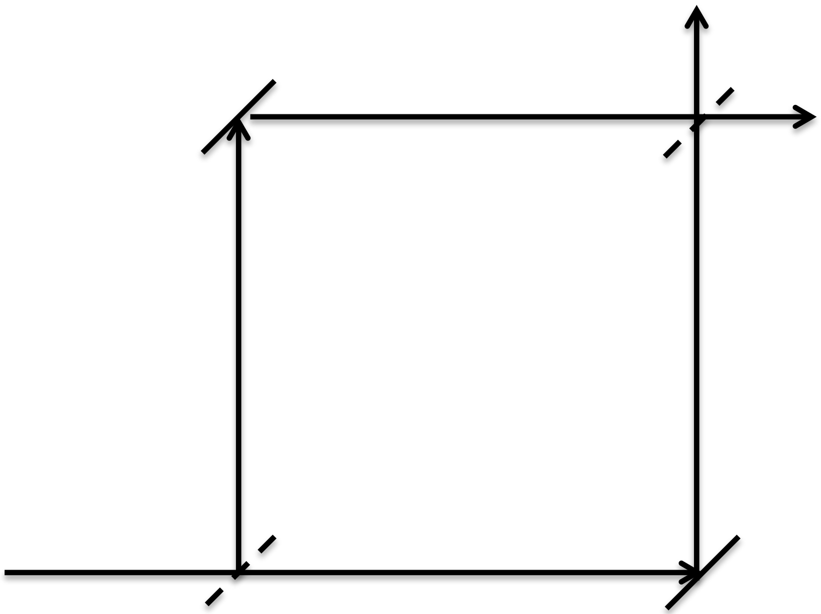
so that the localization term has no effect at all. The effects occur to second order only in a_0/a_L

One then finds a localization rate: $\gamma \lesssim \gamma_L \left(\frac{a_0}{a_L}\right)^2 N^2$

Small atom: $\gamma \leq 10^{-24} N^2$

Molecule: $\gamma \leq 10^{-20} N^2$

Interference experiment



Localization rate: $$\gamma \simeq \gamma_L \, N^2$$

if $N < 10^7$ the localization rate remains negligible, and interference effects still occur

Bose-Einstein condensates

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Interference of an Array of Independent Bose-Einstein Condensates

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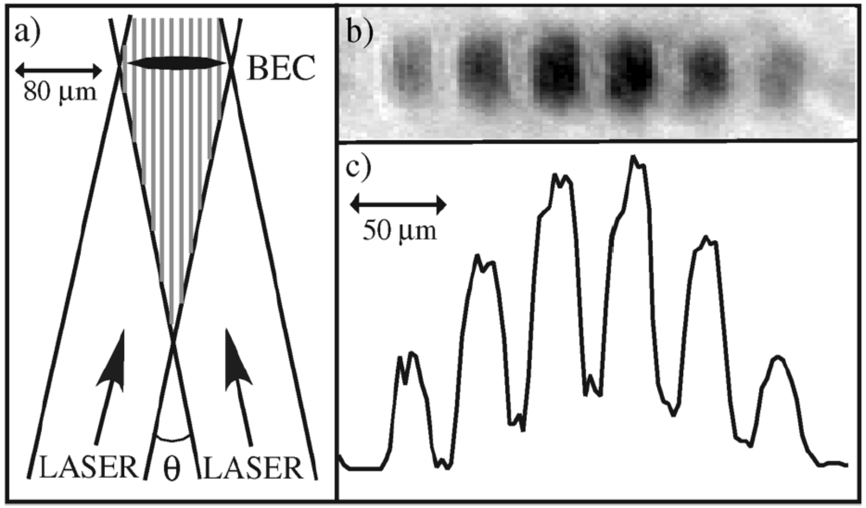

$$\gamma \simeq 10^{-6}$$

Macroscopic system

$$\gamma \simeq \gamma_L \; N_B \; N_P$$

$$N_P = 10^{20}, \, N_B = 10^{11},$$

$\gamma \simeq 10^{15}$! very fast collapse

Now, what is real?

Two points of view are possible:

Pure Bohmian: all the positions are real and directly observable. The proposed dynamics then suppresses the difficulties arising from the empty waves. It provides a more plausible guiding field.

What is directly observable is the field/wave function, in the spirit of CSL theory. Not that the real field propagates in configuration space, nor ordinary space; this is very different from a classical field.

The Bohmian average density $N_B\left(\mathbf{r},t\right)$ then plays the role of a « dark attractor » for the field, with no microscopic effect but a strong macroscopic effect.

Conclusion

A unified dynamics is obtained, which applies in all situations, but requires the introduction of 2 new parameters.

This dynamics provides predictions that are in principle experimentally testable. Here, we do not have a heating effect, as opposed to CSL and GRW. But interferences of large objects shoud disappear at a certain size. European project MAQRO.

The main interest of this dynamics is to provide a proof of existence, and to refute pseudo impossibilities: it shows that, if one wants, it is perfectly possible to account for all experimental data with a simple unified dynamics, and without the need of an elaborate interpretation.

But, depending on one's personal taste, one may prefer:
- to keep the linear form of the Schrödinger equation; the price to pay is the need for a subtle interpretation
- or accept to modify the basic equation, in order to allow for a much simpler interpretation.