Contribution ID: **22** Type: **not specified**

Pierangelo Marcati: "Existence and Stability of almost finite energy weak solutions to the Quantum Euler Maxwell system"

Friday, January 20, 2023 9:00 AM (55 minutes)

We prove the existence of global in time, finite energy, weak solutions to a quantum magneto hydrodynamic system [7] (QMHD) with large data, modeling a charged quantum fluid interacting with a self generated electromagnetic field. The analysis of QMHD relies upon the use of Madelung transformations. The rigorous derivation requires nontrivial smoothing estimates, which are obtained by assuming slightly higher regularity for the electromagnetic potential. These assumptions are motivated by the nonlinear dependence of the hydrodynamic system in terms of the underlying wave function dynamics, which is supercritical with respect to the bare energy bounds. [2,3,4]
Due to quantum effects on the dispersive properties of QMHD, our approach requires neither smallness nor high regularity, unlike a large amount of existing literature for Euler Maxwell's classical system [9,8]. In fact, the difficulty posed by the presence of the nonlinear electromagnetic force field (Lorentz) severely restricts the possibility to get existence and stability results in the general framework of finite energy solutions. In the classical case the dispersion is not able to deal with the transport of a nontrivial vorticity, therefore almost GWP holds in a life span, reciprocal of the amplitude of the vorticity. GWP can be proved in the irrotational case, where in any case smallness and high regularity assumptions are needed.
For a quantum MHD system the irrotationality and the presence of a highly nonlinear quantum stress tensor induce much stronger dispersive properties, as a byproduct of a close relationship with the classical Maxwell Schrödinger system. Therefore the core argument is shifted to the analysis of the nonlinearities related to the formulation of the hydrodynamic variables through the Madelung transformations. The analysis carried out shows that it is necessary to go through nontrivial smoothing estimates and these require us to assume regularity conditions, just above the energy norms, for the initial data of the Maxwellian electromagnetic potential. In the same regime of regularity, with the help of suitable local smoothing estimates, we also prove stability of both the hydrodynamic variables and the Lorentz force associated with the electromagnetic field. These results can be found in [6]

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